國立交通大學

統計學研究所

碩 士 論 文

多選題排序應用的演算法

Algorithms for Ranking Responses in Multiple Response Questions

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中 華 民 國 一百 年 六 月

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摘　　要

問卷調查在很多研究中是一項常用的調查工具，其中複選題是問卷中常見到的題型。近年來，很多研究提出了一些模型和方法針對複選題的資料做分析，但複選題選項排序的問題是目前主要有興趣的議題。Wang (2008) 提出了一些方法用來檢定任兩個選項被選到的機率是否相等，然而，當選項的個數太多時，根據這些方法來做排序將會使得排序過程變得複雜費時。因此，在本篇文章裡，我們提出了一個演算法進而寫成一個程式，使得不管選項個數的多寡，都能迅速排序出結果。此外，為了減少排序上不一致性問題的情況發生，我們提出採用 False Discovery Rate 的檢定準則來做排序。根據我們模擬的結果，證明這樣的方法可以減少不一致性現象的出現。

Algorithms for Ranking Responses in Multiple Response Questions

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ABSTRACT

In many studies, the questionnaire is a common tool for surveying. A multiple response question is a commonly used question designed in a questionnaire. Recently, many studies proposed models and approaches for analyzing data of a multiple response question. Ranking responses problem may be the primary issue in the analysis of a multiple response question. Wang (2008) proposed methodologies for testing the equality of selected probabilities for two responses. Since it is possible that the number of responses is large, it leads to a complicated situation to rank the responses based on these approaches. In this study, we develop algorithms for ranking responses for any response number. In addition, to diminish the ranking inconsistent situation, we propose adopting the false discovery rate testing criterion for ranking. A simulation study shows it can reduce the frequency of ranking inconsistent phenomenon.

Keywords: single response question; multiple response question; Wald test; generalized score test; likelihood ratio test; ranking consistency

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柯佳伶　謹誌于

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1 Introduction

Questionnaires are important tools for surveying in many studies. They are especially important in marketing or management studies. There are usually two kinds of questions: single response questions and multiple response questions. The analyses of single response questions have been investigated in the literature. However, the analyses of multiple response questions have not been discussed in depth like single response questions until recently. Umesh (1995) first discussed the problem of analyzing multiple response questions. Subsequently, Loughin and Scherer (1998), Decady and Thomas (1999) and Bilder, Loughin and Nettleton (2000) proposed several methods for testing marginal independence between a single response question and a multiple response question. Agresti and Liu (1999,2001) discussed the modeling of multiple response questions. These studies mainly focused on the analysis of the dependence between a single response question and a multiple response question. However, in practice, most researchers are also interested in ranking responses to a question according to the probabilities of responses being chosen. In fact, the ranking responses problem may be the primary issue in the analysis of a survey.

Wang (2008) propose several testing methods to test the equality of the probabilities of responses being selected in a multiple response question, including the wald test, score test and the likelihood ratio test. A ranking guide is giving in Wang (2008) based on the testing results to rank the responses. However, in real applications, it is common that there are more than 10 responses in a multiple response question. When the number of responses is large, it is not straightforward to apply the guide to rank the responses. In this study, we mainly provide a rule and programming to rank the responses for any number of responses.

In addition, except adopting the wald test for ranking, we propose using false discovery rate criterion to rank responses. As pointed out in Wang (2008), a reasonable approach should have the ranking consistent property. However, the wald test, score test and likelihood ratio test do not have the ranking consistent property. To reduce the frequency of ranking inconsistency, the false discovery rate criterion is used to to be an alternative approach. From a simultaneous study, we find that the false discovery rate method can diminish the frequency of ranking inconsistent situation occurrence. Therefore, it is a potential competitor for ranking responses.

We first illustrate the model using the following example. For instance, a company is designing a marketing survey to help develop a body lotion product. The researchers will design a multiple response question and list several factors, including price, effect, brand, ingredients and smell that could attract consumers to purchase it. Suppose that a group of individuals are surveyed on purchasing a body lotion. They are asked to fill out questionnaires which list all the questions that we wish to address to each respondent. The following is a multiple response question in the questionnaire:

Question 1. Which factors are important to you when considering the purchase of a body lotion? (1) price (2) effect (3) brand (4) ingredients (5) smell.

Assume that according to the number of each response being chosen, most respondents are more concerned about the price and the effect than the other factors of the product. Then we may rank the response "price" first and "effect" second according to the number of responses selected. However, only basing on the response selected numbers to rank responses is not statistically significant and we cannot confidently claim that the factor "price" is more important than the factor "effect". In this study, we are interested in developing algorithms to rank all responses based on statistical testing approaches.

