Recovery of laser beam propagation through a turbulent amplifier using phase conjugation

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Abstract. The recovery of an optical beam, distorted by a turbulent medium with random gain (loss), using phase conjugation is investigated. Turbulence broadens the beamwidth and the random gain enhances the broadening effect. Meanwhile, the backscattering enhancement effect occurs because there is a coherent addition of the forward incident beam and the backscattered beam by a phase conjugate mirror (PCM). For comparison, the beam reflected by a conventional mirror is also considered. The enhancement backscattering effect is stronger for a PCM than for the conventional mirror. It is found that when the random gain exists, the enhancement effect is enhanced and becomes stronger when either the characteristic length or fluctuation strength of active molecule number density of the turbulent medium is increased. The average reflected beam intensity is evaluated using the path integral technique.

Subject terms: random gain (loss); phase conjugate mirrors; path integral technique.

Optical Engineering 32(1), 121–130 (January 1993).

1 Introduction

The phenomenon of optical phase conjugation has been widely studied.[1,2] Basically, optical phase conjugation is a four-wave mixing process that can reverse the wavefront of an incident optical wave. One of the major applications of optical phase conjugation is the recovery of the distorted phase of a wave field that propagates through a scattering medium.[3–5] It can be shown that if there are no depolarization effects in the scattering and the conjugate wave is generated without losses or gains, a total elimination of wave distortions may be achieved even when the random fluctuations of the dielectric permittivity of the medium, which are due to turbulence, exist. However, usually the gain of the medium has to be included because the wave frequency is in the vicinity of the resonant frequency of the scattering medium. An optical wave cannot be recovered completely by phase conjugation because of the existence of random gain.

Recently, more attention has been paid to the effects of turbulent flows in the gain medium of a gas or liquid laser system.[6,7] To understand the laser performance under the influence of random gain, this paper investigates the extent of recovery of an optical beam propagating through a turbulent medium with gain reflected by a phase conjugate mirror (PCM). The geometry of the problem is shown in Fig. 1. For a random medium in gas or liquid phase, random fluctuations of the dielectric permittivity are generated from the irregular distortions of the molecule number density. These irregular distributions produce fluctuations of both the propagation constant and the amplification coefficient. This is quite clear because the real and imaginary part of electric susceptibility are related through the Kramer-Kronig relations.[8] Hence, in our problem, the wave field deteriorates even when the phase conjugate mirror is used. This is because the random gain induces stochastic fluctuations of amplitude that cannot be recovered by the phase conjugation. Because the change of turbulence pattern is so slow during the time period for the wave propagating in the medium, the turbulence can be regarded as being "frozen" during the round trip of the wave in the random medium. Therefore, the optical wave propagates through the same turbulence pattern during the forward and backward trips. To simplify the computation, the molecule number density is assumed to be statistically Gaussian distributed. A beam wave is considered as an initial wave. The average beam intensity is evaluated with the parabolic approximation using the path integral technique.[9,10] The broadening of beamwidth reveals evidence that the distortion of the beam wave cannot be recovered by the PCM. For comparison, the case of an optical wave reflected by a conventional mirror is also investigated. We observe that reflection by a phase conjugate mirror has larger enhancement effects[11,12] than that by a conventional mirror.

The structure of this paper is as follows. The wave equation in a turbulent medium with gain is illustrated in Sec. 2. The formulation of the average intensity of the reflected beam is derived in Sec. 3. The numerical results and discussion are given in Sec. 4, and the conclusion is shown in the last section.

2 Recovery of Optical Waves in a Turbulent Medium with Amplification by Phase Conjugation

The interaction of an electromagnetic field with an atomic transition is accompanied by amplification (absorption) of energy. When an optical frequency is close to a particular

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Paper OA-021 received Jan. 21, 1992; revised manuscript received July 1, 1992; accepted for publication July 27, 1992.

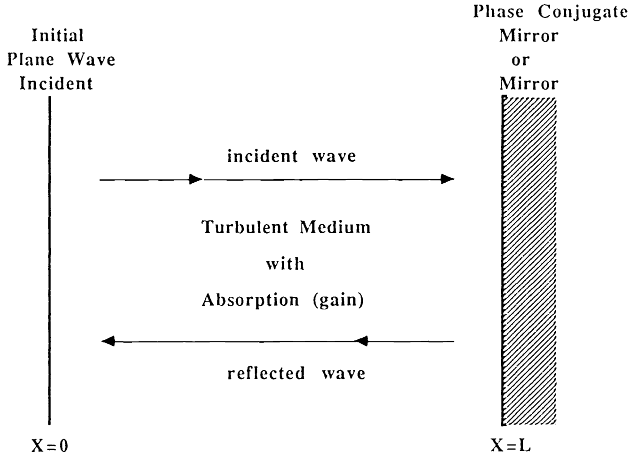

Fig. 1 Configuration for a plane wave at $x=0$ propagating through a turbulent medium with gain (absorption) by a distance L and reflected by a phase conjugate mirror or a conventional mirror.

resonant frequency of the turbulent medium, the effects of random amplification (absorption) are considerable. Hence, the electric susceptibility is then complex and defined[13] by $\chi=\chi_1-j\chi_2$, where

$$\chi_1(\omega)=\beta N\frac{(\omega_0-\omega)T}{1+(\omega_0-\omega)^2T^2} , \quad (1)$$

and

$$\chi_2(\omega)=\pm\beta N\frac{1}{1+(\omega_0-\omega)^2T^2} , \quad (2)$$

because the gain (loss) is homogeneously broadened. Here, N is the number density of active molecules that are responsible for the amplification (absorption), ω is the angular frequency of the propagated wave, ω_0 is the resonance frequency, and T is the relaxation time. Also, β is a positive real constant related to some implicit properties of the medium that are not our concern here. From Eqs. (1) and (2), both χ_1 and χ_2 are linearly proportional to the number density N fluctuating spatially. Therefore, χ_1 and χ_2 must follow the same fluctuation. It is noted that the + and − signs on the right-hand side of Eq. (2) represent the absorption and amplification media, respectively.

When an optical wave propagates in a turbulent medium with gain, the scalar wave equation of the wave field E with source-free condition is given by

$$\nabla^2E+k^2(1+\mu N_1)E=0 , \quad (3)$$

where $N_1=(N-\langle N\rangle)/\langle N\rangle$ is the relative fluctuation of the number density; μ is a complex constant related to β, ω_0, T, and $\langle N\rangle$; and k is the complex wavenumber in the background medium and can be approximately estimated because of the facts that $\langle\chi_1\rangle$, $\langle\chi_2\rangle\ll 1$:

$$k=\frac{\omega}{c}(1+\langle\chi_1\rangle+j\langle\chi_2\rangle)^{1/2}\cong\frac{\omega}{c}+j\omega\frac{\langle\chi_2\rangle}{2c} . \quad (4)$$

