

Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering

<http://pig.sagepub.com/>

Failure analysis of a frangible laminated composite canister cover

J H Wu, W T Wang and T Y Kam

Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 1999 213: 187

DOI: 10.1243/0954410991532945

The online version of this article can be found at:

<http://pig.sagepub.com/content/213/3/187>

Published by:



<http://www.sagepublications.com>

On behalf of:



[Institution of Mechanical Engineers](http://www.imeche.org)

Additional services and information for *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering* can be found at:

Email Alerts: <http://pig.sagepub.com/cgi/alerts>

Subscriptions: <http://pig.sagepub.com/subscriptions>

Reprints: <http://www.sagepub.com/journalsReprints.nav>

Permissions: <http://www.sagepub.com/journalsPermissions.nav>

Citations: <http://pig.sagepub.com/content/213/3/187.refs.html>

>> [Version of Record](#) - Mar 1, 1999

[What is This?](#)

Failure analysis of a frangible laminated composite canister cover

J H Wu, W T Wang and T Y Kam*

Department of Mechanical Engineering, National Chiao Tung University, Hsin-Chu, Taiwan

Abstract: The static burst strength of a frangible laminated composite canister cover subjected to uniform pressure is studied via both theoretical and experimental approaches. The frangible canister cover, which is fabricated with four plate-like laminated composite parts, is designed to fail in a predetermined pattern. The stress distribution in the canister cover is determined using the finite element method and the failure of the cover is identified on the basis of a failure criterion. A number of laminated composite canister covers were fabricated and subjected to static burst pressure testing. The failure modes of the frangible covers are studied and the experimental results are used to verify the theoretical predictions. Close agreements between the experimental and theoretical results have been observed. The canister covers that had been tested did fail in accordance with the predetermined pattern.

Keywords: laminated composite materials, finite element method, failure analysis, frangible cover, structure, failure criterion

NOTATION

a_{ij}	components of laminate compliance matrix
A_{ij}	components of laminate stiffness matrix
E_1	lamina Young's modulus in fibre direction
\bar{E}_1	effective lamina Young's modulus in fibre direction
F_{ij}, F_i	strength parameters
G_{12}	lamina shear modulus
\bar{G}_{12}	effective lamina shear modulus
h	laminate thickness
Q_{ij}	lamina stiffness constants
S	shear strength
S'	adhesive shear strength
X_C	lamina compressive strength
X_T	lamina tensile strength
X'_T	adhesive tensile strength
ε_i	strain components
$\bar{\varepsilon}_i$	average strain components
ν_{12}	Poisson's ratio
σ_i	stress components
$\bar{\sigma}_i$	average stress components
σ_{\max}	maximum principal stress
τ_{\max}	maximum shear stress

1 INTRODUCTION

A missile is usually stored in a launch tube or canister filled with inert gas. Each end of the canister is sealed with a cover which can prevent the inert gas from leaking out and protect the encanistered missile from attack by foreign objects. In the past, different types of canister covers have been designed and fabricated [1–6]. For instance, canister covers through which missiles exit from missile launchers have used rigid doors or covers that are ruptured by explosive means prior to missile launch. A break-apart diaphragm ruptured by the missile as it exits the canister has also been fabricated. These canister covers, though popular, may have different disadvantages, e.g. complexity of the covers, difficulty in manufacturing, costliness, weakness to differential pressures encountered, etc. It is noted that these disadvantages can directly affect the reliability of the covers. Therefore, the design of highly reliable yet economical canister covers is still a goal to be strived for.

Due to the fact that they possess many qualities such as a high strength–weight ratio, good corrosion resistance and a long fatigue life, laminated composite materials have been used extensively in the industry. In the past, laminated composite materials have been mainly applied to the fabrication of aerospace and aircraft structures which are generally reliability stringent. Recently, their use has also been extended to the fabrication of canister covers. For instance, Doane [6] has designed a frangible fly-through diaphragm for missile launch canisters using glass/epoxy laminae. The composite canister diaphragm was designed

The MS was received on 6 April 1999 and was accepted after revision for publication on 8 June 1999.

*Corresponding author: Department of Mechanical Engineering, National Chiao Tung University, Hsin-Chu 30050, Taiwan, Republic of China.

to fail in a predetermined pattern when impacted by the missile nose cone during launch. The disadvantage of this type of cover is that it may damage the surface of the missile during the fly-through process. Though canister covers are of great importance to missile systems, due to security reasons papers on the analysis of canister covers are not often found in the literature.

In this paper, a new design of frangible canister cover made of laminated composite materials is presented. The static burst strength of the canister cover is studied via both theoretical and experimental approaches. A finite element model of the frangible cover is established and used for analysing the stress distribution in the cover. The theoretical burst pressure of the canister cover is determined using a phenomenological failure criterion. Static burst strength tests of a number of laminated composite canister covers were performed. Experimental results are used to validate the suitability of the design and verify the accuracy of the theoretical method for burst strength prediction.

