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SHYI-MING CHEN
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AGGREGATING FUZZY OPINIONS IN THE GROUP DECISION-MAKING ENVIRONMENT

SHYI-MING CHEN

Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

This paper presents a new method for dealing with fuzzy opinion aggregation in group decision-making problems. The proposed method has the following advantages: (1) The experts' estimates do not necessarily have a common intersection at α-level cuts, where $\alpha \in (0, 1]$. (2) It can perform fuzzy opinion aggregation in a more efficient manner. (3) It does not need to use the Delphi method to adjust trapezoidal fuzzy numbers given by experts.

Some researchers (Bardossy et al., 1993; Chen & Lin, 1995; Chen et al., 1989; Hsu & Chen, 1996; Ishikawa et al., 1993; Kacprzyk & Fedrizzi, 1988; Kacprzyk et al., 1992; Lee, 1996; Nurmi, 1981; Spillman et al., 1980; Tanino, 1984, 1990; Xu & Zhai, 1992; Chen, 1997) have focused on the fuzzy opinion aggregation problem in the multicriteria group decision-making (MCDM) environment based on fuzzy set theory (Zadeh, 1965) to combine the individual opinions of experts, where each expert usually has its own opinion or estimated rating under each criterion for each alternative. Thus, to find a group consensus function

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Address correspondence to Professor Shyi-Ming Chen, Department of Computer and Information Science, National Chiao Tung University Hsinchu, Taiwan, Republic of China. E-mail: symchen@cis.nctu.edu.tw

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for aggregating these estimates ratings to a common opinion is an important issue in handling multicriteria group decision-making problems. Because in multicriteria decision making with group decision-making problems, the estimates of experts of a criterion for an alternative may involve subjectiveness, imprecision, and vagueness, fuzzy set theory can provide us with a useful way to deal with the fuzziness of human judgments.

Kacprzyk et al. (1992) showed how fuzzy logic with linguistic quantifiers can be used in group decision making. Tanino (1984) discussed fuzzy preference orderings in group decision making. Bardossy et al. (1993) represented expert opinions or imprecise estimates of a physical variable by using fuzzy numbers and developed five techniques for combining these fuzzy numbers into a single fuzzy number estimate. The guidelines for the choice of combination technique are also provided in Bardossy et al. Ishikawa et al. (1993) proposed the max-min Delphi method and fuzzy Delphi method via fuzzy integration. Xu and Zhai (1992) presented extensions of the analytic hierarchy process in a fuzzy environment, where each expert represents it's subjective judgment by an interval value rating of each criterion for each alternative. Lee (1996) presented a method for group decision making using fuzzy set theory for evaluating the rate of aggregative risk in software development. Nurmi (1981) presented some approaches to collective decision making with fuzzy preference relations. Hsu and Chen (1996) presented a similarity aggregation method (SAM) for aggregating individual fuzzy opinions into a group fuzzy consensus opinion, where the estimates of experts are represented by positive trapezoidal fuzzy numbers.

However, there are some drawbacks of the method presented in Hsu and Chen (1996), shown as follows:

1. It requires that the experts' estimates have a common intersection at some α-level cut, where $\alpha \in (0, 1]$. If the initial estimates of the ith expert and the jth expert have no intersection, then it must use the Delphi method (Satty, 1980) or get more information to adjust the trapezoidal fuzzy number given by each expert to obtain a common intersection at the α-level cut. However, applying the Delphi method to adjust the trapezoidal fuzzy numbers given by the experts will take a large amount of time to perform the operations.
2. It requires a large amount of time to calculate the degree of agreement between experts' estimates because it uses a complicated

similarity measure function S to calculate the degree of agreement of the subjective estimate $\tilde{R}_i$ of expert E_i and subjective estimate $\tilde{R}_j$ of expert E_j,

$$S\left(\tilde{R}_i, \tilde{R}_j\right) = \frac{\int_u \left(\min\left\{f_{\tilde{R}_i}(u), f_{\tilde{R}_j}(u)\right\}\right) du}{\int_u \left(\max\left\{f_{\tilde{R}_i}(u), f_{\tilde{R}_j}(u)\right\}\right) du}$$

where $\tilde{R}_i$ and $\tilde{R}_j$ are positive trapezoidal fuzzy numbers and the membership functions of the trapezoidal fuzzy numbers $\tilde{R}_i$ and $\tilde{R}_j$ are $f_{\tilde{R}_i}$ and $f_{\tilde{R}_j}$, respectively.