Note that $\langle\cdot\rangle$ represents the ensemble average. Here, c is the speed of light in free space. Definitely, except for the factor j, the second term on the right-hand side of Eq. (4) is the average gain coefficient α, i.e.,

$$\alpha=\frac{\omega\langle\chi_2\rangle}{2c} . \quad (5)$$

Then according to Eqs. (1), (2), and (4), μ is given by

$$\mu\simeq\langle\chi_1\rangle+j\langle\chi_2\rangle . \quad (6)$$

3 Formulation of the Average Intensity of the Reflected Beam

To evaluate the laser field in a turbulent amplifier, the parabolic approximation is used. Therefore, Eq. (3) becomes

$$\nabla_T^2K-2jk\frac{\partial K}{\partial x}+k^2\mu N_1K=-\delta(\mathbf{r}-\mathbf{r}_0) , \quad (7)$$

where the complex K, which represents the field amplitude at position $\mathbf{r}$ originated from the point source at $\mathbf{r}_0$, can be solved by using the path integral technique[14] to yield

$$K(0,\boldsymbol{\rho}_0;x_f,\boldsymbol{\rho})=\int D\boldsymbol{\rho}'\exp\left\{\frac{-jk}{2}\int_0^{x_f}\left[\left(\frac{\mathrm{d}\boldsymbol{\rho}'(x')}{\mathrm{d}x'}\right)^2+\mu N_1(x',\boldsymbol{\rho}')\right]\mathrm{d}x'\right\} , \quad (8)$$

where $\mathbf{r}_0=(0,\boldsymbol{\rho}_0)$ and $\mathbf{r}=(x_f,\boldsymbol{\rho})$ with x_f being the propagation distance along the x direction. Using Eq. (8), an initial beam $U_0(\boldsymbol{\rho}_0)$ propagating from the plane $x=0$ to the plane $x=L$ is solved and given by

$$U(x=0;x=L,\boldsymbol{\rho})=\iint_{-\infty}^{\infty}\mathrm{d}\boldsymbol{\rho}_0U_0(\boldsymbol{\rho}_0)\int D\boldsymbol{\rho}'\exp\left(\frac{-jk}{2}\int_0^L\left\{\left[\frac{\mathrm{d}\boldsymbol{\rho}'(x')}{\mathrm{d}x'}\right]^2+\mu N_1(x',\boldsymbol{\rho}')\right\}\mathrm{d}x'\right) . \quad (9)$$

After the beam wave is reflected by the PCM and propagating in the negative x direction to the initial plane, the reflected wave field at $x=0$ is written as

$$U_F(x=L;x=0,\boldsymbol{\rho})=\iint_{-\infty}^{\infty}U^*(0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\,\mathrm{d}^2\boldsymbol{\rho}_1 . \quad (10)$$

With Eqs. (8) and (9), Eq. (10) becomes

$$U_F(x=L;x=0,\boldsymbol{\rho})=\iiiint_{-\infty}^{\infty}U_0^*(\boldsymbol{\rho}_0)K^*(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)\times K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\,\mathrm{d}^2\boldsymbol{\rho}_0\,\mathrm{d}^2\boldsymbol{\rho}_1 . \quad (11)$$

The average received beam wave $\langle U_F\rangle$ is given by

$$\langle U_F(\boldsymbol{\rho})\rangle = \iiiint_{-\infty}^{\infty} U_0^*(\boldsymbol{\rho}_0)\langle K^*(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1) \times K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\rangle \, d^2\boldsymbol{\rho}_0 \, d^2\boldsymbol{\rho}_1 \ , \tag{12}$$

where $\langle K^*K\rangle$ represents the two position autocorrelation of the random propagator K. Using Markov's approximation, $\langle K^*K\rangle$ is evaluated by[14]

$$\langle K^*(0,\boldsymbol{\rho}_2;L,\boldsymbol{\rho}_{02})K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}_{01})\rangle = K_0^*(0,\boldsymbol{\rho}_2;L,\boldsymbol{\rho}_{02}) \times K_0(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}_{01}) \exp\left\{\frac{-L}{4}\int_0^1 du \times [\mathrm{Re}(k^2\mu^2)A_N(0) - |k\mu|^2 A_N(|u\boldsymbol{\rho}_0' + (1-u)\boldsymbol{\rho}_f'|)]\right\} , \tag{13}$$

where $\boldsymbol{\rho}_0' = \boldsymbol{\rho}_{01} - \boldsymbol{\rho}_2$ and $\boldsymbol{\rho}_f' = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_{02}$ and A_N denotes the integrated correlation function of N_1 defined by

$$A_N(|\boldsymbol{\rho}|) = \int_{-\infty}^{\infty} B_N[(|\boldsymbol{\rho}|^2 + x^2)] \, dx \ , \tag{14}$$

where $B_N(\mathbf{r}) = \langle N_1(\mathbf{r}_1 + \mathbf{r})N_1(\mathbf{r}_1)\rangle$ is the autocorrelation function of N_1. Here, K_0 is the deterministic propagator in the background medium given by

$$K_0(x=0,\boldsymbol{\rho}';x=L,\boldsymbol{\rho}) = \frac{-jk}{2\pi L} \exp\left(\frac{-jk|\boldsymbol{\rho}-\boldsymbol{\rho}'|^2}{2L}\right) . \tag{15}$$

To simplify the computation, the value of the exponent of Eq. (13) is assumed small enough that Eq. (13) can be expanded and approximated by

$$\langle K^*(0,\boldsymbol{\rho}_2;L,\boldsymbol{\rho}_{02})K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}_{01})\rangle = K_0^*(0,\boldsymbol{\rho}_2;L,\boldsymbol{\rho}_{02}) \times K_0(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}_{01})\left\{1 - \frac{L}{4}\int_0^1 du \times [\mathrm{Re}(k^2\mu^2)A_N(0) + |k\mu|^2 A_N(|u\boldsymbol{\rho}_0' + (1-u)\boldsymbol{\rho}_f'|)]\right\} . \tag{16}$$

With the returned beam expressed by Eq. (11), the average beam intensity is given by

$$\langle |U_F|^2\rangle = \int \cdots \int_{-\infty}^{\infty} d^2\boldsymbol{\rho}_0 \, d^2\boldsymbol{\rho}_0' \, d^2\boldsymbol{\rho}_1 \, d^2\boldsymbol{\rho}_2 U_0^*(\boldsymbol{\rho}_0)U_0(\boldsymbol{\rho}_0') \times \langle K^*(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}) \times K(0,\boldsymbol{\rho}_0';L,\boldsymbol{\rho}_2)K^*(L,\boldsymbol{\rho}_2';0,\boldsymbol{\rho})\rangle \ . \tag{17}$$

With assumption of Gaussian distribution of the random propagator K, the fourth moment of K, the integrand of Eq. (17), becomes

$$\langle K^*(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})K(0,\boldsymbol{\rho}_0';L,\boldsymbol{\rho}_2)K^*(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho})\rangle \simeq \langle K^*(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\rangle\langle K(0,\boldsymbol{\rho}_0';L,\boldsymbol{\rho}_2) \times K^*(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho})\rangle + \langle K^*(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(0,\boldsymbol{\rho}_0';L,\boldsymbol{\rho}_2)\rangle \times \langle K^*(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho})K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\rangle \ . \tag{18}$$

The fourth moment of K has factored into the product of the second moment of K, which can be evaluated using Eq. (16). Hence, replacing Eqs. (16) and (18) into Eq. (17), the expression of the average beam intensity is given by

$$\langle |U_F|^2\rangle = P_1(\boldsymbol{\rho}) + \int_0^1 P_2(\boldsymbol{\rho},u) \, du + \int_0^1 P_3(\boldsymbol{\rho},u) \, du + \iint_0^1 P_4(\boldsymbol{\rho},u,u') \, du \, du' + P_1'(\boldsymbol{\rho}) + \int_0^1 P_2'(\boldsymbol{\rho},u) \, du + \int_0^1 P_3'(\boldsymbol{\rho},u) \, du + \iint_0^1 P_4'(\boldsymbol{\rho},u,u') \, du \, du' \ . \tag{19}$$

The detail derivations and expressions for P_j and P_j', $j=1, \ldots, 4$, are given in the Appendix. Notice that to simplify the computation, the incident beam is a Gaussian shape, i.e., $U_0(\boldsymbol{\rho}_0) = \exp[-|\boldsymbol{\rho}_0|^2/2w_0^2]$ with the initial beamwidth w_0 and a Gaussian autocorrelation function of the relative fluctuation of N_1 is assumed, i.e.,

$$B_N(\mathbf{r}) = \langle N_1^2\rangle \exp\left(-\frac{|\mathbf{r}|^2}{l^2}\right) . \tag{20}$$

