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A VARIABLE POWER SYSTEM WITH HOLOGRAPHIC OPTICAL ELEMENTS

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SUMMARY : Based on the diffraction properties of a hololens, the principle of a variable power system is described. The important factors of the system such as cross-talk, aberrations and diffraction efficiency, are discussed. Then an optimum condition for the configuration of a symmetrical off-axis type is derived. It is achieved by combining a Leith-Upatnieks hololens and a Brandt hololens. In order to demonstrate its feasibility, a variable power system is fabricated, and its resolution is evaluated to be better than 30 μm.

Un dispositif à focale variable avec des composants holographiques

RÉSUMÉ : On décrit le principe d'un dispositif à focale variable fondé sur les propriétés de diffraction d'une lentille holographique. Les caractéristiques importantes du système telles que le taux de réjection entre ordres, les aberrations et l'efficacité de diffraction sont discutées. On en déduit une configuration optimale dans une structure symétrique hors d'axe. Celle-ci est réalisée en combinant une lentille holographique de type Leith-Upathieks et une de type Brandt. On réalise un système de ce type pour montrer sa faisabilité, et une résolution meilleure que 30 μm est observée.

1. — *INTRODUCTION*

Although holographic optical elements (HOEs) and conventional optical elements (COEs) utilize different fashions to bend light rays, they have been used to perform similar optical functions. COEs bend light rays by reflection or refraction, and HOEs are through diffraction. Unlike COEs, which require tedious processes for their production, it is simpler and cheaper to manufacture HOEs. In addition, HOEs are compact and light. Therefore, HOEs are emerging as viable alternatives to COEs in some special elements, such as zone plates [1], lenses [2], achromats [3], and beam-splitters [4]. There are also some optical systems with two holographic lenses (hololens) [5], [6], [7]. However, the focal power of all these systems are fixed, and thus their applications are limited.

To improve the limitation, a variable power system is presented. It is based on the diffraction properties of a hololens. The focal power is varied by adjusting the position of one of the two composite hololens. In order to inspect its characteristics, the important factors such as cross-talk problem, aberrations and diffraction efficiency are discussed. Then, an optimum condition for the configuration of a symmetrical off-axis type is derived. It is achieved by combining a Lieth-Upatnieks hololens and a Brandt hololens. In order to demonstrate its feasibility, a variable power system is manufactured. In addition, its resolution is evaluated with a resolution test chart. The result shows its resolution is better than 30 μm. This system can be applied to bandwidth filtering, coherent signal transformation and multi-signal output.

2. — THE PRINCIPLE OF A VARIABLE POWER SYSTEM

The diffraction properties of an HOE depend on the geometries of recording and reconstruction. The relevant coordinates (R_q, α_q) are the positions of the point sources ; where R_q $(q = o, r, c, i\ )$ are the distances from the point sources of (object, reference, reconstruction, image) to the center of the hologram, α_q are the off-axis angles of the waves. The other revelant parameters, ϕ_q denote the phases of the waves at the hologram plane. Based on the paraxial wave approximation, the relevant readout equations for an HOE are given by [8]

$$\phi_i = \phi_c \pm \mu (\phi_o - \phi_r) , \tag{1}$$

$$\frac{1}{R_i} = \frac{1}{R_c} \pm \mu \left(\frac{1}{R_o} - \frac{1}{R_r} \right) , \tag{2}$$

$$\sin \alpha_i = \sin \alpha_c \pm \mu (\sin \alpha_o - \sin \alpha_r) , \tag{3}$$

and

$$F = \frac{1}{f} = \mu \left(\frac{1}{R_o} - \frac{1}{R_r} \right) ; \tag{4}$$

where $\mu = \frac{\lambda_c}{\lambda_r}$ is the ratio between the reconstruction and recording wavelength, and is assumed to be unity in the present analysis. The $\pm$ refers to $+1$ and -1 orders of the diffracted images of the hologram. Here, only the real image is of interest, and the minus sign is used hereafter instead of the ± 1 in the equations. F and f is the hologram focal power and focal length, respectively. Moreover, the aberrations introduced by hololens can be written as [9]