Thus, it is necessary to develop a new method for dealing with the fuzzy opinion aggregation problem in a more flexible and more efficient manner.

In this paper, we present a new method for dealing with the fuzzy opinion aggregation problem. The proposed method can overcome the drawbacks of the one presented in Hsu and Chen (1996) due to the fact that

1. The experts' estimates do not necessarily have a common intersection at the α level, where $\alpha \in (0, 1]$. Thus, it is more flexible than the one presented in Hsu and Chen.
2. It does not need to use the Delphi method to adjust trapezoidal fuzzy numbers given by experts.
3. It can calculate the degree of similarity between the subjective estimates of experts in a more efficient manner. Thus, it can perform fuzzy opinion aggregation in a more efficient manner.

BASIC CONCEPTS OF FUZZY SET THEORY

The theory of fuzzy sets was proposed by Zadeh in 1965. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set $\tilde{A}$ of U is a set of ordered pairs $\{(u_1, f_{\tilde{A}}(u_1)), (u_2, f_{\tilde{A}}(u_2)), \ldots, (u_n, f_{\tilde{A}}(u_n))\}$, where $f_{\tilde{A}}$ is the membership function of the fuzzy set A, $f_{\tilde{A}}: U \rightarrow [0, 1]$, and $f_{\tilde{A}}(u_i)$ indicates the degree of membership of u_i in the fuzzy set $\tilde{A}$. A fuzzy set $\tilde{A}$ of the universe of discourse U is called a normal fuzzy set if $\exists u_i \in U$, $f_{\tilde{A}}(u_i) = 1$. If for all u_1, u_2 in U,

$$f_{\tilde{A}}(\lambda u_1 + (1 - \lambda)u_2) \geqslant \text{Min}(f_{\tilde{A}}(u_1), f_{\tilde{A}}(u_2)) \tag{1}$$

then the fuzzy set $\tilde{A}$ is called a convex fuzzy set. A fuzzy number is a fuzzy subset in the universe of discourse U that is both normal and convex. For example, Figure 1 shows a fuzzy number $\tilde{A}$ of the universe of discourse U. A standardized fuzzy number is a fuzzy number defined in the universe of discourse U, where $U = [0, 1]$.

A trapezoidal fuzzy number $\tilde{M}$ of the universe of discourse U can be characterized by a quadruple (a, b, c, d) shown in Figure 2.

In the following, we briefly review the defuzzification technique of trapezoidal fuzzy numbers from Chen (1994, 1996) and Kauffman and Gupta (1988). Consider the trapezoidal fuzzy number $\tilde{M}$ shown in Figure 3, where e is the defuzzification value of the trapezoidal fuzzy number $\tilde{M}$. From Figure 3, we can see that

$$\tfrac{1}{2}(b - a)(1) + (e - b)(1) = \tfrac{1}{2}(d - c)(1) + (c - e)(1)$$

$$\Rightarrow \tfrac{1}{2}(b - a)(1) + (e - b) = \tfrac{1}{2}(d - c) + (c - e)$$

$$\Rightarrow (e - b) - (c - e) = \tfrac{1}{2}(d - c) - \tfrac{1}{2}(b - a)$$

$$\Rightarrow 2e = \tfrac{1}{2}(d - c) - \tfrac{1}{2}(b - a) + b + c$$

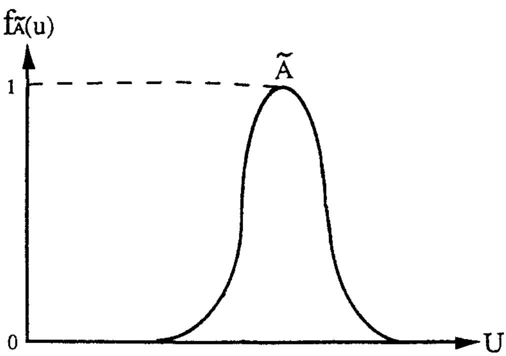

Figure 1. A fuzzy number.