2 FRANGIBLE LAMINATED COMPOSITE COVER

The geometry of the frangible laminated composite canister covers under consideration is shown in Fig. 1. The cover is

a convex structure which is fabricated with four plate-like laminated composite parts and a laminated composite frame. All the laminated composite components are adhesively bonded together along their edges to form the cover. The canister cover is designed in such a way that when subjected to an impulsive pressure, the adhesive bonds between the components will be broken and the severed plate-like laminated composite parts blown away from the opening of the frame. A schematic description of the burst-open process is shown in Fig. 2. Based on this idea, other types of frangible covers can be designed. Figure 3 shows another possible design of the frangible cover.

The laminated composite components of the canister cover are fabricated with two types of glass fabric/epoxy plies, namely glass fabric A/epoxy and glass fabric B/epoxy. All the glass fabric/epoxy plies are balanced orthotropic materials for which the in-plane Young's moduli in material (fibre and matrix) directions, i.e. E_1 and E_2 , are the same. The laminated composite components are fabricated in such a way that the stack of glass fabric B/epoxy plies is sandwiched in between those of glass fabric A/epoxy plies. The material properties of different cured glass fabric/epoxy plies are listed in Table 1. Referring to the lamina material directions, the stress-strain relations of a balanced orthotropic lamina can be expressed as [7]