$$\Delta = \phi_i - \phi_d = \frac{2\pi}{\lambda} \left(-\frac{1}{8} \rho^4 S + \frac{1}{2} \rho^3 C - \frac{1}{2} \rho^2 A \right) \tag{5}$$

where ρ is the effective recording radius on the hologram, and S, C and A are the coefficients of spherical aberration, coma and astigmatism, respectively. They are given by

$$S = \frac{1}{R_c^3} - \frac{1}{R_i^3} - \left(\frac{1}{R_o^3} - \frac{1}{R_r^3} \right) , \tag{6}$$

$$C = \frac{\sin \alpha_c}{R_c^2} - \frac{\sin \alpha_i}{R_i^2} - \left(\frac{\sin \alpha_o}{R_o^2} - \frac{\sin \alpha_r}{R_r^2} \right) , \tag{7}$$

and

$$A = \frac{\sin^2 \alpha_c}{R_c} - \frac{\sin^2 \alpha_i}{R_i} - \left(\frac{\sin^2 \alpha_o}{R_o} - \frac{\sin^2 \alpha_r}{R_r} \right) . \tag{8}$$

Based on the above formulas of the diffraction properties of a hololens, the principle of a variable power system will be described in the following.

The basic schematic diagram of a variable power system, which consists of two hololenses HL_1 and HL_2, is shown in *figure 1*. For convenience, HL_1 is fixed and only HL_2 can be moved. So, the distance d between HL_1 and HL_2 may be changed. Here, HL_1 transforms the input plane wave into an ideal spherical wave, and this output is used to reconstruct HL_2. Because the position of HL_2 is changeable, thus the reconstruction light may be slightly different from the recording light. Therefore, the focal power can be varied, but there will be with some aberrations.

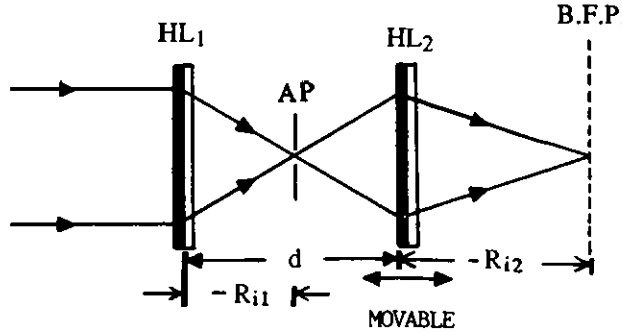

FIG. 1. — *The basic schematic diagram of a variable power system with holographic optical elements. AP : aperture, B.F.P. : back focal plane, HL : hololens.*

The curvature of the wave diffracted from HL_1 and HL_2 can be derived from Eq. (2) and written as

$$\frac{1}{R_{i1}} = - \left(\frac{1}{R_{o1}} - \frac{1}{R_{r1}} \right) , \tag{9}$$

and

$$\frac{1}{R_{i2}} = \frac{1}{R_{i1} + d} - \left(\frac{1}{R_{o2}} - \frac{1}{R_{r2}} \right) , \tag{10}$$

respectively. Obviously, R_{i2} is the back focal length of the system. Hence the back focal power and the effective focal power can be written as

$$F_{\text{b.f.p.}} = -\frac{1}{R_{i2}} = -\frac{1}{R_{i1} + d} + F_2 , \tag{11}$$

and

$$F_{\text{e.f.p.}} = (1 - dF_1) F_{\text{b.f.p.}} , \tag{12}$$

respectively. From Eq. (11) and Eq. (12), it is clear that the focal power of this system depends on d.

