Analytical Model for Wireless Communication Systems

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Abstract—In this paper, a user capacity model, combining both the modulation and coding scheme (MCS) distribution model and resource allocation model, is developed to analyze the system performance of wireless communication systems. From the model, we can intuitively calculate the system capacity from MCS distribution. An example based on relay networks is used to quantify the benefit of deploying relay stations.

Index Terms—Resource management, mobile communication, relay networks.

I. INTRODUCTION

IN future wireless mobile communication systems, some advanced technologies including multi-hop relaying, multiple-input multiple-output (MIMO), femtocell, and etc., are proposed to achieve high transmission rates. With extra implementation costs, it is important to be able to quantify the performance under various deployment scenarios. Issues related to radio resource allocation are addressed in [1]. In [2,3], orthogonal frequency division multiple access (OFDMA) system based resource allocation is proposed. From [4], the impacts on broadcast channel and capacity are investigated. [5] proposes a rate-and-power based resource allocation algorithm for OFDMA downlink systems. In our work, instead of focusing on the resource allocation algorithms, a system resource usage model is established to provide the engineering sense for system deployment and resource allocation. The remaining paper is organized as follows: The proposed analytical model is presented in Section II. In Section III, a two-hop relay network is used as an example to show how to apply the analytical model. Some analysis results are also discussed in Section III. Conclusion is drawn in Section IV.

II. PROPOSED USER CAPACITY MODEL

The proposed model contains two components including the MCS distribution model and resource allocation model.

A. MCS distribution model

The probability that a mobile station (MS) on the i-level MCS is defined as:

$$P_{MCS,i} = Pr\{MS \ on \ i\text{-}level \ MCS \ Coverage\} \quad (1)$$

In wireless, with the changing signal quality, the associated MCS assignment will also be changed. To show that, a two dimensional Markov Chain is used. From Fig. 1, the state
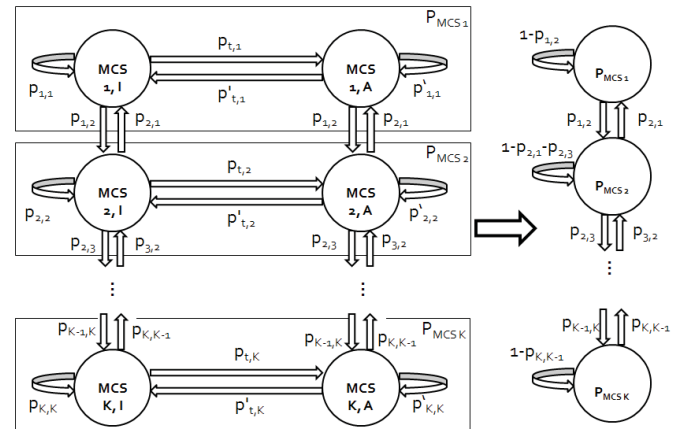
Fig. 1. MCS transition model.

label contains two parameters: The integer number represents the MCS level, and the letter of "I" and "A" means "Idle" and "Active", respectively. For example, if a MS is transmitting or receiving data based on 64QAM-1/2 coding MCS, then it is labeled as "MCS, 1, A" state. If the MS is in the 16QAM-1/2 coding coverage but is idle, then it is labeled as "MCS, 2, I" state. Due to the MCS setting is mostly based on average channel performance, we assume that the state only jumps to its adjacent states. The state transition probability from i-level to j-level MCS, is $p_{i,j}$ and $p'_{i,j}$ for "Idle" and "Active" modes, respectively. And the state transition probability at i-level MCS between "Idle" and "Active" are $p_{t,i}$ and $p'_{t,i}$. Since the moving MS from one MCS coverage to another MCS coverage is independent from whether it is in "Idle" or "Active" mode, $p_{i,j}$ and $p'_{i,j}$ will be equal. With that, we can prove that the two dimensional Markov chain can be deducted to one dimensional Markov chain; a general proof is provided in Appendix.

B. Resource allocation model

The normalized probability of $P_{MCS,i,A}$, ρ_i, can be calculated as:

$$\rho_i = \frac{P_{MCS,i,A}}{\sum_{j=1}^{K} P_{MCS,j,A}} \quad (2)$$

With the ρ-factor, we can investigate the transmission efficiency of radio resources. For the user capacity study, based on a long-term observation, the distribution of MCS, ρ_1, ρ_2, ... ρ_k, is stable. For i-level MCS, the number of associated information bits carried by one resource unit (RU) is defined asbi. We assume that the total number of MSs is N and the maximum number of RU in each downlink sub-frame is U_{DL}.