Here, $\langle N_1^2\rangle$ and l represent the mean-square fluctuation of N_1 and the scale length of the turbulent medium, respectively. When the phase conjugate mirror is replaced by a conventional mirror, the field U_F', reflected by the mirror, can be expressed by Eq. (10) except that the conjugate sign of the function U on the right side is excluded, i.e.,

$$U_F'(x=L;x=0,\boldsymbol{\rho}) = \iint_{-\infty}^{\infty} U(0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}) \, d^2\boldsymbol{\rho}_1 \ . \tag{21}$$

With Eqs. (8), (9), and (21), $\langle U_F'\rangle$ becomes

$$U_F'(\boldsymbol{\rho}) = \iiiint_{-\infty}^{\infty} U_0(\boldsymbol{\rho}_0) \langle K(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\rangle \, d^2\boldsymbol{\rho}_0 \, d^2\boldsymbol{\rho}_1 \ . \tag{22}$$

Using Markov's approximation, the integrand $\langle K(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho})\rangle$ is evaluated and given by

$$\langle K(0,\boldsymbol{\rho}_1;L,\boldsymbol{\rho}_{01})K(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho}_{02})\rangle = K_0^*(0,\boldsymbol{\rho}_1;L,\boldsymbol{\rho}_{01}) \times K_0(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho}_{02}) \exp\left\{\frac{-L}{4}\int_0^1 du \times [\mathrm{Re}(k^2\mu^2)A_N(0) - |k\mu|^2 A_N(|u\boldsymbol{\rho}_0' + (1-u)\boldsymbol{\rho}_f'|)]\right\} . \tag{23}$$

Also, Eq. (23) is expanded and approximated by

$$\langle K(0,\boldsymbol{\rho}_1;L,\boldsymbol{\rho}_{01})K(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho}_{02})\rangle = K_0(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_{01}) \times K_0(L,\boldsymbol{\rho}_2;0,\boldsymbol{\rho}_{02}) \times \left\{1-\frac{L}{4}\int_0^1 \mathrm{d}u \times [\mathrm{Re}(k^2\mu^2)A_N(0)+|k\mu|^2A_N(|u\boldsymbol{\rho}_0'+(1-u)\boldsymbol{\rho}_f'|)]\right\} . \quad (24)$$

With the returned beam expressed by Eq. (21), the average beam intensity is given by

$$\langle|U_F'|^2\rangle = \int\cdots\int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\ \mathrm{d}^2\boldsymbol{\rho}_0'\ \mathrm{d}^2\boldsymbol{\rho}_1\ \mathrm{d}^2\boldsymbol{\rho}_2 U_0(\boldsymbol{\rho}_0)U_0^*(\boldsymbol{\rho}_0') \langle K(0,\boldsymbol{\rho}_0;L,\boldsymbol{\rho}_1)K(L,\boldsymbol{\rho}_1;0,\boldsymbol{\rho}) \times K^*(0,\boldsymbol{\rho}_0';L,\boldsymbol{\rho}_2)K^*(L,\boldsymbol{\rho}_2';0,\boldsymbol{\rho})\rangle . \quad (25)$$

Using Eq. (18), Eq. (25) becomes

$$\langle|U_F|^2\rangle = M_1(\boldsymbol{\rho}) + \int_0^1 M_2(\boldsymbol{\rho},u)\ \mathrm{d}u + \int_0^1 M_3(\boldsymbol{\rho},u)\ \mathrm{d}u + \int\int_0^1 M_4(\boldsymbol{\rho},u,u')\ \mathrm{d}u\ \mathrm{d}u' + M_1'(\boldsymbol{\rho}) + \int_0^1 M_2'(\boldsymbol{\rho},u)\ \mathrm{d}u + \int_0^1 M_3'(\boldsymbol{\rho},u)\ \mathrm{d}u + \int\int_0^1 M_4'(\boldsymbol{\rho},u,u')\ \mathrm{d}u\ \mathrm{d}u' . \quad (26)$$

The detailed expressions for M_j and M_j', $j=1, \cdots, 4$, are also given in the Appendix. The terms without integration on the right sides of Eq. (19) and (26), i.e., P_1, P_1', M_1, and M_1', represent the coherent beam intensity, and the rest terms, with one-fold or two-fold integration, represent the incoherent beam intensity.[15] The former intensity, which is from the first term in the square bracket of Eq. (13), is from the autocorrelation of each individual random propagator. The latter one comes from the cross-correlation of the two propagators.

4 Numerical Result and Discussion

The average intensity of the laser beam and its peak value are calculated and illustrated by figures for both cases in this section. The random gain effects on the recovery of light by using a PCM are examined. Meanwhile, the phenomenon of backscattering enhancement occurs and is discussed. Here, a CO_2 laser amplifier and the following parameter values are considered: wavelength of the laser beam $\lambda=10.6$ μm and the average gain coefficient $\alpha=2\ \mathrm{m}^{-1}$. Here, we estimate $\chi_2=6.4\times10^{-6}$ and let $\chi_1=\chi_2$. With Eqs. (4) and (6), the complex constant $k=6.28\times10^5+j2\ \mathrm{m}^{-1}$ and $\mu=6.4\times10^{-6}+j6.4\times10^{-6}$.

In Fig. 2, the average intensity of the half-beam profile is illustrated as a function of the transversal length $|\boldsymbol{\rho}|$. To examine broadening of the beamwidth of each beam, the value of each profile is normalized by its peak value. Here, $L=10$ m, $\langle N_1^2\rangle=0.3$ and $l=0.01$ m. Two dotted curves I and II represent the case when the beam reflected by a mirror with $\alpha=0$ and $2\ \mathrm{m}^{-1}$, respectively. Curve II shows a larger beamwidth than that of curve I, simply because the random

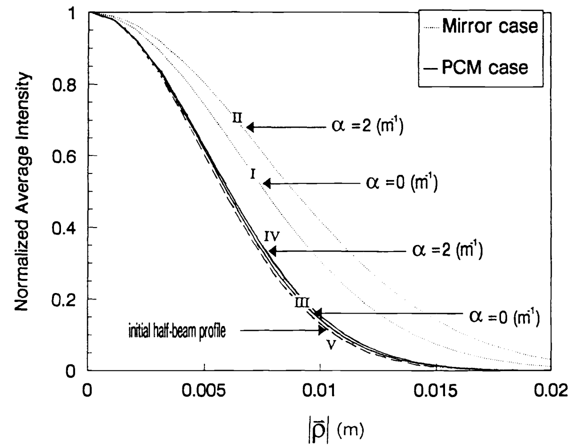

Fig. 2 Normalized average intensity of half-beam profile versus the transversal length $|\boldsymbol{\rho}|$ for the conventional mirror case (dotted lines) and the PCM case (solid lines) for two α values: 0 and 2 m^{-1}. For comparison, the initial half-beam profile is plotted as the curve *V*. The broadening of beamwidths for the PCM and conventional mirror cases are shown.

fluctuations of amplitude are stronger when $\alpha=2\ \mathrm{m}^{-1}$ than when $\alpha=0$. The larger amplitude fluctuations decorrelate the rays that synthesize the beam, i.e., broaden the beamwidth. For the PCM case, the same phenomenon is observed by comparing curves III and IV, which represent situations when $\alpha=0$ and $2\ \mathrm{m}^{-1}$, respectively. Note that the beamwidth for the mirror case is larger than that of the PCM case, because in the PCM case, the phase distortions, which can evolve into the amplitude fluctuations, are suppressed. Hence, the amplitude fluctuations become weaker, which leads to less broadening of the beamwidth.