First, we focus on ranking two specific responses that we are interested in. For the general case, assuming that a multiple response question has k responses, $v_1, \cdots, v_k$, and we interview n respondents. Each respondent is asked to choose at least one and at most s answers for this question, where $0 < s \leq k$. If $s = 1$, it is a single response question. There are a total of $c = C_1^k + \cdots + C_s^k$ possible kinds of answers that respondents will choose. Let $n_{i_1 \cdots i_k}$ denote the number of respondents selecting the responses v_h and not selecting $v_{h'}$ if $i_h = 1$ and $i_{h'} = 0$, and $p_{i_1 \cdots i_k}$ denotes the corresponding probability. For example, when $k = 5$, n_{01001} denote the number of respondents selecting the second and the fifth responses and not slecting the other responses. Thus, the pmf function of $n^* = \{n_{i_1\cdots i_k}, i_j = 0 \text{ or } 1, \text{ and } \sum_{j=1}^{k} i_j \leq s\}$ is

$$f_s(n_{i_1\cdots i_k}) = I(\sum_{j=1}^{k} i_j \leq s) \frac{n!}{\prod\limits_{i_j=0 \ or \ 1} n_{i_1\cdots i_k}!} \prod_{i_j=0 \ or \ 1} p_{i_1\cdots i_k}^{n_{i_1\cdots i_k}}, \tag{1}$$

where $I(\cdot)$ denotes the indicator function. Let m_j denote the sum of the number $n_{i_1\cdots i_k}$ such that the jth response is selected, and π_j denote the corresponding probability, that is $m_j = \sum\limits_{i_j=1} n_{i_1\cdots i_k}$ and $\pi_j = \sum\limits_{i_j=1} p_{i_1\cdots i_k}$. Note π_j is called a marginal probability of response j. Also let m_{jl} denote the sum of the number $n_{i_1,\cdots i_k}$ such that the jth and lth responses are selected, and π_{jl} denote the corresponding probability. Then $m_{jl} = \sum\limits_{i_j=i_l=1} n_{i_1\cdots i_k}$ and $\pi_{jl} = \sum\limits_{i_j=i_l=1} p_{i_1\cdots i_k}$.

For ranking the importance of two specified responses, say response 1 and response 2 in Question 1 from the survey data, we will consider the two-sided test:

$$H_0 : \pi_1 = \pi_2 \ vs \ H_1 : \pi_1 \neq \pi_2, \tag{2}$$

which is equivalent to

$$H_0^* : \pi_1 - \pi_{12} = \pi_2 - \pi_{12} \ vs \ H_1^* : \pi_1 - \pi_{12} \neq \pi_2 - \pi_{12}. \tag{3}$$

If (2) is rejected, then we can rank the response with larger m_j first.

The methods for testing (2) are given in Wang (2008). In this study, we propose an algorithm based on these testing approaches to rank the responses. This algorithm can successfully rank responses to several clusters.

Although the algorithm based on testing approaches proposed in Wang (2008) can successfully rank the responses. The testing approaches suffer the drawback of the ranking inconsistent property (Wang 2008). Since these testing approaches have ranking inconsistent property, we intend to diminish the frequency of ranking inconsistent situations. Therefore, we propose using a false discovery rate testing method to replacing the testing method. From a simulation study, the false discovery rate criterion is shown to successfully reduce the frequency of ranking inconsistent situations.

This thesis is organized as follows. In Section 2, the three testing methods for testing (2) are reviewed. The proposed algorithm based on testing approaches in Wang (2008) is given in Section 3. We review the ranking inconsistent property and propose a false discovery rate method in Section 4. Finally, a real data example is given to illustrate the proposed methods.

2 Preliminaries

Before proposing a rule for ranking all responses in a multiple response question, we have to know how to solve the problem of ranking the two specific responses. In this section, we will review the literature which was presented by Hsiuying Wang (2008). In the literature, the professor first proposed three methods for testing whether there are significant differences in two specific responses, i.e. it was used for testing (2).

2.1 Wald Test

A Wald test is a test based on a statistic of the form

$$Z_n = \frac{W_n - (\pi_1 - \pi_2)}{S_n},$$

where W_n is an estimator of $\pi_1 - \pi_2$, and S_n is a standard error for W_n. An unbiased estimator of $p_{i_1,\cdots,i_k}$ is $n_{i_1,\cdots,i_k}/n$, which is also a maximum likelihood estimator (MLE). Let $\hat{\pi_1} = m_1/n, \hat{\pi_2} = m_2/n$ and $\hat{\pi}_{12} = m_{12}/n$. We can use $\hat{\pi_1} = m_1/n$ and $\hat{\pi_2} = m_2/n$ as estimators of π_1 and π_2 respectively,and we have

$$Var\,(\hat{\pi_1} - \hat{\pi_2}) = \begin{cases} \pi_1(1-\pi_1)/n + \pi_2(1-\pi_2)/n + 2\pi_1\pi_2/n & \text{if } s = 1 \\ \\ (\pi_1 - \pi_{12})(1 - \pi_1 + 2\pi_2 - \pi_{12})/n + \\ (\pi_2 - \pi_{12})(1 - \pi_2 + \pi_{12})/n & \text{otherwise.} \end{cases} \tag{4}$$

Under the null hypothesis H_0 in (2) and based on the central limit theorem, the statistics

$$\frac{\hat{\pi_1} - \hat{\pi_2}}{\sqrt{Var(\hat{\pi_1} - \hat{\pi_2})}} \tag{5}$$

converges in distribution to a standard normal random variable when n is large. Since π_1, π_2 and π_{12} are unknown, we can use $\hat{\pi_1}$, $\hat{\pi_2}$ and $\hat{\pi}_{12}$ to substitute π_1, π_2 and π_{12} in (4).

Thus, for testing (2), H_0 is rejected if the absolute value of (5) is greater than $z_{\frac{\alpha}{2}}$, where $z_{\frac{\alpha}{2}}$ is the upper $\alpha/2$ cutoff point of the standard normal distribution.