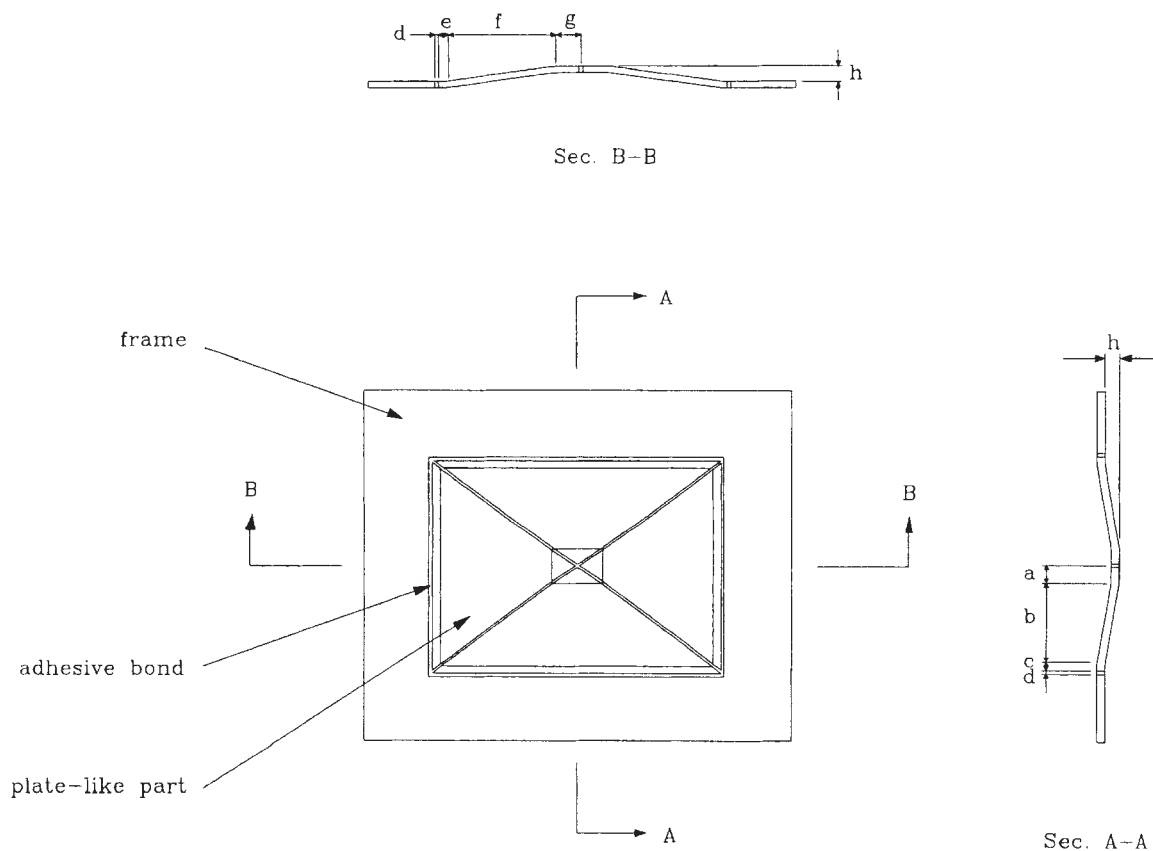


Fig. 1 Geometry of the frangible canister cover CA

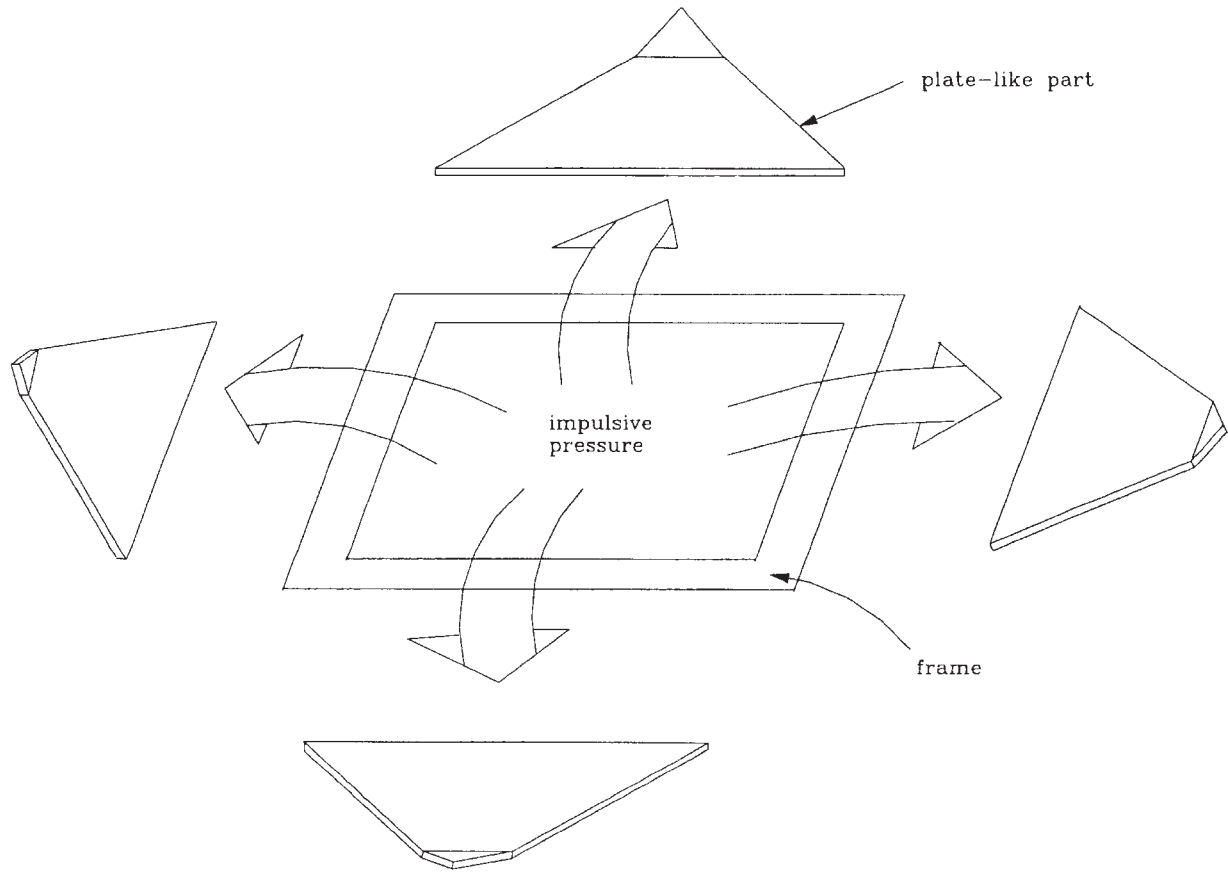


Fig. 2 Schematic description of the burst-open process for cover CA

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{11} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \quad (1)$$

where σ_1, σ_2 are normal stresses, σ_6 is the shear stress, $\varepsilon_1, \varepsilon_2$ are normal strains and ε_6 is the engineering shear strain. The components of the lamina stiffness matrix, Q_{ij} , are obtained as

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}^2} \\ Q_{12} &= \frac{\nu_{12} E_1}{1 - \nu_{12}^2} \\ Q_{66} &= G_{12} \end{aligned} \quad (2)$$

where ν_{12} is Poisson's ratio and G_{12} is the in-plane shear modulus. The in-plane stress-strain relations of a balanced orthotropic laminate are expressed as

$$\begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{Bmatrix} = \frac{1}{h} \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} \quad (3)$$

where $\bar{\sigma}_i$ are average stresses and h is the laminate thickness. The components of the laminate stiffness matrix, A_{ij} , are obtained as

$$A_{ij} = \int_{-h/2}^{h/2} Q_{ij} dz, \quad i, j = 1, 2, 6 \quad (4)$$

By inversion, equation (3) can be rewritten as

$$\begin{Bmatrix} \bar{\varepsilon}_1 \\ \bar{\varepsilon}_2 \\ \bar{\varepsilon}_6 \end{Bmatrix} = h \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{12} & a_{11} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_6 \end{Bmatrix} \quad (5)$$

where a_{ij} are the components of the laminate compliance matrix. Once a_{ij} are known, the effective engineering constants of the balanced orthotropic laminate can be obtained as

$$\begin{aligned} \bar{E}_1 &= \frac{1}{a_{11} h} \\ \bar{\nu}_{12} &= -\frac{a_{12}}{a_{11}} \\ \bar{G}_{12} &= \frac{1}{a_{66} h} \end{aligned} \quad (6)$$