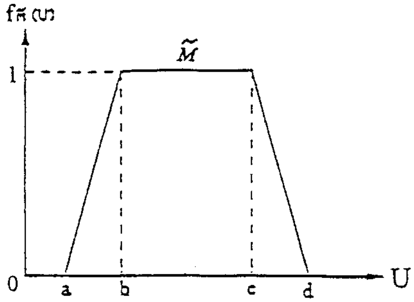

Figure 2. A trapezoidal fuzzy number.

$$\Rightarrow 2e = \frac{d - c - b + a + 2b + 2c}{2}$$

$$= \frac{a + b + c + d}{2}$$

$$\Rightarrow e = \frac{a + b + c + d}{4} \qquad (2)$$

SIMILARITY MEASURES

Zwich et al. (1987) have made a comparative analysis of 19 similarity measures among fuzzy sets. In Chen and Lin (1995) we made a comparison of similarity of fuzzy sets. In the following, we introduce a method for measuring the degree of similarity between trapezoidal fuzzy num-

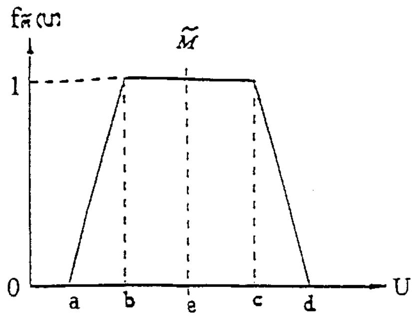

Figure 3. Defuzzification of a trapezoidal fuzzy number.

bers (Chen & Lin, 1995). Let $\hat{A}$ and $\hat{B}$ be two standardized trapezoidal fuzzy numbers,

$$\hat{A} = (a_1, b_1, c_1, d_1)$$

$$\hat{B} = (a_2, b_2, c_2, d_2)$$

where $0 \leqslant a_1 \leqslant b_1 \leqslant c_1 \leqslant d_1 \leqslant 1$ and $0 \leqslant a_2 \leqslant b_2 \leqslant c_2 \leqslant d_2 \leqslant 1$. Then the degree of similarity between the standardized trapezoidal fuzzy numbers $\hat{A}$ and $\hat{B}$ can be measured by the similarity function S,

$$S(\hat{A}, \hat{B}) = 1 - \frac{|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|}{4} \tag{3}$$

where $S(\hat{A}, \hat{B}) \in [0, 1]$. The larger the value of $S(\hat{A}, \hat{B})$, the greater the similarity between the standardized trapezoidal fuzzy numbers $\hat{A}$ and $\hat{B}$.

Let $\hat{A}$ and $\hat{B}$ be two standardized trapezoidal fuzzy numbers,

$$\hat{A} = (a_1, b_1, c_1, d_1)$$

$$\hat{B} = (a_2, b_2, c_2, d_2)$$

where $0 \leqslant a_1 \leqslant b_1 \leqslant c_1 \leqslant d_1 \leqslant 1$ and $0 \leqslant a_2 \leqslant b_2 \leqslant c_2 \leqslant d_2 \leqslant 1$. Then it is obvious that $S(\hat{A}, \hat{B}) = S(\hat{B}, \hat{A})$.

Let x and y be two real values, where $x \in [0, 1]$ and $y \in [0, 1]$. It is obvious that the real values x and y can be represented by standardized trapezoidal fuzzy numbers x and y, respectively, where $x = (x, x, x, x)$ and $y = (y, y, y, y)$. By applying formula (3), the degree of similarity between the real values x and y can be evaluated as follows:

$$\begin{aligned} S(x, y) &= 1 - \frac{|x - y| + |x - y| + |x - y| + |x - y|}{4} \\ &= 1 - |x - y| \end{aligned} \tag{4}$$

It is obvious that this result coincides with the one shown in Chen et al. (1989).