2.2 Generalized Score Test

In Section 2.1, π_1, π_2 and π_{12} in $Var(\hat{\pi_1} - \hat{\pi_2})$ are replaced by $\hat{\pi_1}$, $\hat{\pi_2}$ and $\hat{\pi}_{12}$ in the test statistic. In this section, we consider the variance under the null hypothesis in (2), that is, $\pi_1 = \pi_2$. Thus, we have

$$Var_{\pi_1=\pi_2}(\hat{\pi_1} - \hat{\pi_2}) = \begin{cases} 2\pi_1/n & \text{if } s = 1 \\ 2(\pi_1 - \pi_{12})/n. & \text{otherwise.} \end{cases} \tag{6}$$

By the central limit theorem, under H_0, the statistic

$$(\hat{\pi_1} - \hat{\pi_2})/\sqrt{Var_{\pi_1=\pi_2}(\hat{\pi_1} - \hat{\pi_2})}$$

coverges to a standard normal distribution when n is large. We can use $(\hat{\pi_1} + \hat{\pi_2})/2$ and $\hat{\pi}_{12}$ as substitutes for π_1 and π_{12} in the variance. Hence, for testing (2), the null hypothesis is rejected if

$$\begin{cases} \frac{\sqrt{n}*|\hat{\pi_1}-\hat{\pi_2}|}{\sqrt{\hat{\pi_1}+\hat{\pi_2}}} > z_{\alpha/2} & \text{if } s = 1, \\ \frac{\sqrt{n}*|\hat{\pi_1}-\hat{\pi_2}|}{\sqrt{(\hat{\pi_1}+\hat{\pi_2}-2\hat{\pi}_{12})}} > z_{\alpha/2} & \text{if } 1 < s \leq k. \end{cases} \tag{7}$$

This approach is similar to the score test of testing a marginal probability equal to a specified value. Hence we call this approach a generalized score test.

2.3 Likelihood Ratio Test

The third approach is the likelihood ratio test (LRT). For testing $H_0 : \pi_1 = \pi_2$, let

$$\Lambda_{12} = \frac{L(\hat{\hat{p}}_{i_1\cdots i_k})}{L(\hat{p}_{i_1\cdots i_k})}, \tag{8}$$

where L is the likelihood function, and $\hat{\hat{p}}_{i_1\cdots i_k}$ and $\hat{p}_{i_1\cdots i_k}$ denote the MLE of $p_{i_1\cdots i_k}$ under the restricted parameter space $\pi_1 = \pi_2$ and the whole parameter space, respectively. Thus, we have

$$\hat{p}_{i_1\cdots i_k} = n_{i_1\cdots i_k}/n.$$

When $s = 1$,

$$\hat{\hat{p}}_{i_1\cdots i_k} = \begin{cases} (n_{100\cdots0} + n_{010\cdots0})/(2n) & \text{if } i_1 = 1 \\ (n_{100\cdots0} + n_{010\cdots0})/(2n) & \text{if } i_2 = 1 \\ n_{i_1\cdots i_k}/n & \text{otherwise,} \end{cases} \tag{9}$$

which is easily to be interpreted because under $\pi_1 = \pi_2$, $\hat{\hat{p}}_{10\cdots0}$ and $\hat{\hat{p}}_{20\cdots0}$ should be equal to $(\hat{p}_{10\cdots0} + \hat{p}_{010\cdots0})/2$.

When $1 < s \leq k$, by solving the equations of derivatives of the likelihood ratio functions with respect to $p_{i_1\cdots i_k}$ being zero, we have

$$\hat{\hat{p}}_{i_1\cdots i_k} = \begin{cases} S \cdot n_{i_1\cdots i_k}/(2n(\sum_{i_1=1,i_2=0} n_{i_1\cdots i_k})) & \text{if } i_1 = 1, i_2 = 0 \\ S \cdot n_{i_1\cdots i_k}/(2n(\sum_{i_1=0,i_2=1} n_{i_1\cdots i_k})) & \text{if } i_1 = 0, i_2 = 1 \\ n_{i_1\cdots i_k}/n & \text{otherwise,} \end{cases} \tag{10}$$

where

$$S = \sum_{i_1=1,i_2=0} n_{i_1\cdots i_k} + \sum_{i_1=0,i_2=1} n_{i_1\cdots i_k}.$$

According to the asymptotic theory of the likelihood ratio test, $-2log\Lambda_{12}$ has a limiting distribution with one degree of freedom. For testing (2), H_0 is rejected if

$$-2log\Lambda_{12} > \chi^2_{1,\alpha},$$

where $\chi^2_{1,\alpha}$ is a upper α cutoff point of chi-square distribution with one degree of freedom.

3 Ranking Rule and Algorithm

In this section, we focus on how to use these three testing methods to rank all responses. Since the method of Likelihood ratio test is more complicated to calculate than the other two methods, we only use the method of Wald test or Score test to rank all responses, But in this paper we focus on using Wald test. First we propose a rule for ranking all responses as follows.

3.1 Rule

Assume that we have k responses, and first we have to compute each $m_j, j = 1, j = 1, \cdots, k$. Let $m_{(j)}$ be the order statistics, that is , $m_{(1)} \leq \cdots \leq m_{(k)}$. Let $v_{(j)}$ be the response corresponding to $m_{(j)}$. It is natural to rank the importance of responses in order of $m_{(j)}$. That is, the most influential response is $v_{(k)}$, and the second influential response is $v_{(k-1)}$. However, ranking responses based on the order of m_j is not statistically significant. The proposed testing methods in Section 2 can be used to rank the responses. If the hypothesis $\pi_{(k)} = \pi_{(k-1)}$ is rejected, where $\pi_{(r)}$ denotes the marginal probability corresponding to $v_{(r)}$, then we may claim that $v_{(k)}$ is the most influential response. If it is not rejected, then we compare $v_{(k)}$ with $v_{(j)}, j \leq k-2$ sequentially. A ranking rule is proposed in this study. The flowchart of the rule is given in Figure 1 of Appendix A.