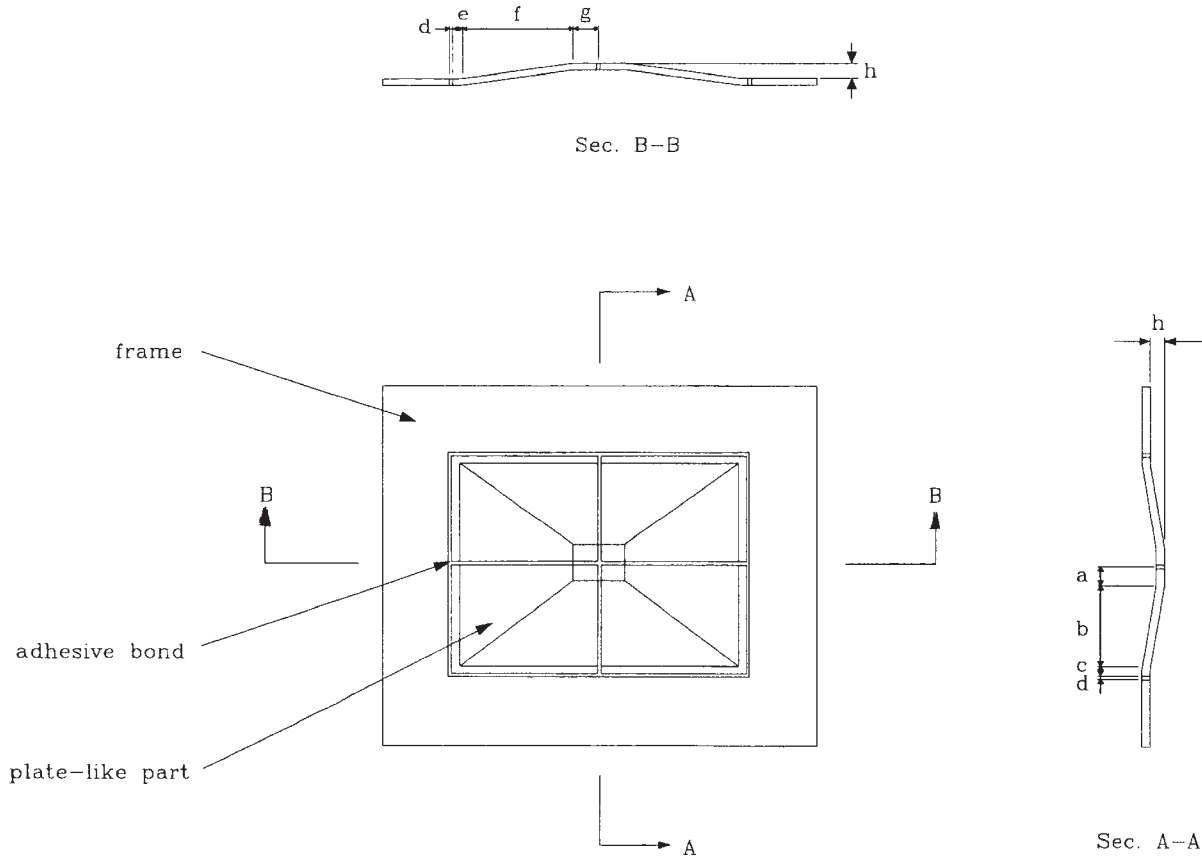


Fig. 3 Geometry of the frangible canister cover CB

Table 1 Properties of different composite laminae

Material type	Material constant*			Strength parameter†			Lamina thickness (mm)
	E_1 (GPa)	G_{12} (GPa)	ν_{12}	X_T (MPa)	X_C (MPa)	S (MPa)	
Glass fabric A/epoxy	19.70	1.03	0.106	184.33	251.56	57.48	0.22
Glass fabric B/epoxy	24.95	2.00	0.143	369.00	504.03	44.91	0.44

*Coefficients of variation are about 5%.

†Coefficients of variation are about 8%.

where \bar{E}_1 , $\bar{\nu}_{12}$, \bar{G}_{12} are the effective in-plane longitudinal Young's modulus, Poisson's ratio and shear modulus respectively. If expressed in terms of A_{ij} , the above effective engineering constants are obtained as

$$\bar{E}_1 = \frac{A_{11}}{mh}$$

$$\bar{\nu}_{12} = \frac{A_{12}}{A_{11}}$$

$$\bar{G}_{12} = \frac{A_{66}}{h}$$

(7)

with

$$m = \left(1 - \frac{A_{12}^2}{A_{11}^2} \right)^{-1} \tag{8}$$

3 FAILURE ANALYSIS OF FRANGIBLE COVER

The finite element method formulated on the basis of the theory of linear elasticity is used to evaluate the stress distribution in the composite canister cover. Due to symmetry, only a quarter of the cover is considered in the finite element analysis. Since the width, d , of the adhesive bond is much smaller than its depth, it will be more

appropriate to model the adhesive bond using three-dimensional brick elements than two-dimensional plate elements. Therefore, all the components of quarter cover CA are modelled by eight-node brick and six-node wedge elements, while those of quarter cover CB are modelled by eight-node brick elements. When constructing the finite element mesh, a number of layers of elements are considered in the thickness directions of the components and the aspect ratios of the elements are chosen in such a way that no numerical instability occurs.