The peak value of average intensity of the backscattered beam at plane $x=0$, which is expressed by the notation $\langle I\rangle_p$, is plotted as a function of mean-square fluctuation N_1, i.e., $\langle N_1^2\rangle$, in Fig. 3. Here, $\alpha=0\ \mathrm{m}^{-1}$, $L=1$ m, and $l=0.01$ m. The dotted and solid lines are drawn for the conventional mirror and PCM cases, respectively. The latter case has larger $\langle I\rangle_p$ than that of the conventional mirror case, which means that the PCM can recover the phase fluctuations of the backscattered beam, i.e., reduce the lateral scattering. Therefore, larger backscattering enhancement is observed. It gives a larger value of $\langle I\rangle_p$ because the incident and back-reflected beams are more correlated. Actually, with the PCM, the forward incident beam is the time reversal field of the backward reflected beam when $\alpha=0\ \mathrm{m}^{-1}$. In this figure, the values of $\langle I\rangle_p$ for both curves are slightly decreased as $\langle N_1^2\rangle$ is increased, because stronger fluctuations of active molecule density lead to larger lateral scattering. This implies that the incident and backscattered waves are more incoherent, i.e., the enhancement effects are decreased. Hence, the value of $\langle I\rangle_p$ is decreased.

In Fig. 4, the value of $\langle I\rangle_p$ is plotted as a function of $\langle N_1^2\rangle$ for $\alpha=2\ \mathrm{m}^{-1}$, and $L=1$ m, and $l=0.01$ m. The dotted and solid lines represent the cases of waves reflected by a conventional mirror or a PCM, respectively. The values of $\langle I\rangle_p$ for both curves increase when $\langle N_1^2\rangle$ increases, because of the amplitude fluctuations that cannot be recovered by the PCM because these fluctuations are caused by the random gain. This has been observed in the case of a wave

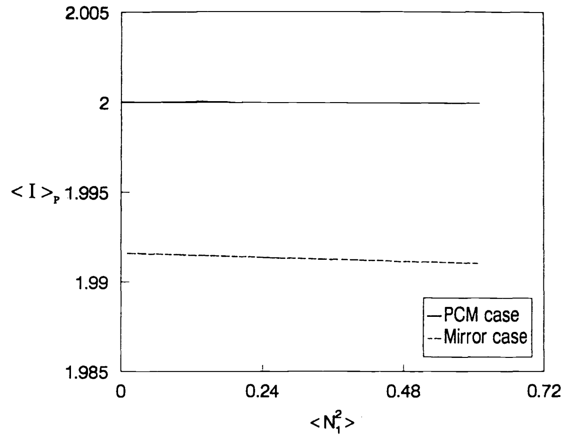

Fig. 3 Peak value of the average intensity of the backscattered beam at plane $x=0$, denoted by $\langle I\rangle_p$, plotted as a function of the mean-square fluctuation $\langle N_1^2\rangle$ for the PCM and conventional mirror cases when $\alpha=0\ \mathrm{m}^{-1}$. The values of $\langle I\rangle_p$ for both cases decrease slightly when $\langle N_1^2\rangle$ increases.

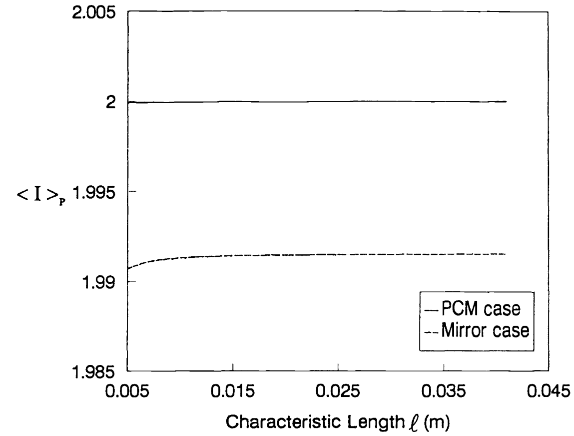

Fig. 5 Peak value of the average intensity of the backscattered beam at plane $x=0$, i.e., $\langle I\rangle_p$, as a function of l, for the conventional mirror and PCM cases. The values of $\langle I\rangle_p$ for both cases increase slightly as l increases.

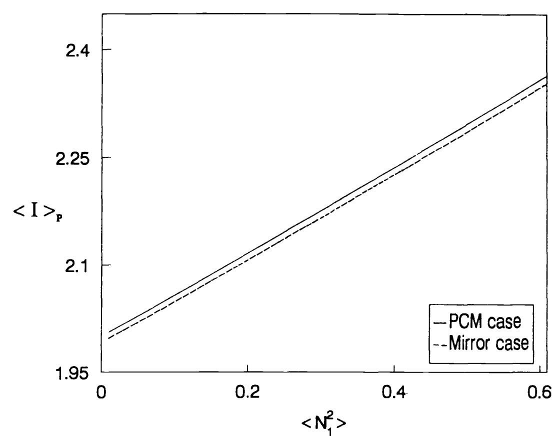

Fig. 4 Peak value of the average intensity of a backscattered beam at $x=0$, denoted by $\langle I\rangle_p$, versus mean-square fluctuation $\langle N_1^2\rangle$ for the case of a beam reflection by a conventional mirror or by a PCM with $\alpha=2\ \mathrm{m}^{-1}$. The value of $\langle I\rangle_p$ for both cases increases when $\langle N_1^2\rangle$ increases.

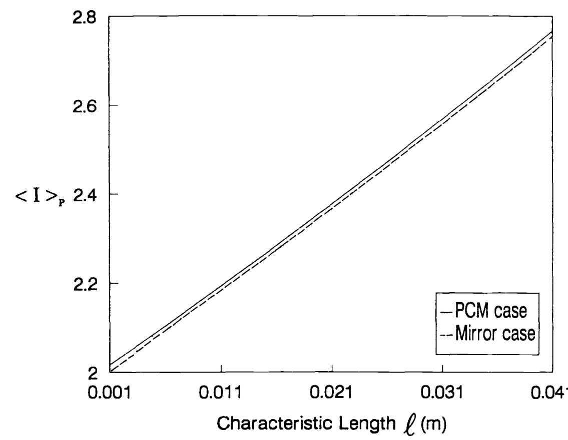

Fig. 6 Peak value of the average intensity $\langle I\rangle_p$ at plane $x=0$ drawn as a function of l for the conventional mirror and PCM cases. The values of $\langle I\rangle_p$ for both cases increase when l increases.

propagating in a turbulent medium with random gain.[17] It has been found that stronger fluctuations of N_1, i.e., the increases of $\langle N_1^2\rangle$, result in an increase of the average intensity, which is attributed to the redistribution of the coherent intensity, incoherent intensity, and gain energy when the wave propagates in a turbulent amplifier. When $\langle N_1^2\rangle$ increases, an increase of $\langle I\rangle_p$ caused by the energy redistribution can overcome the decrease of $\langle I\rangle_p$ that results from the reduction of backscattering enhancement. Comparing Figs. 3 and 4 shows that the case when $\alpha=2\ \mathrm{m}^{-1}$ has a larger value of peak average intensity than that when $\alpha=0\ \mathrm{m}^{-1}$, simply because of the existence of random gain.

In Fig. 5, the peak value of average intensity of the backscattered beam at plane $x=0$, i.e., $\langle I\rangle_p$, is plotted as a function of characteristic length l. Here, $\alpha=0\ \mathrm{m}^{-1}$, $L=1$ m, and $\langle N_1^2\rangle=0.3$. The values of $\langle I\rangle_p$ in both cases, the wave reflected by the conventional mirror and by the PCM, are slightly increased, because the increase of characteristic length causes incident and backscattered waves to be more coherent, i.e., to have stronger enhancement backscattering effects, which lead to a larger value of $\langle I\rangle_p$.

In Fig. 6, $\langle I\rangle_p$ is evaluated as a function of l when $\alpha=2\ \mathrm{m}^{-1}$. Note that the value of $\langle I\rangle_p$ increases as $\langle N_1^2\rangle$ increases. By comparing with the results shown in Figs. 4 and 5, we can conclude that the increasing trend of $\langle I\rangle_p$ in Fig. 6 is also mainly caused by the existence of the random gain. The random gain not only strengthens the enhancement effects but also makes these effects stronger when the characteristic length increases. The larger l yields a stronger correlation between the incident and backscattered waves, i.e., larger $\langle I\rangle_p$.