3. — DESIGN AND ANALYSIS

Although some important factors such as cross-talk, aberrations and diffraction efficiency are not concerned with the focal power directly, they can influence seriously on the optical qualities of the system. They will be discussed in the following and an optimum configuration will be derived.

3. — 1. Cross-talk

When the output signal superposes on the directly transmitted light, the system is said to have a cross-talk. It will make the signal-to-noise ratio (SNR)

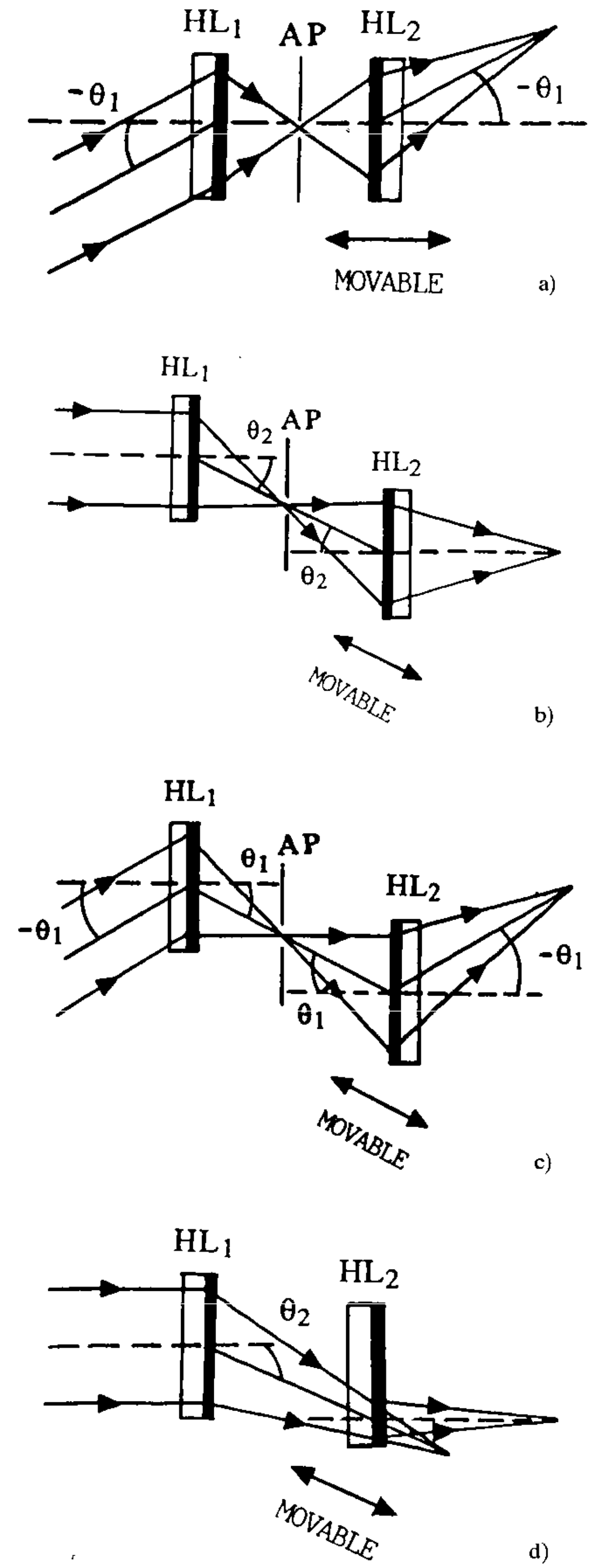

FIG. 2. — *The four possible configurations of a variable power system : (a) unsymmetrical (recording) on-axis type, (b) total off-axis type, (c) symmetrical (recording) off-axis type, and (d) interlace off-axis type (where both θ_1 and $\theta_2 > 0$).*

reduced. To avoid the cross-talk problem, four possible configurations are designed, as shown in *figure 2.* They are unsymmetrical (recording) on-axis type, total off-axis type, symmetrical (recording) off-axis type and interlace off-axis type. All of HL_1 in *figure 2* are Leith-Uptanieks hololenses [10]. The geometries of recording and reconstruction for HL_1 are shown in *figure 3.* On the other hand, the HL_2 in *figure 2* (*d*) is a Stroke hololens [11] and other $HL_2\,s$ are Brandt hololenses [12]. The geometries of recording and reconstruction for $HL_2\,s$ are shown in *figure 4* and *figure 5,* respectively.