A schematical description of the finite element mesh and the boundary conditions of the quarter cover (cover CB) are shown in Fig. 4. The material constants for the elements of the laminated composite components are determined on the basis of the concept of effective engineering constants, as mentioned in the previous section. Furthermore, since the length–thickness ratios of the laminated components are large, it is reasonable to assume that the stressess in the thickness direction are negligible. For simplicity, it is assumed that the material constants in the thickness direction have the following properties:

$$\bar{E}_3 = \bar{E}_1, \quad \bar{\nu}_{13} = \bar{\nu}_{12}, \quad \bar{G}_{13} = \bar{G}_{12} \quad (9) \quad \text{with}$$

where $\bar{E}_3, \bar{\nu}_{13}, \bar{G}_{13}$ are effective Young’s modulus, Poisson’s ratio and transverse shear modulus in the thickness direction respectively. On the other hand, the adhesive is assumed to be isotropic. Different types of adhesive have been used to bond the cover components and their material properties are listed in Table 2.

In the failure analysis of the canister cover, two phenomenological failure criteria, namely the Tsai–Wu and maximum stress criteria, are used to study the incipient failures in the components. In particular, the incipient failure in the laminated composite components is identified using the Tsai–Wu criterion [8], which is an improved and simplified version of a tensor polynomial failure theory for anisotropic materials that has been suggested earlier by Gol’denblat and Kopnov [9]. For the case of plane stress, the Tsai–Wu criterion is expressed as

$$F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + F_1\sigma_1 + F_2\sigma_2 = 1 \quad (10)$$

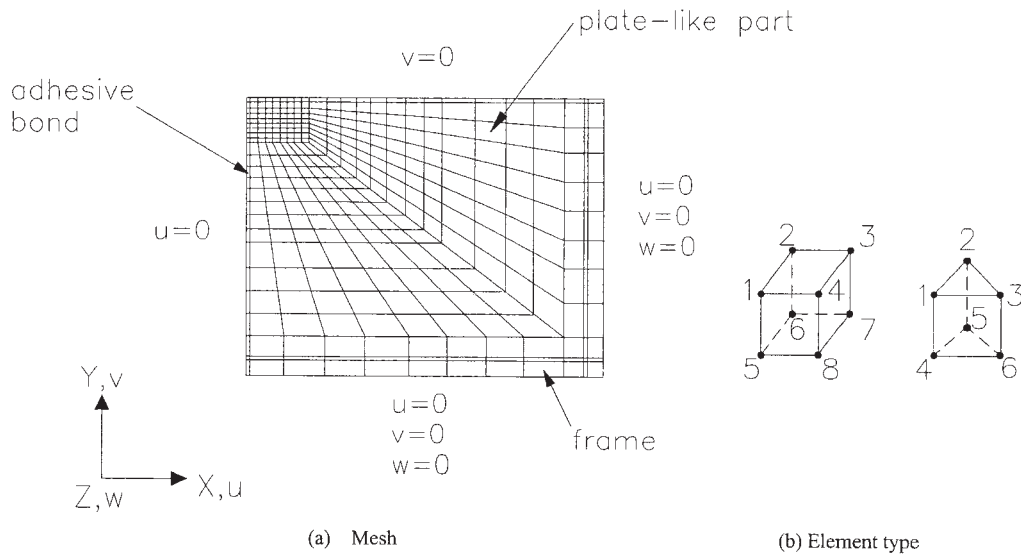


Fig. 4 Schematic description of the finite element model for cover CB

Table 2 Properties of different types of adhesive

Adhesive type	Material constant*			Strength parameter†	
	E (GPa)	G (GPa)	ν	X _T (MPa)	S (MPa)
I	1.264	0.498	0.269	31.57	6.837
II	1.251	0.496	0.261	21.5	4.125
III	1.231	0.491	0.253	18.3	4.524
IV	1.157	0.464	0.247	12.4	4.860
V	2.152	0.784	0.373	35.7	11.68

*Coefficients of variation are about 5%.