We use Question 1 as an example to illustrate the rule. For example, $m_1 = 47, m_2 = 35, m_3 = 17, m_4 = 19, m_5 = 30$, and then $m_{(1)} = 17, m_{(2)} = 19, m_{(3)} = 30, m_{(4)} = 35, m_{(5)} = 47$. It is nature to rank the importance of responses in order of $m_{(j)}$. We may claim that the factor of price is more importan than the factor of effect for consumers to purchase. However, ranking response only based on the magnitudes of m_j, is not statistically significant. Hence, we follow the proposed rule and use one of the three methods to rank all responses. First, we rank response $v_{(5)}$ and response $v_{(4)}$, i.e. rank the response (1) and response (2). If $H_0 : \pi_{(5)} = \pi_{(4)}$ is not rejected, but $H_0 : \pi_{(5)} = \pi_{(3)}$ is rejected, it means that the response (1) and response (2) are equally important, but they are more important than response (5). And then we will compare $\pi_{(3)}$ with $\pi_{(2)}$. If $H_0 : \pi_{(3)} = \pi_{(2)}$ is rejected, and then testing $H_0 : \pi_{(2)} = \pi_{(1)}$. If it is not rejected, the result of ranking is to rank response (1) and (2) first, response (5) second, and response (2) and (3) third. We denote the ranking notations for the above result as (1) 1 (2) 1 (3) 3 (4) 3 (5) 2 . Hence, we can know that response (1)-price and response (2)-effect are top priorities when consumers purchase a body lotion, response (5)-smell is the second important factor, and response (3)-brand and response (4)-ingredients are relatively unimportant for consumers.

Following the rule we can rank the importance of all response in a multiple response question, but it is more complicated and not easy to do when there are many responses in a multiple response question. Hence, we will provide an algorithm for ranking. The algorithm is a R code which is included in the Appendix C. The manual for using the code is also provided in the Appendix B.

4 Ranking Inconsistency

According to the rule of ranking responses, a reasonable test should have the following property: if $H_0 : \pi_{(j)} = \pi_{(i)}, i \leq j$ is rejected by test, then $H_0 : \pi_{(j)} = \pi_{(g)}, g < i$ should also be rejected by the test with the same level because $|m_{(j)} - m_{(i)}| < |m_{(j)} - m_{(g)}|$. We call this property ranking consistency. If a test has ranking consistent property, we call it a ranking consistent test. The algorithm we proposed in Section 3 is under the assumption of ranking consistency. However, in the multiple response question case, this ranking consistent property is not valid for all data, but the frequency of the ranking inconsistent phenomenon occurrence is low according to the simulation results.

4.1 False Discovery Rate

To reduce the frequency of ranking inconsistent phenomenon occurrence we propose using false discovery rate approach. Assume that there are k responses and we are interested in testing

$$
\begin{aligned}
H_{0k-1} : \pi_{(k)} = \pi_{(k-1)} \quad &vs \quad H_{1k-1} : \pi_{(k)} \neq \pi_{(k-1)} \\
H_{0k-2} : \pi_{(k)} = \pi_{(k-2)} \quad &vs \quad H_{1k-2} : \pi_{(k)} \neq \pi_{(k-2)} \\
&\cdots \\
H_{01} : \pi_{(k)} = \pi_{(1)} \quad &vs \quad H_{11} : \pi_{(k)} \neq \pi_{(1)} \qquad (11)
\end{aligned}
$$

Since it is a multiple hypothesis testing, we try to use false discovery rate (FDR) method to improve the problem of ranking inconsistency.

False discovery rate (FDR) control is a statistical method used in multiple hypothesis testing to correct for multiple comparisons. In a list of rejected hypotheses, FDR controls the expected proportion of incorrectly rejected null hypotheses (Type I errors). The false discovery rate is given by $\mathrm{E}[\frac{V}{V+S}]$ and one wants to keep this value below a threshold α.

Table 1: Random variables related to m hypothesis tests.

	Null hypothesis is true	**Alternative hypothesis is true**	**Total**
Declared significant	V	S	R
Declared non-significant	U	T	$m-R$
Total	m_0	$m-m_0$	m

- m is the total number hypotheses tested.
- m_0 is the number of true null hypotheses.
- $m-m_0$ is the number of true alternative hypotheses.
- V is the number of false positives (Type I error).
- S is the number of true positives.
- T is the number of false negatives (Type II error).
- U is the number of true negatives.
- In m hypothesis tests of which m_0 are true null hypotheses, R is an observable random variable, and S, T, U, and V are unobservable random variables.

The FDR procedure proposed by Benjamini and Hochberg (1995) ensures that its expected value $\mathrm{E}[\frac{V}{V+S}]$ is less than a given α. This procedure is valid when the m tests are independent. Let $H_1, \cdots, H_m$ be the null hypotheses and $p_1, \cdots, p_m$ be their corresponding p-values. Let $p_{(1)}, \cdots, p_{(m)}$ denote the order statistics. For a given α, find the largest c such that

$$p_{(c)} \leq \frac{c}{m}\alpha$$

Then reject all $H_{(i)}$ for $i = 1, \cdots, c$.