Let $\tilde{A}$ be a positive trapezoidal fuzzy number in the universe of discourse U, where

$$U = [0, m]$$

$$\tilde{A} = (a, b, c, d)$$

and $0 \leqslant a \leqslant b \leqslant c \leqslant d \leqslant m$. Then, the positive trapezoidal fuzzy number $\tilde{A}$ can be translated into the standardized trapezoidal fuzzy number $\hat{A}$ shown as follows:

$$\hat{A} = \left(\frac{a}{m}, \frac{b}{m}, \frac{c}{m}, \frac{d}{m}\right)$$

where $0 \leqslant a/m \leqslant b/m \leqslant c/m \leqslant d/m \leqslant 1$ and the membership function curve of the standardized trapezoidal fuzzy number $\hat{A}$ is as shown in Figure 4. In this case, the standardized trapezoidal fuzzy number $\hat{A}$ is defined in the universe of discourse U, where $U = [0, 1]$.

A NEW METHOD FOR HANDLING FUZZY OPINION AGGREGATION PROBLEMS

In the following, we present a new method for handling fuzzy opinion aggregation problems. The algorithm essentially is a modification of the one presented in Hsu and Chen (1996). Let U be the universe of discourse, $U = [0, m]$. Assume that each expert E_i $(i = 1, 2, \ldots, n)$ constructs a positive trapezoidal fuzzy number $\tilde{R}_i = (a_i, b_i, c_i, d_i)$ to represent the subjective estimate of the rating to a given criterion and

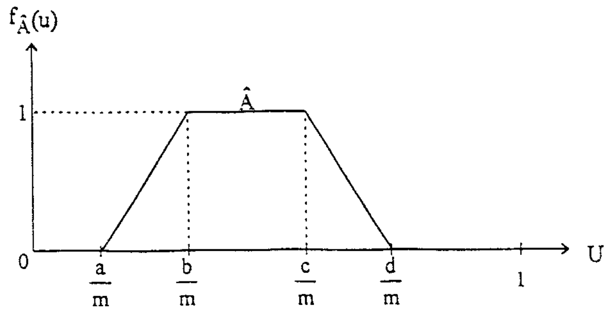

Figure 4. A standardized trapezoidal fuzzy number A.

alternative, where $0 \leq a_i \leq b_i \leq c_i \leq d_i \leq m$. Furthermore, assume that the degree of importance of export E_i $(i = 1, 2, \ldots, n)$ is w_i, where $w_i \in [0, 1]$ and $\Sigma_{i=1}^{n} w_i = 1$. The algorithm is presented as follows:

Step 1: Translate each trapezoidal fuzzy number $\tilde{R}_i = (a_i, b_i, c_i, d_i)$ given by expert E_i into standardized trapezoidal fuzzy number $\hat{R}_i$ $(i = 1, 2, \ldots, n)$, where

$$\hat{R}_i = \left(\frac{a_i}{m}, \frac{b_i}{m}, \frac{c_i}{m}, \frac{d_i}{m}\right)$$

$$= (a_i^*, b_i^*, c_i^*, d_i^*)$$

and $0 \leq a_i^* \leq b_i^* \leq c_i^* \leq d_i^* \leq 1$.

Step 2: Based on formula (3), calculate the degree of agreement $S(\hat{R}_i, \hat{R}_j)$ of the opinions between each pair of experts E_i and E_j, where $S(\hat{R}_i, \hat{R}_j) \in [0, 1]$, $1 \leq i \leq n$, $1 \leq j \leq n$, and $i \neq j$.

Step 3: Calculate the average degree of agreement $A(E_i)$ of expert E_i $(i = 1, 2, \ldots, n)$, where

$$A(E_i) = \frac{1}{n - 1} \sum_{\substack{j=1 \\ j \neq i}}^{n} S\left(\hat{R}_i, \hat{R}_j\right) \tag{6}$$

Step 4: Calculate the relative degree of agreement $RA(E_i)$ of expert E_i $(i = 1, 2, \ldots, n)$, where

$$RA(E_i) = \frac{A(E_i)}{\Sigma_{i=1}^{n} A(E_i)} \tag{7}$$

Step 5: Assume that the weight of the degrees of importance of the experts and the weight of the relative degree of agreement of the experts are y_1 and y_2, respectively, where $y_1 \in [0, 1]$ and $y_2 \in [0, 1]$. Calculate the consensus degree coefficient $C(E_i)$ of expert E_i $(i = 1, 2, \ldots, n)$, where

$$C(E_i) = \frac{y_1}{y_1 + y_2} * w_i + \frac{y_2}{y_1 + y_2} * RA(E_i). \tag{8}$$