5 Conclusion

In this paper, the path integral technique is used to evaluate the average beam intensity after propagation through a turbulent amplifier and reflection by a conventional mirror or a PCM. Turbulence in the amplifier broadens the beamwidth and the random gain enhances this phenomenon in the conventional mirror case. In the PCM case, with the recovery of phase distortions, the beamwidth broadening reduces a lot compared with that of the conventional mirror case.

The enhancement backscattering effects, which are caused by the coherent addition of the forward incident and backscattered beams, are examined by evaluating the peak value of average beam intensity. The larger value of average beam intensity represents stringer enhancement effects. It is found that the PCM case has stronger enhancement effects than the conventional mirror case, because the PCM can reduce the lateral scattering in the backward direction, i.e., it makes the incident and backscattering beams more coherent. It is also discovered that an increase of fluctuation strength of active molecule density and a decrease of characteristic length of turbulence lead to a reduction of the enhancement effects when there is no random gain. However, if the random gain exists, the enhancement backscattering effects are stronger compared with cases without random gain. These effects become larger when either the characteristic length or $\langle N_1^2 \rangle$ increase.

6 Appendix

In this appendix, the final expressions for P_j, P_j', M_j, and M_j', $j=1, \ldots, 4$, for the PCM and conventional mirror cases are derived for numerical computations. To determine these expressions, Eqs. (16) and (18) are substituted into Eq. (17) to yield

$$
\begin{aligned}
P_1(\boldsymbol{\rho}) = {} & C_1 \exp\left[|\boldsymbol{\rho}|^2\left(\frac{k_2}{L}\right)\right] \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
& \times \exp\left[-|\boldsymbol{\rho}_0|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right) - |\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}\right) - |\boldsymbol{\rho}_0'|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right) \right. \\
& \qquad \left. - |\boldsymbol{\rho}|^2\left(-\frac{k_2}{L}\right)\right] \\
& \times \exp\left[\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right) + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right) + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_1}{L}\right) \right. \\
& \qquad \left. + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right] \\
& \times \exp\left[-j|\boldsymbol{\rho}_0|^2\left(-\frac{k_1}{2L}\right) - j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right) + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right) \right. \\
& \qquad \left. + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right] ,
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
P_2(\boldsymbol{\rho},u) = {} & C_2 \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
& \times \exp\left\{|\boldsymbol{\rho}|^2\left[\frac{k_2}{L}-\frac{(1-u)^2}{l^2}\right]\right\} \\
& \times \exp\left\{-|\boldsymbol{\rho}_0|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u)^2}{l^2}\right] - |\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}\right) - |\boldsymbol{\rho}_0'|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right) \right. \\
& \qquad - |\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_2}{L}\right) \\
& \qquad \left. + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_0\cdot\boldsymbol{\rho}\left[\frac{2(1-u)^2}{l^2}\right]\right\} \\
& \times \exp\left[-j|\boldsymbol{\rho}_0|^2\left(-\frac{k_1}{2L}\right) - j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right) + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right) \right. \\
& \qquad \left. + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right] ,
\end{aligned}
\tag{28}
$$

$$
P_3(\boldsymbol{\rho},u) = P_2(\boldsymbol{\rho},u) , \tag{29}
$$

$$
\begin{aligned}
P_4(\boldsymbol{\rho},u,u') = {} & C_3 \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
& \times \exp\left\{|\boldsymbol{\rho}|^2\left[\frac{k_2}{L}-\frac{(1-u)^2}{l^2}-\frac{(1-u')^2}{l^2}\right]\right\} \\
& \times \exp\left\{-|\boldsymbol{\rho}_0|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u)^2}{l^2}\right] - |\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}\right) \right. \\
& \qquad - |\boldsymbol{\rho}_0'|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u')^2}{l^2}\right] - |\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}\right) \\
& \qquad + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_2}{L}\right) \\
& \qquad \left. + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_0'\cdot\boldsymbol{\rho}\left[\frac{2(1-u')^2}{l^2}\right] + \boldsymbol{\rho}_0\cdot\boldsymbol{\rho}\left[\frac{2(1-u)^2}{l^2}\right]\right\} \\
& \times \exp\left[-j|\boldsymbol{\rho}_0|^2\left(-\frac{k_1}{2L}\right) - j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right) + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right) \right. \\
& \qquad \left. + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right] ,
\end{aligned}
\tag{30}
$$

$$
P_1'(\boldsymbol{\rho}) = P_1(\boldsymbol{\rho}) , \tag{31}
$$

$$
\begin{aligned}
P_2'(\boldsymbol{\rho},u) = {} & C_2 \exp\left[|\boldsymbol{\rho}|^2\left(\frac{k_2}{L}\right)\right] \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
& \times \exp\left[-|\boldsymbol{\rho}_0|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right) - |\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}\right) \right. \\
& \qquad - |\boldsymbol{\rho}_0'|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right) - |\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}\right) + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_2}{L}\right) \\
& \qquad + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right) + \boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2\left(\frac{2u^2}{l^2}\right) + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{-k_2}{L}\right) \\
& \qquad \left. + \boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(\frac{-k_2}{L}\right)\right] \\
& \times \exp\left[-j|\boldsymbol{\rho}_0|^2\left(-\frac{k_1}{2L}\right) - j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right) + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right) \right. \\
& \qquad \left. + j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{k_1}{L}\right) + j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right] ,
\end{aligned}
\tag{32}
$$

$$P_3'(\boldsymbol{\rho},u')=C_2\exp\left[|\boldsymbol{\rho}|^2\left(\frac{k_2}{L}\right)\right]\int\cdots\int_{-\infty}^{\infty}d^2\boldsymbol{\rho}_0\,d^2\boldsymbol{\rho}_1\,d^2\boldsymbol{\rho}_0'\,d^2\boldsymbol{\rho}_2$$
$$\times\exp\left\{-|\boldsymbol{\rho}_0|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u')^2}{l^2}\right]-|\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}+\frac{u'^2}{l^2}\right)\right.$$
$$-|\boldsymbol{\rho}_0'|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u')^2}{l^2}\right]-|\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}\right)$$
$$+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left[-\frac{k_2}{L}-\frac{2u(1-u)}{l^2}\right]+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0'\left[\frac{2u(1-u)}{l^2}\right]+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right)$$
$$+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2\left(\frac{2u^2}{l^2}\right)+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0\left[\frac{2u(1-u)}{l^2}\right]$$
$$\left.+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left[-\frac{k_2}{L}-\frac{2u(1-u)}{l^2}\right]+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right)+\boldsymbol{\rho}_0\cdot\boldsymbol{\rho}_0'\left[\frac{2(1-u)^2}{l^2}\right]\right\}$$
$$\times\exp\left[-j|\boldsymbol{\rho}_0|^2\left(-\frac{k_1}{2L}\right)-j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right)+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right)+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right)\right.$$
$$\left.+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{k_1}{L}\right)+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right]\ ,\qquad(33)$$