4.2 Simulation Result

This simulation study is conducted to prove that the inconsistent situation is reduced by applying FDR criterion for multiple hypothesis testing. The simulation procedure is to generate a set of $p^* = \{p_{i_1 \cdots i_k}, i_j = 0 \text{ or } 1\}$ and then generate a sample from (1) based on the generated p^* . Then we can use the sample to check the ranking consistent property of the usual and FDR criterion. Here we repeat the simulation process 10000 times. According to the result, we can find the ratio of ranking inconsistent phenomenon occurrence to the total sample number 10000 is 0.0012 by using fixed α, while the ratio is 0.0007 by using FDR. Hence we can claim that the FDR criterion can reduce the frequency of ranking inconsistent phenomenon occurrence. Example 5.1 gives an example which is ranking inconsistent using the usual criterion and is ranking consistent using FDR criterion.

Example 5.1: Assume that a multiple response question has five answers. The data are presented in Table 2. The p-value for testing (2) by Wald test are given in Table 3 :

Table 2: The number of the responses be chosen.

n_{00000}	n_{10000}	n_{01000}	n_{00100}	n_{00010}	n_{00001}	n_{11000}	n_{10100}
0	62	38	27	92	11	8	49
n_{10010}	n_{10001}	n_{01100}	n_{01010}	n_{01001}	n_{00110}	n_{00101}	n_{00011}
47	62	39	21	39	17	26	28
n_{11100}	n_{11010}	n_{11001}	n_{10110}	n_{10101}	n_{10011}	n_{01110}	n_{01101}
1	4	3	49	53	50	39	30
n_{01011}	n_{00111}	n_{11110}	n_{11101}	n_{11011}	n_{10111}	n_{01111}	n_{11111}
27	24	4	2	2	56	36	54

Table 3: The statistics and p-values.

	$H_{01}:\pi_{(5)}=\pi_{(4)}$	$H_{02}:\pi_{(5)}=\pi_{(3)}$	$H_{03}:\pi_{(5)}=\pi_{(2)}$	$H_{04}:\pi_{(5)}=\pi_{(1)}$
test statistics	1.975531	1.927710	2.108682	9.248402
p-value	0.0482	0.0541	0.0350	0
rank p-value	$P_{(3)}$	$P_{(4)}$	$P_{(2)}$	$P_{(1)}$

First we consider the usual case that the Type I error is fixed to be 0.05 (α=0.05) For testing H_{01}, since the p-value $0.0482 < \alpha$, H_{01} is rejected. We expect that H_{02} should also be rejected. However, for testing H_{02}, the p-value $0.0541 > \alpha$, which does not reject H_{02}. Hence, the test is ranking inconsistency.

Then we apply FDR method in the example. Since $P_{(1)} < \frac{1}{4} \times 0.05 = 0.0125$, but $P_{(2)} > \frac{2}{4} \times 0.05 = 0.025$, so according to FDR method, only H_{04} was rejected. Hence, we can claim that the test is ranking consistency.

From example 5.1, we can know that FDR is a better method for solving the problem of ranking inconsistency.

However, the algorithm based on false discovery rate is more complicated than the usual testing criterior and is still under investigation.

5 A Real Data Example

In this section, we use a real data example to illustrate the proposal rule. This example is a survey of 49609 first-year college students in Taiwan about their preferences in their college study. The data set can be accessed at http://srda.sinica.edu.tw . We list one of the multiple response questions in the questionnaire as follows:

Question: What kind of experience do you expect to receive during the period of college study? (Please select at least one response)

1. Read over the Chinese and foreign classic literature.
2. Travel around Taiwan.
3. Present academic papers in conferences.
4. Lead large-scale activities.
5. Be on a school team.
6. Be a cadre of student associations.
7. Participate internship programs.
8. Fall in love.
9. Have sexual experience.
10. Travel around the world.
11. Make many friends.
12. Others.

We are interested in ranking all responses of this multiple response question according to students' preference. The population is all first-year college students in Taiwan. The projectors sampled 49609 first-year college students form the population to fill out the questionnaire. Since there are many missing data in the whole data set, we have to delete the missing data. Then real sample size is 3388. Since we have all data, we can obtain the ranks of the twelve responses by the order of m_j. Note that from the whole data set, the numbers of respondents selecting the twelve responses we denoted by $m_1 = 623, m_2 = 1889, m_3 = 338, m_4 = 913, m_5 = 637, m_6 = 1134, m_7 = 1596, m_8 = 1660, m_9 = 531, m_{10} = 1556, m_{11} = 2699, m_{12} = 118$. Since it's risky to rank all responses basing only on the order of m_j, we have to use the rule and program for ranking all responses.

To implement our program, we need to construct survey data and set α=0.05. Input these parameters into the function in our program, we can obtain the result of ranking all responses is (1) 7 (2) 2 (3) 9 (4) 6 (5) 7 (6) 5 (7) 3 (8) 3 (9) 8 (10) 4 (11) 1 (12) 10 . From the result we can claim that the first-year college students prefer to "make many friends", " travel around Taiwan", "participate internship programs", and "fall in love”, where ”participate internship programs” and ”fall in love” are equally important and ranked third for the students.

From the this real data example, we can verify that the program we proposed is feasible and convenient in practice when we want to rank all responses in a multiple response question.