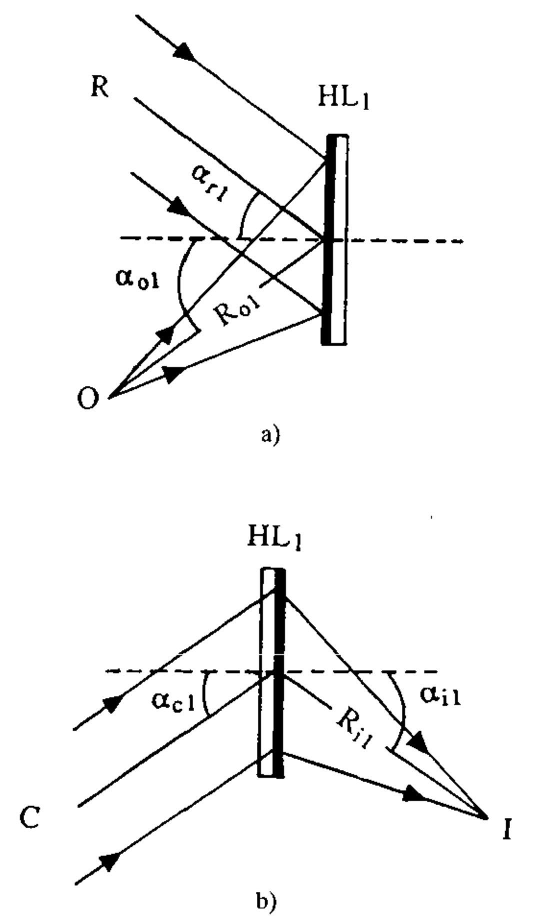

FIG. 3. — *The geometries of recording and reconstruction of a Lieth-Upatnieks hololens, (a) recording parameters : $R_{r1} = \infty$, $R_{o1} = R_1 (> 0)$, $\alpha_{r1} = \theta_1$, $\alpha_{o1} = -\theta_2$; (b) reconstruction parameters : $R_{c1} = \infty$, $R_{i1} = -R_1$, $\alpha_{c1} = -\theta_1$, $\alpha_{i1} = \theta_2$.*

3. — 2. Aberrations

Because the reconstruction light of HL_1 is the same as the recording light, its output is an ideal spherical wave. The aberrations of this system come from HL_2. The coefficients of the aberrations for all types of the hololenses are derived and summarized