Then we can get the following inequality:

$$\sum_{i=1}^{K} \frac{\rho_i N R \tau T_f}{\eta_i} \leq \tau U_{DL} \tag{3}$$

where τ is total frames transmitted, R is the average required data rate, and T_f is the frame duration.

The inequality (3) can be rewritten as:

$$NRT_f \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i} \leq U_{DL} \tag{4}$$

If there are M different applications and each has the average required data rate of R_j and mixed traffic ratio of for application j, then the inequality (4) can be extended as:

$$NT_f \cdot \sum_{j=1}^{M} \{\beta_j R_j \cdot \sum_{i=1}^{K} \frac{\rho_{j,i}}{\eta_i}\} \leq U_{DL} \tag{5}$$

where $\rho_{j,i}$ denotes the normalized station probability of MCS i for application j. Apparently, the inequality (3-5) indicates that resource constraints will result in a capacity limit. For further performance analysis, we can estimate the user capacity and the maximum system throughput from the proposed model once we know MCS distribution. From inequality (5), the user capacity N^* can then be calculated with the mixture of traffic types and the distribution in MCS:

$$N^* = \frac{U_{DL}}{T_f \cdot \sum_{j=1}^{M} \{\beta_j R_j \cdot \sum_{i=1}^{K} \frac{\rho_{j,i}}{\eta_i}\}} \tag{6}$$

In the next section, we will show how to compare the user capacity and system throughput of a wireless system with and without the deploying of relay stations.

III. AN EXAMPLE FOR RELAY NETWORK

In this section, the user capacity of a wireless two-hop relay network is considered as an example to demonstrate the use of the user capacity model. Without restricting the deployment method of relay stations (RSs), we assume that the proportion of users served by RSs is Ṡo, the downlink resource plane can be partitioned into two: one is the resource allocation for the MSs served by macro base station (BS), and the other is the resource allocation for the MSs served by RSs. From inequality (4), the resource allocation for the MSs served by BS and RSs can be written as:

$$NRT_f(1-\alpha) \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i} + NRT_f \alpha \cdot (\sum_{i=1}^{K} \frac{\rho_{RM,i}}{\eta_i} + \sum_{i=1}^{K} \frac{\rho_{BR,i}}{\eta_i}) \tag{7}$$

Here ρ_i is the MCS distribution for BS to MS link, $\rho_{RM,i}$ is for RS to MS link, and $\rho_{BR,i}$ is for BS to RS link, respectively. Finally, the resource allocation inequality can be written as:

$$NRT_f[(1-\alpha) \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i} + \alpha \cdot (\sum_{i=1}^{K} \frac{\rho_{RM,i}}{\eta_i} + \sum_{i=1}^{K} \frac{\rho_{BR,i}}{\eta_i})] \leq U_{DL} \tag{8}$$

The first term represents the average number of resource unit used by BS-MS link. The second term presents the average number of resource unit used by RS-MS link, and the third term presents the average number of resource unit used by BS-RS link. For the pure BS network, the resource
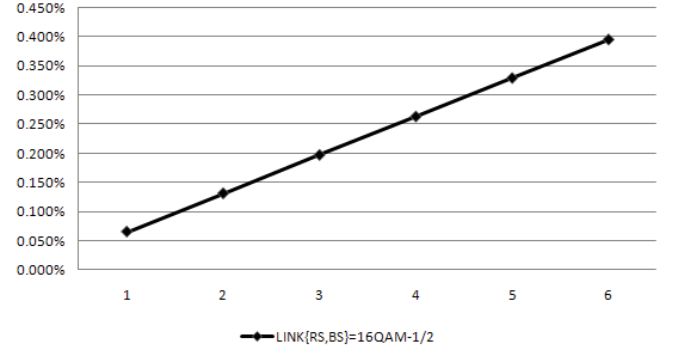

Fig. 2. System throughput gain with BS-RS link of 16QAM-1/2 MCS.