6 Conclusion

The questionnaire is an important tool for surveying. Ranking the responses in a multiple response question is an important issue for analyzing data and is the main information that the researchers intend to obtain from the surveying. The conventional method is to rank the responses depending on the numbers of responses being selected, which does not associate with a statistical method to provide a statistically significant ranking approach. Wang (2008) provided several approaches to rank the responses, but did not provide a general approach and code to implement the methods for any response number. It is common that the response number is large such as 10 or more. In this case, a useful computing code for ranking the responses would be very helpful for the ranking problem. In this study, a R code for ranking the responses is provided which can be used to ranking responses with a large number. In addition, an improved methods using the false discovery rate criterion is provided which can diminish the ranking inconsistency phenomenon.

Appendix

A Flowchart of The Ranking Rule

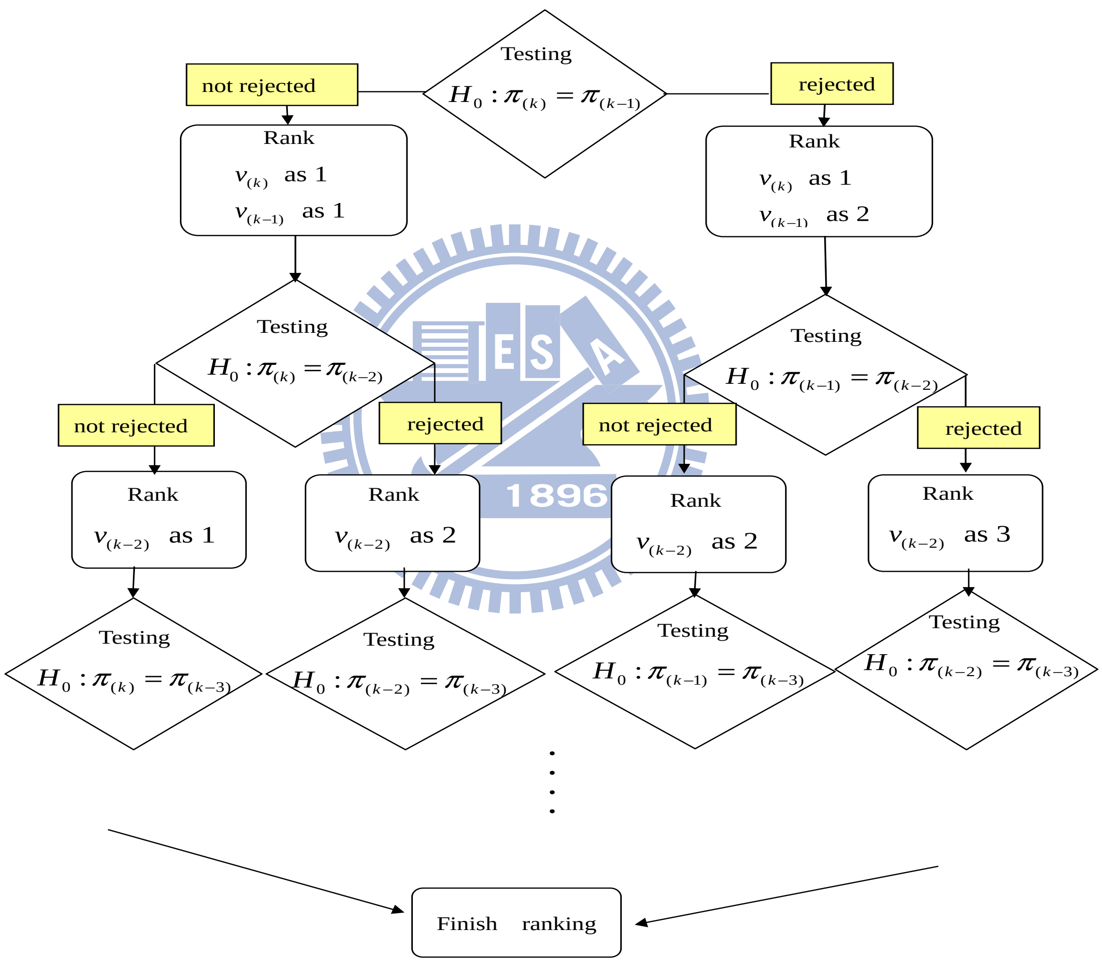


Figure 1: Flowchart of the ranking rule

Appendix

B R Code User Manual

1. To analyze your own data, the form of the data must be saved as the following format in Excel:

Table 4: The data format.

respondent	**response**				
	1	**2**	**3**	$\cdots$	**k**
1	1	0	0	$\cdots$	1
2	0	1	0	$\cdots$	0
3	1	1	1	$\cdots$	0
.			$\cdots$		
.			$\cdots$		
.			$\cdots$		
n	0	0	1	$\cdots$	1