and

$$P_4'(\boldsymbol{\rho},u,u')=C_3\exp\left[|\boldsymbol{\rho}|^2\left(\frac{k_2}{L}\right)\right]\int\cdots\int_{-\infty}^{\infty}d^2\boldsymbol{\rho}_0\,d^2\boldsymbol{\rho}_1\,d^2\boldsymbol{\rho}_0'\,d^2\boldsymbol{\rho}_2$$
$$\times\exp\left\{-|\boldsymbol{\rho}_0|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u')^2}{l^2}\right]-|\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}+\frac{u'^2}{l^2}\right)\right.$$
$$-|\boldsymbol{\rho}_0'|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u')^2}{l^2}\right]-|\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}+\frac{u'^2}{l^2}\right)$$
$$+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left[-\frac{k_2}{L}-\frac{2u'(1-u')}{l^2}\right]+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0'\left[\frac{2u'(1-u')}{l^2}\right]$$
$$+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right)+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2\left(\frac{2u^2}{l^2}+\frac{2u'^2}{l^2}\right)+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0\left[\frac{2u'(1-u')}{l^2}\right]$$
$$\left.+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left[-\frac{k_2}{L}-\frac{2u'(1-u')}{l^2}\right]+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_2}{L}\right)+\boldsymbol{\rho}_0\cdot\boldsymbol{\rho}_0'\left[\frac{2(1-u')^2}{l^2}\right]\right\}$$
$$\times\exp\left[-j|\boldsymbol{\rho}_0|^2\left(-\frac{k_1}{2L}\right)-j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right)+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right)+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right)\right.$$
$$\left.+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(\frac{k_1}{L}\right)+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right]\ ,\qquad(34)$$

where

$$C_1=\frac{|k|^4}{16\pi^4L^4}\left[1-\frac{L}{4}\langle N_1^2\rangle l\sqrt{\pi}\ \mathrm{Re}(k^2\mu^2)\right]^2\ ,\qquad(35)$$

$$C_2=\frac{|k|^4}{16\pi^4L^4}\left[1-\frac{L}{4}\langle N_1^2\rangle l\sqrt{\pi}\ \mathrm{Re}(k^2\mu^2)\right]$$
$$\times\left(\frac{L}{4}\langle N_1^2\rangle l\sqrt{\pi}|k|^2|\mu|^2\right)\ ,\qquad(36)$$

and

$$C_3=\frac{|k|^4}{16\pi^4L^4}\left(\frac{L}{4}\langle N_1^2\rangle l\sqrt{\pi}|k|^2|\mu|^2\right)^2\ .\qquad(37)$$

In the conventional mirror case, the detailed derivations and expressions for M_j and M_j', $j=1,\ \ldots,\ 4$, can be determined by following a procedure that is the same as that for the PCM case, except that the conjugate sign of factor of U in the integrand on the right-hand side of Eq. (11) is excluded, and are given by

$$M_1(\boldsymbol{\rho})=C_1\exp\left[|\boldsymbol{\rho}|^2\left(\frac{k_2}{L}\right)\right]\int\cdots\int_{-\infty}^{\infty}d^2\boldsymbol{\rho}_0\,d^2\boldsymbol{\rho}_1\,d^2\boldsymbol{\rho}_0'\,d^2\boldsymbol{\rho}_2$$
$$\times\exp\left[-|\boldsymbol{\rho}_0|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right)-|\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}\right)-|\boldsymbol{\rho}_0'|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right)\right.$$
$$\left.-|\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}\right)\right]$$
$$\times\exp\left[\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(-\frac{k_1}{L}\right)+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_1}{L}\right)+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right]$$
$$\times\exp\left[-j|\boldsymbol{\rho}_0|^2\left(\frac{k_1}{2L}\right)-j|\boldsymbol{\rho}_1|^2\left(\frac{k_1}{L}\right)-j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right)-j|\boldsymbol{\rho}_2|^2\left(\frac{k_1}{L}\right)\right.$$
$$\left.+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(\frac{k_1}{L}\right)+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right)+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_1}{L}\right)+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right]\ ,\qquad(38)$$

$$M_2(\boldsymbol{\rho},u)=C_2\int\cdots\int_{-\infty}^{\infty}d^2\boldsymbol{\rho}_0\,d^2\boldsymbol{\rho}_1\,d^2\boldsymbol{\rho}_0'\,d^2\boldsymbol{\rho}_2\exp\left\{|\boldsymbol{\rho}|^2\left[\frac{k_2}{L}-\frac{(1-u)^2}{l^2}\right]\right\}$$
$$\times\exp\left\{-|\boldsymbol{\rho}_0|^2\left[\frac{1}{2w_0^2}-\frac{k_2}{2L}+\frac{(1-u)^2}{l^2}\right]-|\boldsymbol{\rho}_1|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}\right)\right.$$
$$-|\boldsymbol{\rho}_0'|^2\left(\frac{1}{2w_0^2}-\frac{k_2}{2L}\right)-|\boldsymbol{\rho}_2|^2\left(-\frac{k_2}{L}+\frac{u^2}{l^2}\right)+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left[-\frac{k_2}{L}-\frac{2u(1-u)}{l^2}\right]$$
$$+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left[-\frac{k_2}{L}+\frac{2u(1-u)}{l^2}\right]+\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2\left(\frac{2u^2}{l^2}\right)+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0\left[\frac{2u(1-u)}{l^2}\right]$$
$$\left.+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_2}{L}\right)+\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left[-\frac{k_2}{L}-\frac{2u(1-u)}{l^2}\right]+\boldsymbol{\rho}_0\cdot\boldsymbol{\rho}\left[\frac{2(1-u)^2}{l^2}\right]\right\}$$
$$\times\exp\left[-j|\boldsymbol{\rho}_0|^2\left(\frac{k_1}{2L}\right)-j|\boldsymbol{\rho}_1|^2\left(\frac{k_1}{L}\right)-j|\boldsymbol{\rho}_0'|^2\left(\frac{k_1}{2L}\right)-j|\boldsymbol{\rho}_2|^2\left(\frac{k_1}{L}\right)\right.$$
$$\left.+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0\left(\frac{k_1}{L}\right)+j\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}\left(\frac{k_1}{L}\right)+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0'\left(-\frac{k_1}{L}\right)+j\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}\left(-\frac{k_1}{L}\right)\right]\ ,\qquad(39)$$

$$M_3(\boldsymbol{\rho},u)=M_2(\boldsymbol{\rho},u)\ ,\qquad(40)$$

$$\begin{aligned}M_4(\boldsymbol{\rho},u,u') &= C_3 \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
&\times \exp\left\{ |\boldsymbol{\rho}|^2 \left[\frac{k_2}{L} - \frac{(1-u)^2}{l^2} - \frac{(1-u')^2}{l^2} \right] \right\} \\
&\times \exp\left\{ -|\boldsymbol{\rho}_0|^2 \left[\frac{1}{2w_0^2} - \frac{k_2}{2L} + \frac{(1-u)^2}{l^2} \right] - |\boldsymbol{\rho}_1|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} + \frac{u'^2}{l^2} \right) \right. \\
&\quad - |\boldsymbol{\rho}_0'|^2 \left[\frac{1}{2w_0^2} - \frac{k_2}{2L} + \frac{(1-u')^2}{l^2} \right] - |\boldsymbol{\rho}_2|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} + \frac{u'^2}{l^2} \right) \\
&\quad + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left[-\frac{k_2}{L} - \frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0' \left[\frac{2u'(1-u')}{l^2} \right] \\
&\quad + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left[-\frac{k_2}{L} + \frac{2u(1-u)}{l^2} - \frac{2u'(1-u')}{l^2} \right] + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2 \left(\frac{2u^2}{l^2} + \frac{2u'^2}{l^2} \right) \\
&\quad + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0 \left[\frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left[-\frac{k_2}{L} - \frac{2u'(1-u')}{l^2} \right] \\
&\quad + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left[-\frac{k_2}{L} - \frac{2u(1-u)}{l^2} + \frac{2u'(1-u')}{l^2} \right] \\
&\quad \left. + \boldsymbol{\rho}_0 \cdot \boldsymbol{\rho} \left[\frac{2(1-u)^2}{l^2} \right] + \boldsymbol{\rho}_0 \cdot \boldsymbol{\rho} \left[\frac{2(1-u')^2}{l^2} \right] \right\} \\
&\times \exp\left[-j|\boldsymbol{\rho}_0|^2 \left(\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_1|^2 \left(\frac{k_1}{L} \right) - j|\boldsymbol{\rho}_0'|^2 \left(-\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_2|^2 \left(-\frac{k_1}{L} \right) \right. \\
&\quad \left. + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left(-\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(-\frac{k_1}{L} \right) \right] ,\end{aligned} \tag{41}$$