allocation inequality is (4). The user capacity, N*, is calculated in (6) with M=1 (for single application). And the user capacity of the two-hop relay network is (9).

$$N^* = \frac{U_{DL}}{RT_f[(1-\alpha) \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i} + \alpha \cdot (\sum_{i=1}^{K} \frac{\rho_{RM,i}}{\eta_i} + \sum_{i=1}^{K} \frac{\rho_{BR,i}}{\eta_i})]} \tag{9}$$

Moreover, we can estimate the maximum system throughput. Since we assume that the required average data rate of each user is R. The maximum system throughput (MST) (the multiplication of the user number and transmission rates) of the pure BS network and two-hop relay network are calculated in (10) and (11), respectively.

$$MST = \frac{U_{DL}}{T_f \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i}} \tag{10}$$

$$MST = \frac{U_{DL}}{T_f[(1-\alpha) \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i} + \alpha \cdot (\sum_{i=1}^{K} \frac{\rho_{RM,i}}{\eta_i} + \sum_{i=1}^{K} \frac{\rho_{BR,i}}{\eta_i})]} \tag{11}$$

Although the deployment of RSs can help MSs in the cell edge and improve their performance. It also results in transmission redundancy in which BS will first transmit data to RS and then the RS will redirect the data to MS. This redundancy in transmission could potentially degrade the system performance. Compared (10) and (11), the condition that the relay network has better system performance than the pure BS network is shown in (12).

$$(1-\alpha) \cdot \sum_{i=1}^{K} \frac{\rho_i}{\eta_i} + \alpha \cdot (\sum_{i=1}^{K} \frac{\rho_{RM,i}}{\eta_i} + \sum_{i=1}^{K} \frac{\rho_{BR,i}}{\eta_i}) \leq \sum_{i=1}^{K} \frac{\rho_{PureBS,i}}{\eta_i} \tag{12}$$

To investigate the performance, we set a simple example for considering only a geographic distribution of MCS as a baseline performance to show the influence of number of RSs and the link quality in between BS and RS. In the analysis, the BS cell radius is 1000 m and the RSs are equally separated and located at the ring of 2/3 cell radius. Three MCSs including 64QAM-1/2, 16QAM-1/2, and QPSK-1/2 are considered. For BS, each MCS has the coverage radius of 200m, 600m, and 1000m, respectively. And for each RS, only 64QAM-1/2 and
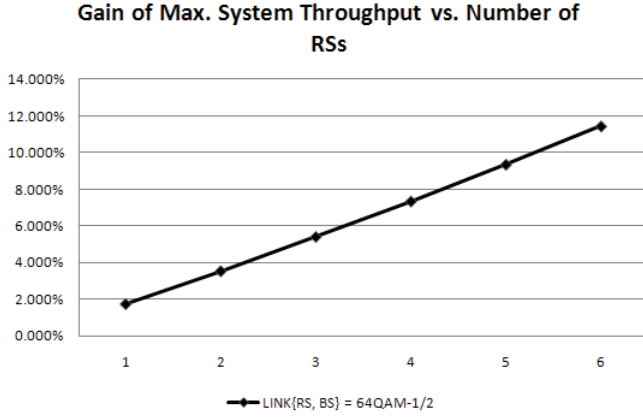
Fig. 3. System throughput gain with BS-RS link of 64QAM-1/2 MCS.

16QAM-1/2 are considered at the coverage radius of 120 m and the coverage radius of 333.33 m.

Figure 2 shows the gain of the system throughput as compared to the Pure BS case. We can see that the improvement is less than 1% even with 6 RSs deployed. But from Fig. 3, there is more than 10% of improvement, if the link between RS and BS can be improved to 64QAM-1/2. Apparently, having a better link in between the BS and RS has more impacts on the system performance as compared to adding more RS stations.

IV. CONCLUSION

In this paper, a user capacity model is developed based on MCS distribution model and resource allocation model. The model captures the effect of various applications and most important the MCS distribution. From the two-hop relay example, we can quickly quantify the gain with the changes of RS station number and the link quality in between BS and RS. Furthermore, the capacity model can be adopted by considering QoS constraints, scheduling and advanced technologies like MIMO and beam forming which can potentially change the MCS distribution and the transmitted rates per RU.