- **Note:**

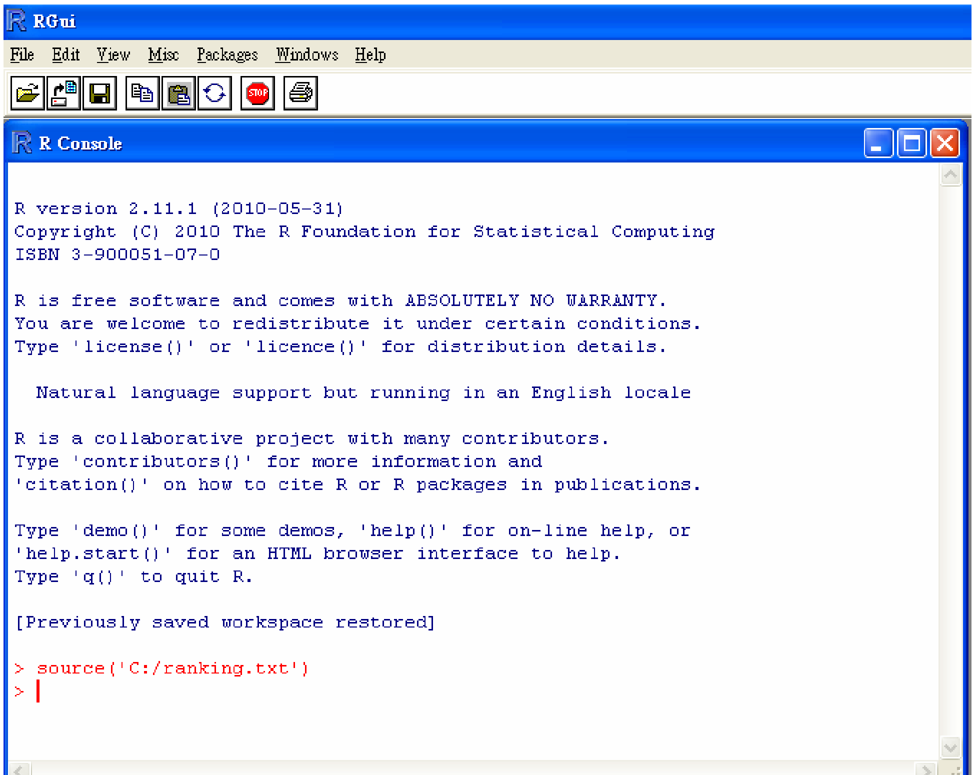
  (i) In the table, rows denote the numbers of respondents, and columns denote the numbers of responses. The notation 0 in Table B.1 denotes that the responses isn't chosen and the notation 1 denotes that the response is chosen.

  (ii) The data must be saved in **.csv** format. For example, data.csv .

2. How to run the R-codes

   (i) Open R .

   (ii) Save the R-codes in C: as a name "ranking.txt". type **source("C:/ranking.txt")**.



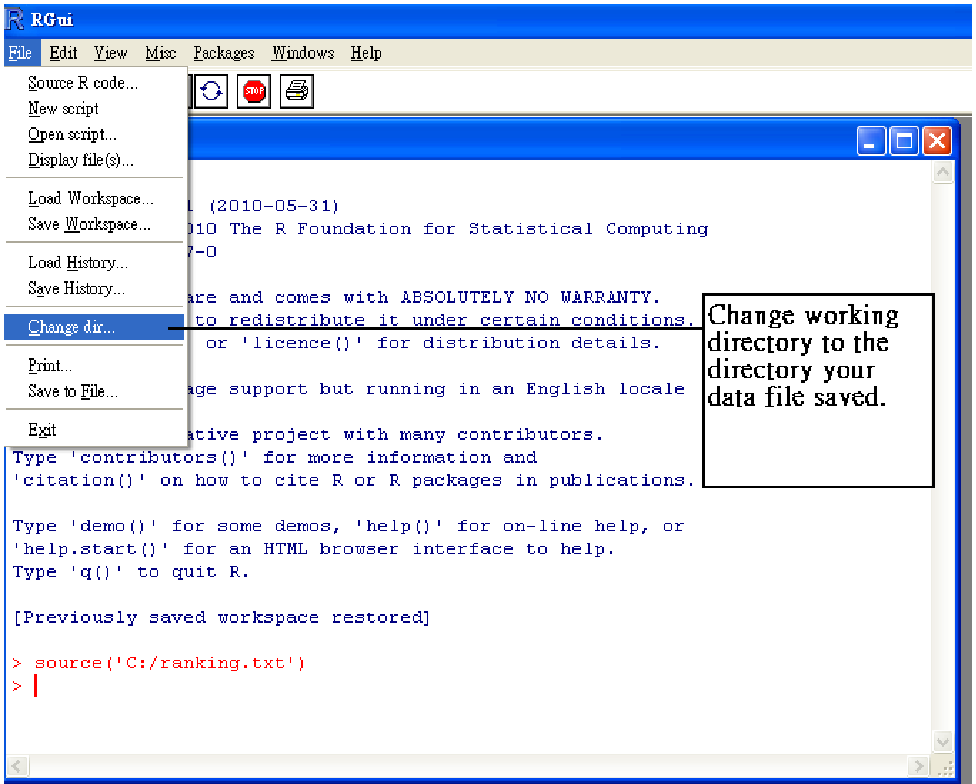
3. Read the data file and rank the data.