$$M_1'(\boldsymbol{\rho}) = M_1(\boldsymbol{\rho}) , \tag{42}$$

$$\begin{aligned}M_2'(\boldsymbol{\rho},u) &= C_2 \exp\left[|\boldsymbol{\rho}|^2 \left(\frac{k_2}{L} \right) \right] \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
&\times \exp\left[-|\boldsymbol{\rho}_0|^2 \left(\frac{1}{2w_0^2} - \frac{k_2}{2L} \right) - |\boldsymbol{\rho}_1|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} \right) - |\boldsymbol{\rho}_0'|^2 \left(\frac{1}{2w_0^2} - \frac{k_2}{2L} \right) \right. \\
&\quad - |\boldsymbol{\rho}_2|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} \right) + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left(-\frac{k_2}{L} \right) + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(-\frac{k_2}{L} \right) \\
&\quad \left. + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2 \left(\frac{2u^2}{l^2} \right) + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left(-\frac{-k_2}{L} \right) + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(\frac{-k_2}{L} \right) \right] \\
&\times \exp\left[-j|\boldsymbol{\rho}_0|^2 \left(\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_1|^2 \left(\frac{k_1}{L} \right) - j|\boldsymbol{\rho}_0'|^2 \left(\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_2|^2 \left(\frac{k_1}{L} \right) \right. \\
&\quad \left. + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left(-\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(-\frac{k_1}{L} \right) \right] ,\end{aligned} \tag{43}$$

$$\begin{aligned}M_3'(\boldsymbol{\rho},u) &= C_2 \exp\left[|\boldsymbol{\rho}|^2 \left(\frac{k_2}{L} \right) \right] \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
&\times \exp\left\{ -|\boldsymbol{\rho}_0|^2 \left[\frac{1}{2w_0^2} - \frac{k_2}{2L} + \frac{(1-u)^2}{l^2} \right] - |\boldsymbol{\rho}_1|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} \right) \right. \\
&\quad - |\boldsymbol{\rho}_0'|^2 \left[\frac{1}{2w_0^2} - \frac{k_2}{2L} + \frac{(1-u)^2}{l^2} \right] - |\boldsymbol{\rho}_2|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} \right) \\
&\quad + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left[-\frac{k_2}{L} - \frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0' \left[\frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(-\frac{k_2}{L} \right) \\
&\quad + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2 \left(\frac{2u^2}{l^2} \right) + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0 \left[\frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left[-\frac{k_2}{L} - \frac{2u(1-u)}{l^2} \right] \\
&\quad \left. + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(-\frac{k_2}{L} \right) + \boldsymbol{\rho}_0 \cdot \boldsymbol{\rho}_0' \left[\frac{2(1-u)^2}{l^2} \right] \right\} \\
&\times \exp\left[-j|\boldsymbol{\rho}_0|^2 \left(\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_1|^2 \left(\frac{k_1}{L} \right) - j|\boldsymbol{\rho}_0'|^2 \left(-\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_2|^2 \left(-\frac{k_1}{L} \right) \right. \\
&\quad \left. + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left(-\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(-\frac{k_1}{L} \right) \right] ,\end{aligned} \tag{44}$$

$$\begin{aligned}M_4'(\boldsymbol{\rho},u,u') &= C_3 \exp\left[|\boldsymbol{\rho}|^2 \left(\frac{k_2}{L} \right) \right] \int \cdots \int_{-\infty}^{\infty} \mathrm{d}^2\boldsymbol{\rho}_0\, \mathrm{d}^2\boldsymbol{\rho}_1\, \mathrm{d}^2\boldsymbol{\rho}_0'\, \mathrm{d}^2\boldsymbol{\rho}_2 \\
&\times \exp\left\{ -|\boldsymbol{\rho}_0|^2 \left[\frac{1}{2w_0^2} - \frac{k_2}{2L} + \frac{(1-u)^2}{l^2} \right] - |\boldsymbol{\rho}_1|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} + \frac{u'^2}{l^2} \right) \right. \\
&\quad - |\boldsymbol{\rho}_0'|^2 \left[\frac{1}{2w_0^2} - \frac{k_2}{2L} + \frac{(1-u')^2}{l^2} \right] - |\boldsymbol{\rho}_2|^2 \left(-\frac{k_2}{L} + \frac{u^2}{l^2} + \frac{u'^2}{l^2} \right) \\
&\quad + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left[-\frac{k_2}{L} - \frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0' \left[\frac{2u(1-u)}{l^2} \right] \\
&\quad + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(-\frac{k_2}{L} \right) + \boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_2 \left(\frac{2u^2}{l^2} + \frac{2u'^2}{l^2} \right) + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0 \left[\frac{2u(1-u)}{l^2} \right] \\
&\quad \left. + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left[-\frac{k_2}{L} - \frac{2u(1-u)}{l^2} \right] + \boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(-\frac{k_2}{L} \right) + \boldsymbol{\rho}_0 \cdot \boldsymbol{\rho}_0' \left[\frac{2(1-u)^2}{l^2} \right] \right\} \\
&\times \exp\left[-j|\boldsymbol{\rho}_0|^2 \left(\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_1|^2 \left(\frac{k_1}{L} \right) - j|\boldsymbol{\rho}_0'|^2 \left(\frac{k_1}{2L} \right) - j|\boldsymbol{\rho}_2|^2 \left(-\frac{k_1}{L} \right) \right. \\
&\quad \left. + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho}_0 \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_1 \cdot \boldsymbol{\rho} \left(\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho}_0' \left(-\frac{k_1}{L} \right) + j\boldsymbol{\rho}_2 \cdot \boldsymbol{\rho} \left(-\frac{k_1}{L} \right) \right] ,\end{aligned} \tag{45}$$

where $\boldsymbol{\rho}_0$, $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_0'$, and $\boldsymbol{\rho}_2$ are the transversal coordinates.

To finish the integrations with respect to the transversal coordinates of P_j, P_j', M_j, and M_j', $j=1,\ldots,4$, we introduce a formula given by

$$\int \cdots \int_{-\infty}^{\infty} \exp(-a_1|\boldsymbol{\rho}_0|^2 - b_1|\boldsymbol{\rho}_1|^2 - c_1|\boldsymbol{\rho}_0'|^2 - d_1|\boldsymbol{\rho}_2|^2 + e_1\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0$$
$$+ f_1\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0' + g_1\boldsymbol{\rho}_1\cdot\boldsymbol{\rho} + h_1\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_2 + i_1\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0$$
$$+ j_1\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0' + k_1\boldsymbol{\rho}_2\cdot\boldsymbol{\rho} + l_1\boldsymbol{\rho}_0\cdot\boldsymbol{\rho}_0' + o_1\boldsymbol{\rho}\cdot\boldsymbol{\rho}_0'$$
$$+ p_1\boldsymbol{\rho}\cdot\boldsymbol{\rho}_0)\ \exp[-ja_2|\boldsymbol{\rho}_0|^2 - jb_2|\boldsymbol{\rho}_1|^2 - jc_2|\boldsymbol{\rho}_0'|^2 - jd_2|\boldsymbol{\rho}_2|^2 + je_2\boldsymbol{\rho}_1\cdot\boldsymbol{\rho}_0$$
$$+ jg_2\boldsymbol{\rho}_1\cdot\boldsymbol{\rho} + jj_2\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}_0' + jk_2\boldsymbol{\rho}_2\cdot\boldsymbol{\rho}]$$
$$= \frac{\pi^4}{(a_1^2+a_2^2)^{1/2}(b_3^2+b_4^2)^{1/2}(c_5^2+c_6^2)^{1/2}(d_7^2+d_8^2)^{1/2}} \exp\left\{|\boldsymbol{\rho}|^2\left[\frac{a_1p_1^2}{4(a_1^2+a_2^2)}\right.\right.$$
$$+ \frac{b_3g_3^2 - b_3g_4^2 + 2b_4g_3g_4}{4(b_3^2+b_4^2)} + \frac{c_5o_5^2 - c_5o_6^2 + 2c_6o_5o_6}{4(c_5^2+c_6^2)}$$
$$\left.\left. + \frac{d_7k_7^2 - d_7k_8^2 + 2d_8k_7k_8}{4(d_7^2+d_8^2)}\right]\right\}$$
$$\times \exp\left[-j\left(\tan^{-1}\frac{a_2}{a_1} + \tan^{-1}\frac{b_4}{b_3} + \tan^{-1}\frac{c_6}{c_5} + \tan^{-1}\frac{d_8}{d_7}\right)\right]$$
$$\times \exp\left\{-j|\boldsymbol{\rho}|^2\left[\frac{a_2p_1^2}{4(a_1^2+a_2^2)} + \frac{b_4g_3^2 - b_4g_4^2 - 2b_3g_3g_4}{4(b_3^2+b_4^2)}\right.\right.$$
$$\left.\left. + \frac{c_6o_5^2 - c_6o_6^2 - 2c_5o_5o_6}{4(c_5^2+c_6^2)} + \frac{d_8k_7^2 - d_8k_8^2 + 2d_7k_7k_8}{4(d_7^2+d_8^2)}\right]\right\}\ , \tag{46}$$