†Coefficients of variation are about 10%.

$$F_1 = \frac{1}{X_T} - \frac{1}{X_C}, \quad F_{12} = -\frac{1}{2\sqrt{X_T^2 X_C^2}}$$

$$F_2 = F_1, \quad F_{66} = \frac{1}{S^2}$$

$$F_{11} = \frac{1}{X_T X_C}, \quad F_{22} = F_{11} \quad (11)$$

where X_T , X_C are the in-plane lamina tensile and compressive strengths in the fibre direction respectively and S is the in-plane shear strength. The value of strength parameter F_{12} has been chosen in such a way that equation (10) has taken on the form of a generalized von Mises criterion for the yielding of isotropic materials [7]. The accuracy of the Tsai–Wu criterion in predicting first-ply failure of laminated composite plates has been verified in previous studies [10–13]. It is noted that the actual stresses rather than the average stresses in each lamina of the laminated composite components are used in the above failure criterion for failure prediction. On the other hand, the incipient failure in the adhesive bonds is identified using the maximum stress failure criterion, which states that failure of the material is assumed to occur if any of the following conditions is satisfied:

$$\sigma_{\max} \geq X'_T$$

or

$$\tau_{\max} \geq S' \quad (12)$$

where σ_{\max} , τ_{\max} are the maximum principal and shear stresses respectively and X'_T , S' are adhesive tensile and shear strengths respectively. It is noted that failure of an adhesive bond line is always induced by the maximum tensile stress developed in the adhesive. The value of the maximum shear stress is small and its effect on the failure of adhesive is minimal. Since the adhesive used in fabricating the frangible covers is brittle, it is thus appropriate to predict adhesive failure using the maximum stress criterion.

The frangible cover is designed in such a way that it will fail in a predetermined pattern. In general, the adhesive bonds are the weakest part in the cover and the strength of the adhesive bonds controls the magnitude of the burst strength of the cover. The strength of the adhesive bonds, on the other hand, depends on the depth of the adhesive bonds (or the thickness of the laminated components) and the tensile strength of the adhesive. It is worth noting that once incipient failure occurs in an adhesive bond line, the failure, which in reality is in the form of a dynamic crack propagating in the direction parallel to the edges of two neighbouring laminated composite components, will make

the frangible cover unstable. Herein, the pressure that causes incipient failure in the cover is treated as the static burst pressure of the cover.

4 EXPERIMENTAL INVESTIGATION

A number of frangible laminated composite canister cover specimens were fabricated and subjected to static burst strength tests. The dimensions of the test specimens are listed in Table 3. The notation used for denoting the layup of a laminated composite component is $[A_n/B_m]_s$, where A and B in the brackets denote the laminae of fabric A/epoxy and fabric B/epoxy, subscripts n and m the numbers of plies and the subscript s outside the brackets symmetric lamination. A schematic description of the experimental set-up for the static burst strength test is shown in Fig. 5. The frame of the cover specimen under testing was properly clamped on the lug at one end of the cylindrical pressure tank while air was pumped into the tank from the other end. The flowrate of the air pumped into the tank was slow enough that no excitation of the specimen was induced. The ultimate pressure that the specimen could sustain was measured via a pressure gauge attached to the tank. At failure, the plate-like laminated composite parts of the cover specimen were broken apart from each other along their edges and were also severed from the frame of the specimen. Furthermore, no debris was generated during the failure process. It is worth noting that debris may be detrimental to some kinds of missiles. Thus, the fact that the frangible covers did not generate debris has demonstrated one of the advantages of the present design. A visual inspection of the failed specimens revealed that failure only occurred at the adhesive bonds and that the plate-like laminated composite parts remained intact. The fact that the cover did fail in a predetermined pattern has thus demonstrated the feasibility of the design.

5 RESULTS AND DISCUSSION

The aforementioned method for failure prediction is used to predict the static burst strength of the frangible laminated composite canister covers that are to be tested. Herein the commercial finite element code NASTRAN [14] is used to perform the finite element analysis of the covers. All the laminated composite components and adhesive bond lines of the frangible covers are modelled by the aforementioned three-dimensional elements. A conver-

Table 3 Dimensions of different cover specimens

Cover specimen	Dimensions (mm)							
	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>
Small scale	10	50	5	1	5	65	15	8
Full scale	50	250	10	2	10	325	75	40

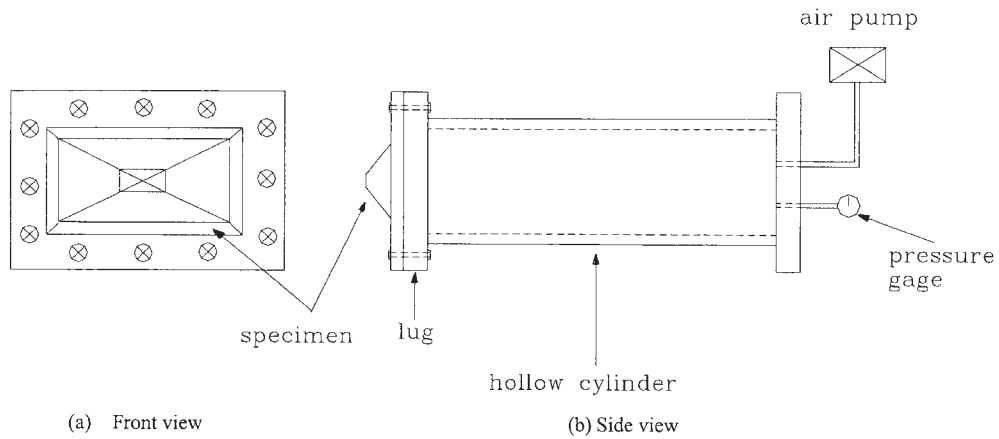


Fig. 5 Experimental set-up for the burst strength test

gence test of the finite element model is first performed to determine the suitable finite element mesh. The failure pressures of the $[A_2/B_{10}]_s$ cover CB have been determined using different types of finite element mesh and the results are listed in Table 4 for illustration. It is noted that the differences among the failure pressures predicted by the various types of finite element mesh are small. For the sake