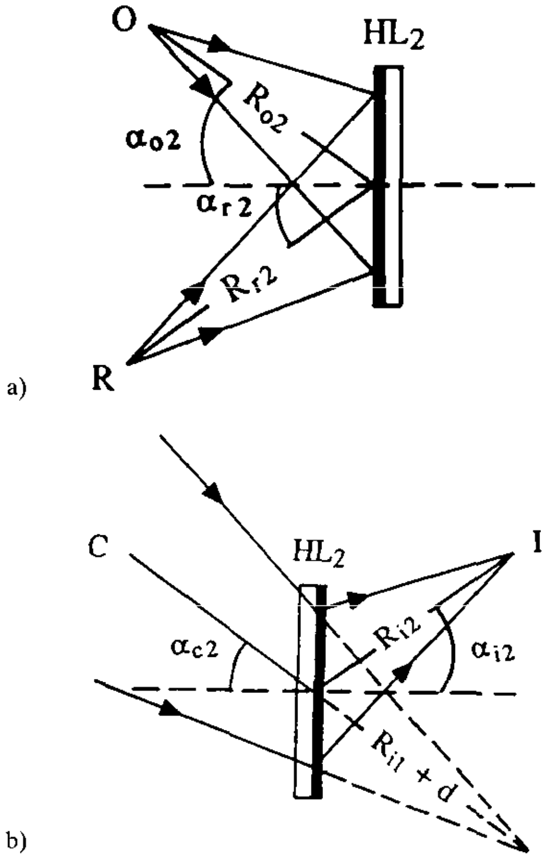

FIG. 4. — *The geometries of recording and reconstruction of a Stroke hololens : (a) recording parameters :* $R_{r2} = R_2$, $R_{o2} = R_3$, $\alpha_{r2} = -\theta_2$, $\alpha_{o2} = \theta_1$; *(b) reconstruction parameters :* $R_{c2} = d - R_1$, $R_{i2} = \left(\frac{1}{d - R_1} - F_2\right)^{-1}$, $\alpha_{c2} = \theta_2$, $\alpha_{i2} = -\theta_1$ *(where both* R_2 *and* $R_3 > 0$*).*

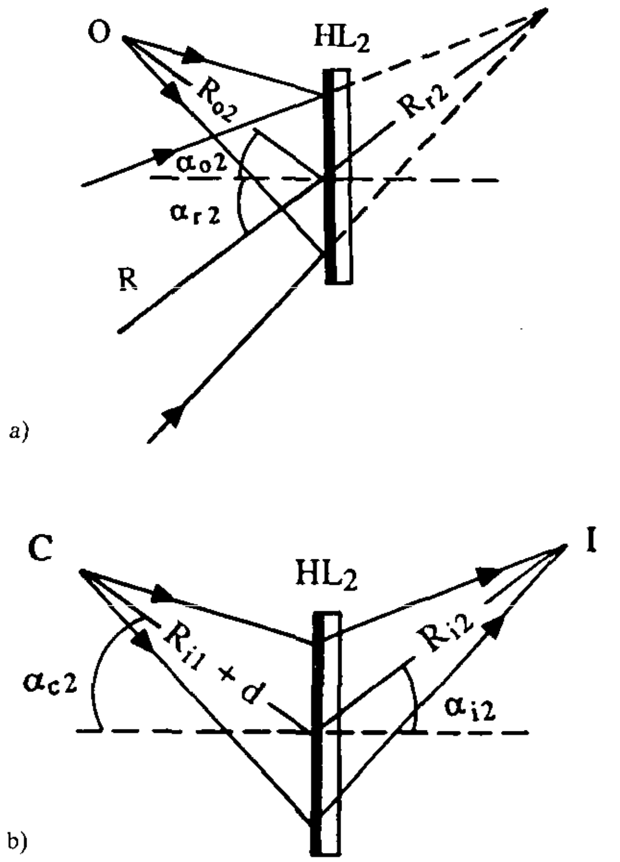

FIG. 5. — *The geometries of recording and reconstruction of a Brandt hololens : (a) recording parameters :* $R_{r2} = -R_2$, $R_{o2} = R_3$, $\alpha_{r2} = -\theta_1$, $\alpha_{o2} = \theta_2$; *(b) reconstruction parameters :* $R_{c2} = d - R_1$, $R_{i2} = \left(\frac{1}{d - R_1} - F_2\right)^{-1}$, $\alpha_{c2} = \theta_2$, $\alpha_{i2} = -\theta_1$.

in *table 1*. To compare the aberrations of each type under similar conditions, the numerical values shown in *table 2* are used. The calculated results are plotted in *figure 6*. From *figure 6*, it is obvious that