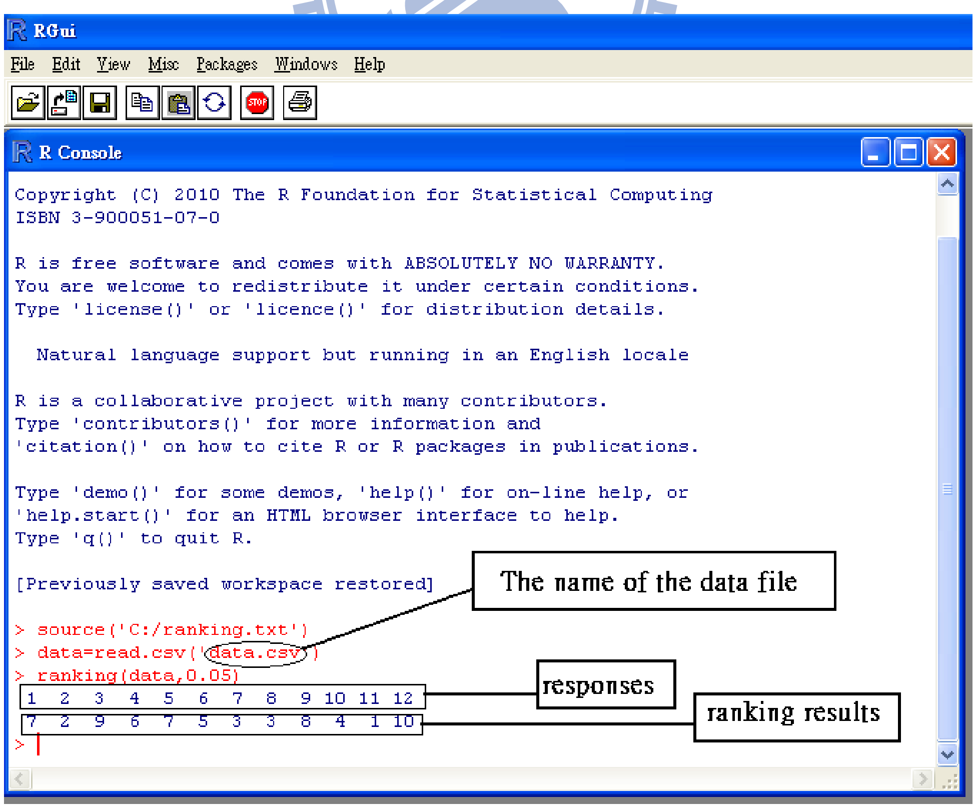
- **Note: First you have to change the directory to the directory your data file saved.**

(i) Read the data you want to analyze. For example, type
data=read.csv('data.csv')

(ii) Then use the function "ranking" to rank all the responses. For example, type **ranking(data,0.05)** , where 0.05 denotes the type I error of the hypothesis testing used for the raking.

- **Note: In the function ranking(data,alpha), where alpha can be set by yourself.**


RGui
File Edit View Misc Packages Windows Help
Source R code...
New script
Open script...
Display file(s)...
Load Workspace...
Save Workspace...
Load History...
Save History...
Change dir...
Print...
Save to File...
Exit
Change working directory to the directory your data file saved.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.
Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
[Previously saved workspace restored]
> source('C:/ranking.txt')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Natural language support but running in an English locale
R is a collaborative project with many contributors.
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Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.
[Previously saved workspace restored]
> source('C:/ranking.txt')
> data=read.csv('data.csv')
> ranking(data,0.05)
1 2 3 4 5 6 7 8 9 10 11 12
7 2 9 6 7 5 3 3 8 4 1 10
The name of the data file
responses
ranking results

Appendix

C R-codes

```
ranking=function(data,alpha){
data=as.matrix(data)
n=length(data[,1])
k=length(data[1,])
m=matrix(0,k,k)
for(i in 1:k)
{
for(j in 1:k)
{
if(i==j) {m[i,j]=sum(data[,j])}
if(i!=j) {m[i,j]=length(which(data[,i]==1&data[,j]==1))}
}
}
p=matrix(0,k,k)
for(i in 1:k)
{
for(j in 1:k)
{
if(i==j) p[i,j]=m[i,j]/n
if(i!=j) p[i,j]=m[i,j]/n
}
}

x=rep(0,k)
for(i in 1:k)
{
x[i]=p[i,i]
}

pi=cbind(x)
names(pi)=c(1:k)
r=rank(pi)

W=matrix(0,k,k)
for(i in 1:k)
{
for(j in 1:k)
{
if(i==j) W[i,j]=0
if(i<j)  W[i,j]=(p[i,i]-p[j,j])*sqrt(n/((p[i,i]-p[i,j])*(1-p[i,i]+2*p[j,j]-p[i,j])+(p[j,j]-p[i,j])*(1-p[j,j]+p[i,j])))
if(i>j)  W[i,j]=-W[j,i]

}
}

d=rep(0,k)
for(i in 1:k)
{
d[i]=which(r==i)
}

z=qnorm(1-alpha/2)
i=k
j=k-1
g=k

if(W[d[i],d[j]]>=z||W[d[i],d[j]]<=(-z))
```

```
{
r[d[k]]=1
r[d[k-1]]=2
t=r[d[k-1]]
}else{
r[d[k]]=1
r[d[k-1]]=1
t=r[d[k-1]]
}

while((k-2)>=1)
{
if(r[d[k-1]]!=r[d[k]])
{
i=k-1
j=j-1
if(W[d[i],d[j]]<=(-z)||W[d[i],d[j]]>=z)
{
r[d[k-2]]=t+1
t=r[d[k-2]]
k=k-1
}else{
r[d[k-2]]=t
t=r[d[k-2]]
k=k-1
g=i
}
}else{
i=g
j=j-1
if(W[d[i],d[j]]<=(-z)||W[d[i],d[j]]>=z)
{
r[d[k-2]]=t+1
t=r[d[k-2]]
k=k-1
}else{
r[d[k-2]]=t
t=r[d[k-2]]
k=k-1
g=i
}
}
}
r
}
```

Reference

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