of saving computing time, mesh type I will be adopted in the failure analyses of the other frangible covers. The maximum principal stress distribution of the quarter $[A_2/B_{10}]_s$ cover CB is shown in Fig. 6. When comparing the principal stresses with the material strengths, it is obvious that points P₁, P₂, P₃ and P₄ in the adhesive bond lines are likely to be the locations where incipient failure

Table 4 Convergence test of the finite element model for the full-scale $[A_2/B_{10}]_s$ cover CB

Mesh type	Number of elements (adhesive bond)		Total number of elements (quarter cover)	Failure pressure (KPa)
	Depth	Width		
I	1	1	2196	68.95
II	2	1	4392	69.60
III	3	1	6588	69.95
IV	3	2	7128	67.85

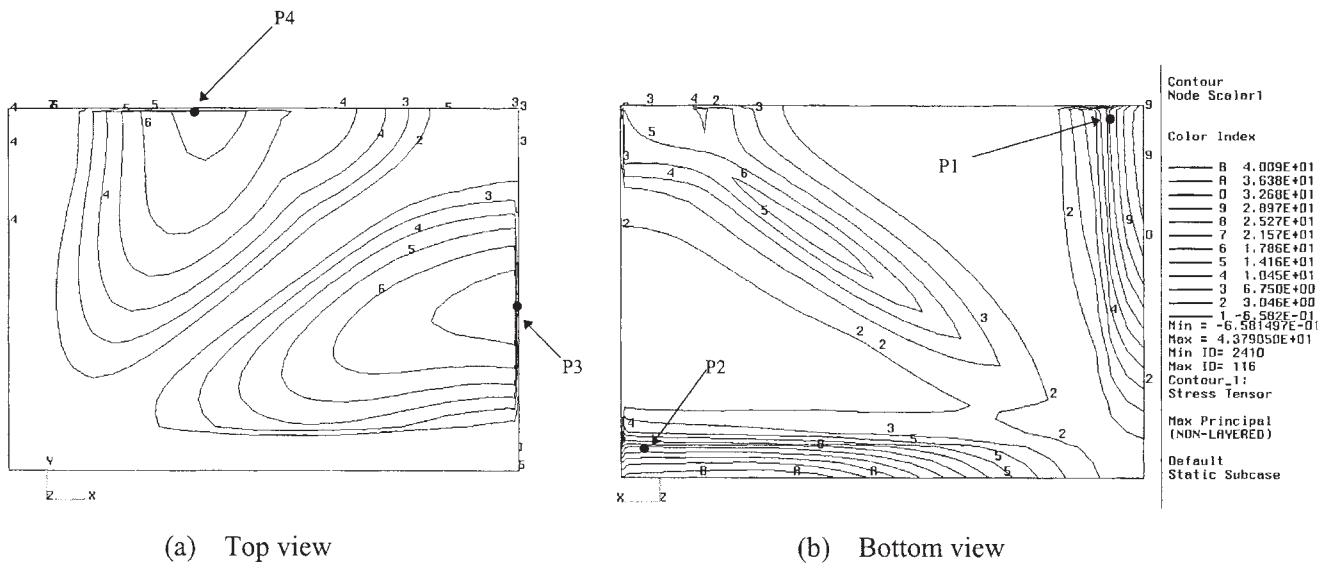


Fig. 6 Distribution of the principal stress in $[A_2/B_{10}]_s$ quarter cover CB

will be incurred in the frangible cover. Using mesh type I, the theoretical failure pressures of the small-scale frangible cover specimens were computed and the results are listed in Table 5 in comparison with the experimental results. It is noted that the differences between the theoretical and experimental results fall in the range from 1.8 to 13.0 per cent. The present method is also used to predict the failure pressures of several full-scale frangible covers. The theoretical failure pressures are listed in Table 6 in comparison with the experimental results. It is again noted that small differences between the theoretical and experimental failure pressures have been observed. The differences between the theoretical and experimental failure pressures for the frangible cover may be caused by the uncertainties involved in the fabrication process and the variations of material properties. It would be more appropriate to analyse the frangible covers via a probabilistic rather than a deterministic approach. Nevertheless, the close agreements between the theoretical and experimental results can still validate the suitability of the proposed method for failure analysis of the present frangible laminated composite canister covers. In view of the results in Tables 5 and 6, it is noted that the depth of the bonding surface, i.e. the thickness of the laminated components, and the strength of the adhesive have important effects on the static burst strength of a frangible cover. The increase in the depth of the bonding surface or the strength of the adhesive can raise the failure pressure of the frangible cover. Since a frangible cover must be of light weight and able to sustain static internal pressure of a certain magnitude, a proper

selection of adhesive is thus vital in designing a reliable frangible cover.