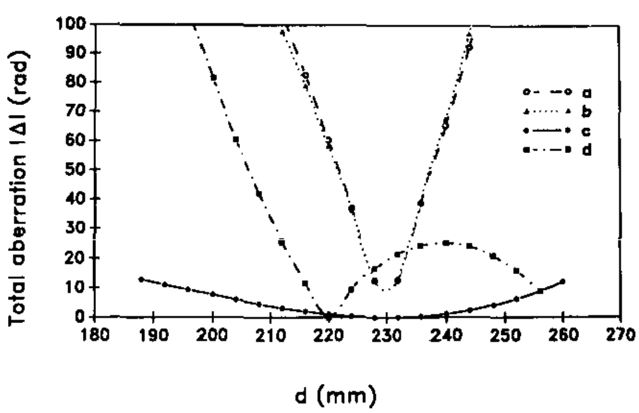

FIG. 6. — *The aberration curves of four possible configurations of a variable power system (Δ, phase retardation as defined by 5), (a) unsymmetrical on-axis type, (b) total off-axis type, (c) symmetrical off-axis type, and (d) interlace off-axis type.*

the symmetrical off-axis type has the smallest aberrations.

In addition, the emulsion shrinkage introduced in the development process also produces aberrations. If the hololens is recorded symmetrically, i.e., the reference light and the object light are symmetrical with respect to the normal to the hologram, then it has the least emulsion shrinkage aberrations [13]. Hence, the symmetrical off-axis type has the smallest aberrations compared with other types.

3. — 3. Diffraction Efficiency

The performance of HOEs and the optical system using HOEs depends particularly on HOEs' diffraction efficiencies. From the coupled wave theory [14], it can be proved that the hololenses fabricated by symmetrical recording process have higher diffraction efficiencies.

Based on the above discussions, it is concluded that the symmetrical off-axis type is the best configuration.

TABLE 1

The aberration coefficients of the four possible configurations.

Types	unsymmetrical on axis $R_{o2} = -R_{r2} = R$	total off-axis $R_{o2} = -R_{r2} = R$	symmetrical off-axis $R_{o2} = -R_{r2} = R$	interlace off-axis $R_{o2} = R_3, R_{r2} = R_2$
S	$6/R\,(1/(d-R_1) - 1/R)^2$			$3\,(1/R_3 - 1/R_2)\,(1/(d-R_1) - 1/R_3)\,(1/(d-R_1) + 1/R_2)$
C	$\sin\theta_1\,(1/(d-R_1) - 3/R)\,(1/(d-R_1) - 1/R)$	$\sin\theta_2\,(1/(d-R_1)^2 - 1/R^2)$	$2\sin\theta_1\,(1/(d-R_1) - 1/R)^2$	$\sin\theta_2\,(1/(d-R_1)^2 - 1/R_2^2)$
A	$\sin^2\theta_1\,(1/R - 1/(d-R_1))$	$\sin^2\theta_2\,(1/(d-R_1) - 1/R)$	0	$\sin^2\theta_2\,(1/(d-R_1) + 1/R_2)$

TABLE 2

The optimum scalar parameters under the similar conditions of four possible configurations.

Types		Ro(mm)	Rr(mm)	αo	αr	αc	αi	ρ(mm)	f#
unsymmetrical on-axis	HL_1	130	∞	0°	32°	-32°	0°	6,2	10,5
	HL_2	100	- 100	32°	0°	0°	-32°	2,4	10,5
total off-axis	HL_1	130	∞	-32°	0°	0°	32°	6,2	10,5
	HL_2	100	-100	0°	-32°	32°	0°	2,4	10,5
symmetrical off-axis	HL_1	130	∞	-16°	16°	-16°	16°	6,2	10,5
	HL_2	100	- 100	16°	-16°	16°	-16°	2,4	10,5
interface off-axis	HL_1	260	∞	-32°	0°	0°	32°	12,4	10,5
	HL_2	200	40	0°	-32°	32°	0°	-2,4	10,5