Step 6: The aggregation result of the fuzzy opinions is $\tilde{R}$, where

$$\tilde{R} = C(E_1) \otimes R_1 \oplus C(E_2) \otimes R_2 \oplus \cdots \oplus C(E_n) \otimes R_n \tag{9}$$

operators $\otimes$ and $\oplus$ are the fuzzy multiplication operator and the fuzzy addition operator, respectively.

In the following, we use an example to illustrate the fuzzy opinion aggregation process.

Example: Assume that experts E_1, E_2, and E_3 construct positive trapezoidal fuzzy numbers $\tilde{R}_1$, $\tilde{R}_2$, and $\tilde{R}_3$ to represent the subjective estimate of the rating to a given criterion and alternative, respectively, where

$$\tilde{R}_1 = (1, 2, 3, 4)$$

$$\tilde{R}_2 = (4, 5, 6, 7)$$

$$\tilde{R}_3 = (7, 8, 9, 10)$$

Assume that the trapezoidal fuzzy numbers $\tilde{R}_1$, $\tilde{R}_2$, and $\tilde{R}_3$ are defined on the universe of discourse U, where $U = [0, 10]$ (i.e., $m = 10$), and assume that the weights of the experts E_1, E_2, and E_3 are 0.4, 0.4, and 0.2, respectively (i.e., $w_1 = 0.4$, $w_2 = 0.4$, and $w_3 = 0.2$). Furthermore, assume that the weight of the degrees of importance of the experts and the weight of the relative degrees of agreement of the experts are 0.9 and 0.6, respectively (i.e., $y_1 = 0.9$ and $y_2 = 0.6$). Based on the proposed algorithm, we can get the following result:

[Step 1] Because $m = 10$, we can translate the trapezoidal fuzzy numbers $\tilde{R}_1$, $\tilde{R}_2$, and $\tilde{R}_3$ into the standardized trapezoidal fuzzy numbers $\hat{R}_1$, $\hat{R}_2$, and $\hat{R}_3$, respectively, where

$$\hat{R}_1 = \left(\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right)$$

$$= (0.1, 0.2, 0.3, 0.4)$$

$$\hat{R}_2 = \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}\right)$$

$$= (0.4, 0.5, 0.6, 0.7)$$

$$\hat{R}_3 = \left(\frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}\right)$$

$$= (0.7, 0.8, 0.9, 1.0)$$

[Step 2] Based on formula (3), we can get the following results:

$$S(\hat{R}_1, \hat{R}_2) = 1 - \frac{|0.1 - 0.4| + |0.2 - 0.5| + |0.3 - 0.6| + |0.4 - 0.7|}{4}$$

$$= 0.7$$

$$S(\hat{R}_2, \hat{R}_1) = 0.7$$

$$S(\hat{R}_1, \hat{R}_3) = 1 - \frac{|0.1 - 0.7| + |0.2 - 0.8| + |0.3 - 0.9| + |0.4 - 1.0|}{4}$$

$$= 0.4$$

$$S(\hat{R}_3, \hat{R}_1) = 0.4$$

$$S(\hat{R}_2, \hat{R}_3) = 1 - \frac{|0.4 - 0.7| + |0.5 - 0.8| + |0.6 - 0.9| + |0.7 - 1.0|}{4}$$

$$= 0.7$$

$$S(\hat{R}_3, \hat{R}_2) = 0.7$$

[Step 3] Based on formula (6), the degrees of agreement of experts E_1, E_2, and E_3 can be evaluated and are equal to $A(E_1)$, $A(E_2)$, and