6 CONCLUSIONS

The design of two new frangible laminated composite canister covers was presented. The frangible canister covers were designed to fail in some predetermined patterns. A finite element model for analysing the mechanical behaviours of the frangible covers was established. The failure pressure of the frangible covers was determined on the basis of a phenomenological failure criterion. Experimental investigations of the failure modes and failure pressure of the frangible covers were performed. The study has shown that the frangible covers did fail in accordance with the predetermined patterns. When compared with the experimental results, the present analytical method could predict reasonably accurate failure pressures for the frangible covers. Therefore, the uses of the effective engineering constants in simulating the material properties of the laminated composite components and the failure criteria in predicting failure in the frangible covers are acceptable. It has also been shown that the bonding area between the laminated composite components and the strength of the adhesive had significant effects on the failure pressure of the frangible cover. It is therefore important to select proper laminate thickness and adhesive in fabricating the frangible covers of light weight, but with relatively high reliability.

Table 5 Failure pressure of frangible canister cover specimens (small scale)

Layup	Cover specimen		Failure pressure (KPa)		Percentage difference $ (I - II)/I \times 100$
	Adhesive type	Cover type	Experimental (I)	Theoretical (II)	
$[A_2/B_2]_s$	II	CA	86.19	90.67	5.2
		CB	96.53	91.23	5.5
	III	CA	75.85	77.20	1.8
		CB	79.29	77.70	2.0
$[A_2/B_3]_s$	II	CA	103.43	116.93	13.0
		CB	110.32	116.42	5.5
	III	CA	93.08	99.54	6.9
		CB	93.08	98.23	5.2
	IV	CA	75.85	67.46	10.0
		CB	62.06	65.49	5.5

Table 6 Failure pressure of frangible canister cover specimens (full scale)

Layup	Cover specimen		Failure pressure (Pa)		Percentage difference $ (I - II)/I \times 100$
	Adhesive type	Cover type	Experimental (I)	Theoretical (II)	
$[A_2/B_5]_s$	I	CA	17.24	18.62	8.00
$[A_2/B_5]_s$	V	CA	24.13	26.89	11.44
$[A_2/B_{10}]_s$	I	CB	62.06	68.95	11.10

The frangible covers should be analysed via a probabilistic approach if the reliability of the frangible covers is to be assessed.

ACKNOWLEDGEMENT

This research was supported by the National Science Council of the Republic of China under Grant NSC 86-2623-D-009-008. Their support is gratefully acknowledged.

REFERENCES

- 1 **Bell, R. E.** Missile weapon system. US Pat. 5239909, 1992.
- 2 **Boeglin, P. H.** Plate-glass fitted with an explosion-cutting device. US Pat. 4333381, 1982.
- 3 **Mussey, R. A.** Launch tube closure. US Pat. 4301708, 1981.
- 4 **Copeland, R. L. and Greene, R. F.** Protective cover for a missile nose cone. US Pat. 3970006, 1976.
- 5 **Krol, U. B.** Frangible cover assembly for missile launchers. US Pat. 3742814, 1971.
- 6 **Doane, W. J.** Frangible fly through diaphragm for missile launch canister. US Pat. 4498368, 1985.
- 7 **Tsai, S. W. and Hahn, H. T.** *Introduction to Composite Materials*, 1980 (Technomic Publishing Company, Westport, Connecticut).
- 8 **Tsai, S. W. and Wu, E. M.** A general theory of strength for anisotropic materials. *J. Composite Mater.*, 1971, **5**, 58–80.
- 9 **Gol'denblat, I. and Kopnov, V. A.** Strength of glass reinforced plastics in the complex stress state. *Mekhanika Polimerov*, 1965, **1**, 70–78. (English translation: *Polym. Mechanics*, 1966, **1**, 54–60).
- 10 **Kam, T. Y. and Lui, Y. W.** First-ply failure strength of laminated composite pressure vessels. *Int. J. Composite Structs*, 1997, **38**(1–4), 447–452.
- 11 **Kam, T. Y., Sher, H. F., Chao, T. N. and Chang, R. R.** Prediction of first-ply failure strengths of laminated composite plates using a finite element approach. *J. Solids Structs*, 1996, **33**(3), 375–398.
- 12 **Kam, T. Y. and Lai, F. M.** Experimental and theoretical predictions of first-ply failure strength of laminated composite plates. *J. Solids Structs*, 1999, **36**(16), 2379–2395.
- 13 **Kam, T. Y. and Chang, E. S.** Reliability formulation for composite laminates subjected to first-ply failure. *Int. J. Composite Structs*, 1997, **38**(1–4), 65–70.
- 14 **Caffrey, J. P. and Lee, J. M.** *MSC/NASTRAN Linear Static Analysis User's Guide, Version 68*, 1994 (The MacNeal-Schwendler Corporation, Los Angeles, California).