APPENDIX A

Here we will show that the two dimensional Markov chain can be deducted to one dimensional Markov chain. As a result, the state probability, $P_{MCS,i}$, depending only on transition probability. The state probability, $P_{MCS,i}$, can be expressed as:

$$P_{MCS,i} \equiv P_{MCS,i,I} + P_{MCS,i,A}$$

For i = 1,

$$P_{MCS,1,I} = p_{1,1} \cdot P_{MCS,1,I} + p_{2,1} \cdot P_{MCS,2,I} + p'_{t,1} \cdot P_{MCS,1,A}$$

$$P_{MCS,1,A} = p'_{1,1} \cdot P_{MCS,1,A} + p_{2,1} \cdot P_{MCS,2,A} + p_{t,1} \cdot P_{MCS,1,I}$$

$$P_{MCS,1} = P_{MCS,1,I} + P_{MCS,1,A}$$

$$= p_{2,1} \cdot (P_{MCS,2,I} + P_{MCS,2,A}) + (p_{1,1} + p_{t,1}) \cdot P_{MCS,1,I} + (p'_{1,1} + p'_{t,1}) \cdot P_{MCS,1,A}$$

given $p_{1,1} + p_{t,1} + p_{1,2} = 1$ and $p'_{1,1} + p'_{t,1} + p_{1,2} = 1$

$$P_{MCS,1} = p_{2,1} \cdot P_{MCS,2} + (1 - p_{1,2}) \cdot P_{MCS,1,I} + (1 - p_{1,2}) \cdot P_{MCS,1,A}$$

$$= p_{2,1} \cdot P_{MCS,2} + (1 - p_{1,2}) \cdot P_{MCS,1}$$

For $1 < i < K$,

$$P_{MCS,i,I} = p_{i-1,i} \cdot P_{MCS,i-1,I} + p_{i,i} \cdot P_{MCS,i,I} + p_{i+1,i} \cdot P_{MCS,i+1,I} + p'_{t,i} \cdot P_{MCS,i,A}$$

$$P_{MCS,i,A} = p_{i-1,i} \cdot P_{MCS,i-1,A} + p'_{i,i} \cdot P_{MCS,i,A} + p_{i+1,i} \cdot P_{MCS,i+1,A} + p_{t,i} \cdot P_{MCS,i,I}$$

$$P_{MCS,i} = P_{MCS,i,I} + P_{MCS,i,A}$$

$$P_{MCS,i} = p_{i-1,i} \cdot P_{MCS,i-1} + p_{i+1,i} \cdot P_{MCS,i+1} + (p_{i,i} + p_{t,i}) \cdot P_{MCS,i,I} + (p'_{i,i} + p'_{t,i}) \cdot P_{MCS,i,A}$$

$$= p_{i-1,i} \cdot P_{MCS,i-1} + p_{i+1,i} \cdot P_{MCS,i+1} + (1 - p_{i,i+1} + p_{i,i-1}) \cdot (P_{MCS,i,I} + P_{MCS,i,A})$$

$$= p_{i-1,i} \cdot P_{MCS,i-1} + p_{i+1,i} \cdot P_{MCS,i+1} + (1 - p_{i,i+1} + p_{i,i-1}) \cdot P_{MCS,i}$$

For $i = K$

$$P_{MCS,K,I} = p_{K-1,K} \cdot P_{MCS,K-1,I} + p_{K,K} \cdot P_{MCS,K,I} + p'_{t,K} \cdot P_{MCS,K,A}$$

$$P_{MCS,K,A} = p_{K-1,K} \cdot P_{MCS,K-1,A} + p'_{K,K} \cdot P_{MCS,K,A} + p_{t,K} \cdot P_{MCS,K,I}$$

$$P_{MCS,K} = P_{MCS,K,I} + P_{MCS,K,A}$$

$$= p_{K-1,K} \cdot P_{MCS,K-1} + (1 - p_{K,K-1}) \cdot P_{MCS,K,I} + (1 - p_{K,K-1}) \cdot P_{MCS,K,A}$$

$$= p_{K-1,K} \cdot P_{MCS,K-1} + (1 - p_{K,K-1}) \cdot (P_{MCS,K,I} + P_{MCS,K,A})$$

For all i, the steady state probability of the new one dimensional Markov chain, $P_{MCS,i}$, depends only on the transition probability and state probabilities. Q.E.D.

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