$A(E_3)$, respectively, where

$$A(E_1) = \frac{0.7 + 0.4}{2} = 0.35$$

$$A(E_2) = \frac{0.7 + 0.7}{2} = 0.7$$

$$A(E_3) = \frac{0.4 + 0.7}{2} = 0.35$$

[Step 4] Based on formula (7), the relative degrees of agreement of experts E_1, E_2, and E_3 can be evaluated and are equal to $RA(E_1)$, $RA(E_2)$, and $RA(E_3)$, respectively, where

$$RA(E_1) = \frac{0.35}{0.35 + 0.7 + 0.35} = 0.25$$

$$RA(E_2) = \frac{0.7}{0.35 + 0.7 + 0.35} = 0.5$$

$$RA(E_3) = \frac{0.35}{0.35 + 0.7 + 0.35} = 0.25$$

[Step 5] Because the weights of the experts E_1, E_2, and E_3 are 0.4, 0.4, and 0.2, respectively (i.e., $w_1 = 0.4$, $w_2 = 0.4$, $w_3 = 0.2$), and because the weight of the degrees of importance of the experts and the weight of the relative degrees of agreement of the experts are 0.9 and 0.6, respectively (i.e., $y_1 = 0.9$ and $y_2 = 0.6$), then based on formula (8) we can get the following results:

$$C(E_1) = \frac{0.9}{0.9 + 0.6} * 0.4 + \frac{0.6}{0.9 + 0.6} * 0.25$$
$$= 0.34$$

$$C(E_2) = \frac{0.9}{0.9 + 0.6} * 0.4 + \frac{0.6}{0.9 + 0.6} * 0.5$$
$$= 0.44$$

$$C(E_3) = \frac{0.9}{0.9 + 0.6} * 0.2 + \frac{0.6}{0.9 + 0.6} * 0.25$$

$$= 0.22$$

[Step 6] Based on formula (9), we can see that the aggregation result of the fuzzy opinions is the trapezoidal fuzzy number $\tilde{R}$, where

$$\tilde{R} = C(E_1) \otimes \tilde{R}_1 \oplus C(E_2) \otimes \tilde{R}_2 \oplus C(E_3) \otimes \tilde{R}_3$$

$$= 0.34 \otimes (1, 2, 3, 4) \oplus 0.44 \otimes (4, 5, 6, 7)$$

$$\oplus 0.22 \otimes (7, 8, 9, 10)$$

$$= (0.34, 0.68, 1.02, 1.36) \oplus (1.76, 2.2, 2.64, 3.08)$$

$$\oplus (1.54, 1.76, 1.98, 2.2)$$

$$= (3.64, 4.64, 5.64, 6.64)$$

The membership function curves of the trapezoidal fuzzy numbers $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{R}_3$ and the aggregation result $\tilde{R}$ are shown in Figure 5.

CONCLUSIONS

In this paper, we have extended the work of Hsu and Chen (1996) to propose a new method for dealing with fuzzy opinion aggregation with group decision-making problems. From the illustrative example pre-

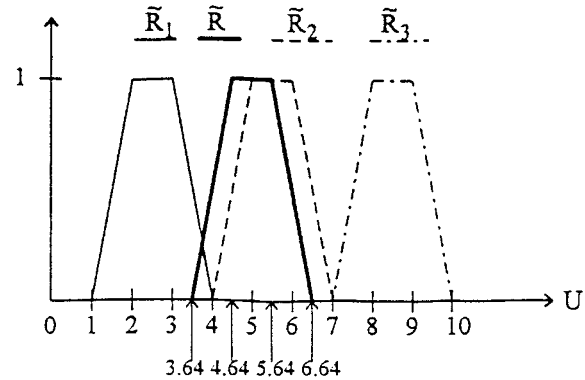

Figure 5. Membership functions of R_1, R_2, R_3, and R.

sented previously, we can see that the proposed method can overcome the drawbacks of the one presented in Hsu and Chen (1996) because

1. The experts' estimates do not necessarily have a common intersection at α-level cuts, where $\alpha \in (0, 1]$. Thus, it is more flexible than the one presented in Hsu and Chen.
2. It can calculate the degree of similarity between the subjective estimates of experts in a more efficient manner. Thus, it can perform fuzzy opinion aggregation in a more efficient manner.
3. It does not need to use the Delphi method to adjust trapezoidal fuzzy numbers given by experts.

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