where

$$b_3 = b_1 - \frac{a_1e_1^2 - a_1e_2^2 + 2a_2e_1e_2}{4(a_1^2+a_2^2)}\ , \tag{47}$$

$$c_3 = c_1 - \frac{a_1l_1^2}{4(a_1^2+a_2^2)}\ , \tag{48}$$

$$d_3 = d_1 - \frac{a_1i_1^2}{4(a_1^2+a_2^2)}\ , \tag{49}$$

$$f_3 = f_1 + \frac{a_1e_1l_1 + a_2e_2l_1}{2(a_1^2+a_2^2)}\ , \tag{50}$$

$$g_3 = g_1 + \frac{a_1e_1p_1 + a_2e_2p_1}{2(a_1^2+a_2^2)}\ , \tag{51}$$

$$h_3 = h_1 + \frac{a_1e_1i_1 + a_2e_2i_1}{2(a_1^2+a_2^2)}\ , \tag{52}$$

$$j_3 = j_1 + \frac{a_1i_1l_1}{2(a_1^2+a_2^2)}\ , \tag{53}$$

$$k_3 = k_1 + \frac{a_1i_1p_1}{2(a_1^2+a_2^2)}\ , \tag{54}$$

$$o_3 = o_1 + \frac{a_1l_1p_1}{2(a_1^2+a_2^2)}\ , \tag{55}$$

$$b_4 = b_2 + \frac{a_2e_1^2 - a_2e_2^2 - 2a_1e_1e_2}{4(a_1^2+a_2^2)}\ , \tag{56}$$

$$c_4 = c_2 + \frac{a_2l_1^2}{4(a_1^2+a_2^2)}\ , \tag{57}$$

$$d_4 = d_2 + \frac{a_2i_1^2}{4(a_1^2+a_2^2)}\ , \tag{58}$$

$$f_4 = \frac{a_1e_2l_1 - a_2e_1l_1}{2(a_1^2+a_2^2)}\ , \tag{59}$$

$$g_4 = g_2 - \frac{a_2e_1p_1 - a_1e_2p_1}{2(a_1^2+a_2^2)}\ , \tag{60}$$

$$h_4 = \frac{a_1e_2i_1 - a_2e_1i_1}{2(a_1^2+a_2^2)}\ , \tag{61}$$

$$j_4 = j_2 - \frac{a_2i_1l_1}{2(a_1^2+a_2^2)}\ , \tag{62}$$

$$k_4 = k_2 - \frac{a_2i_1p_1}{2(a_1^2+a_2^2)}\ , \tag{63}$$

$$o_4 = -\frac{a_2l_1p_1}{2(a_1^2+a_2^2)}\ , \tag{64}$$

$$c_5 = c_3 - \frac{b_3f_3^2 - b_3f_4^2 + 2b_4f_3f_4}{4(b_3^2+b_4^2)}\ , \tag{65}$$

$$d_5 = d_3 - \frac{b_3h_3^2 - b_3h_4^2 + 2b_4h_3h_4}{4(b_3^2+b_4^2)}\ , \tag{66}$$

$$j_5 = j_3 - \frac{b_3f_3h_3 - b_3f_4h_4 + b_4f_3h_4 + b_4f_4h_3}{2(b_3^2+b_4^2)}\ , \tag{67}$$

$$k_5 = k_3 + \frac{b_3g_3h_3 - b_3g_4h_4 + b_4g_3h_4 + b_4g_4h_3}{2(b_3^2+b_4^2)}\ , \tag{68}$$

$$o_5 = o_3 + \frac{b_3g_3f_3 - b_3g_4f_4 + b_4g_3f_4 + b_4g_4f_3}{2(b_3^2+b_4^2)}\ , \tag{69}$$

$$c_6 = c_4 + \frac{b_4f_3^2 - b_4f_4^2 + 2b_3f_3f_4}{4(b_3^2+b_4^2)}\ , \tag{70}$$

$$d_6 = d_4 + \frac{b_4h_3^2 - b_4h_4^2 - 2b_3h_3h_4}{4(b_3^2+b_4^2)}\ , \tag{71}$$

$$j_6 = j_4 + \frac{b_4f_4h_4 - b_4f_3h_3 + b_3f_3h_4 + b_3f_4h_3}{2(b_3^2+b_4^2)}\ , \tag{72}$$

$$k_6 = k_4 + \frac{b_4g_4h_4 - b_4g_3h_3 + b_3g_3h_4 + b_3g_4h_3}{2(b_3^2+b_4^2)}\ , \tag{73}$$

$$o_6 = o_4 + \frac{b_4g_4f_4 - b_4g_3f_3 + b_3g_3f_4 + b_3g_4f_3}{2(b_3^2+b_4^2)}\ , \tag{74}$$

$$d_7 = d_5 - \frac{c_5j_5^2 - c_5j_6^2 + 2c_6j_5j_6}{4(c_5^2+c_6^2)}\ , \tag{75}$$

$$k_7 = k_5 + \frac{c_5j_5o_5 - c_5j_6o_6 + c_6j_5o_6 + c_6j_6o_5}{2(c_5^2+c_6^2)}\ , \tag{76}$$

$$d_8 = d_4 + \frac{c_6 j_5^2 - c_6 j_6^2 - 2c_5 j_5 j_6}{4(c_5^2 + c_6^2)} , \tag{77}$$

and

$$k_8 = k_6 + \frac{c_6 j_5 o_6 - c_6 j_6 o_5 + c_5 j_5 o_6 + c_5 j_6 o_5}{2(c_5^2 + c_6^2)} , \tag{78}$$

Here, all the parameters in Eq. (46) are real numbers. To derive Eq. (46), we have used the integral table.[17] With Eq. (46), we can finish the eight-fold integration with respect to the transversal coordinates in the expressions of P_j, P_j', M_j, and M_j', where $j = 1,\ldots,4$, and have the final expressions for numerical computations for these functions. However, the expression for each function is too lengthy and complicated to be illustrated here.

Acknowledgment

This work was partly supported by the National Science Council of Taiwan under the project number NSC 810404E009546.

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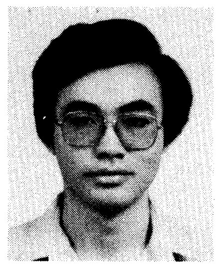

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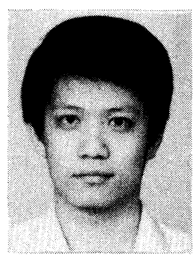

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