4. — FABRICATION AND RESULTS

In order to demonstrate the feasibility, a variable power system of symmetrical off-axis type is fabricated according to the numerical values written in *table 2*. Hololenses are fabricated in 8E75HD plates. A He-Ne laser ($\lambda = 633$ nm) is used for both recording and reconstruction. To get a higher diffraction efficiency, the special bleaching process presented by P. Hariharan *et al.* [15] is used. The diffraction efficiency of a single hololens is 65 %. The emulsion shrinkage during the development process makes focal length errors $\Delta f_1 \doteqdot -2$ mm and $\Delta f_2 \doteqdot -1$ mm. The back focal length is calculated by using Eq. (11) as a function of d and is shown in *figure 7*. If the allowable aberration of the system is assumed to be below 10 rad. ($1.5\,\lambda$), then it is estimated from *figure 6* that d should be within

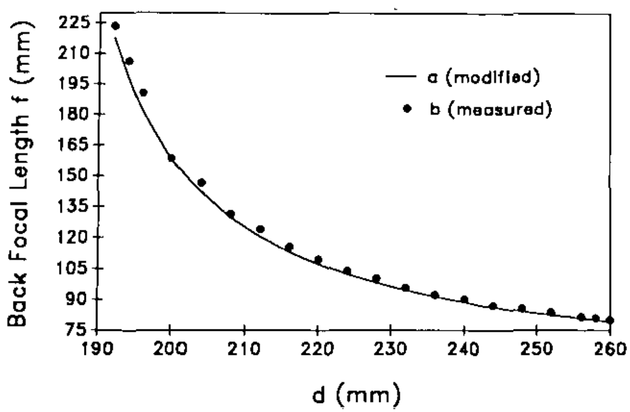

FIG. 7. — *The relation between d and the back focal length of the variable power system with a symmetrical off-axis configuration : (a) theoretical values (the errors due to the shrinkage of the emulsion have been modified) ; (b) measured values.*

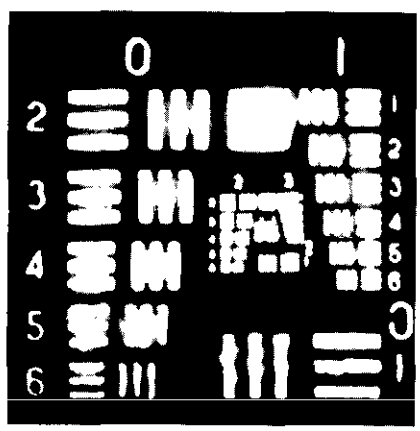

FIG. 8. — *The photograph of the image of a resolution test chart (scale 1 : 1) by the holographic variable power system fabricated under the conditions written in table 2 at* $d = 251.5$ mm.

(194 mm, 256 mm). Hence the back focal length can be varied between the region (80.5 mm, 206.3 mm) as shown in *figure 7*. In experiments, an automatic high-precision pulse stage (model number PS-120XY) manufactured by Japan Chuo Precision Industrial Co. is used to move HL_2 accurately.

Furthermore, a resolution test chart is used to evaluate the resolution of the system. With $f_{\text{b.f.l.}} = 83$ mm, the image of the resolution test chart is taken and shown in *figure 8*. Its resolution is better than 30 μm as read with a Nikon microscope.

5. — *CONCLUSION AND DISCUSSION*

In this paper, a variable power system with two HOEs, one is a Leith-Upatnieks hololens and the other is a Brandt hololens, is presented. To get the best configuration, some important factors such as cross-talk, aberrations and diffraction efficiency, are discussed. Then an optimum configuration is derived. In addition to demonstrate the flexibility, a variable power system is fabricated, and the resolution is evaluated (better than 30 μm).

Because of its variable power, it has a wide range of applications. If a broadband source is used, this system has the function of bandwidth filtering [16], which the conventional optical system does not have. In addition, this system can also be applied to coherent signal transformation and multi-signal output [11].

The measuring error of the focal length also depends on the focus depth. The focus depth is proportional to the *f*-number, and it is about 0.5 mm in this system.

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