International Journal of Control, Automation, and Systems (2010) 8(2):272-278 http://www.springer.com/12555
DOI 10.1007/s12555-010-0213-9

Design of Midcourse Guidance Laws via a Combination of Fuzzy and SMC Approaches

Chun-Hone Chen, Yew-Wen Liang*, Der-Cherng Liaw, Shih-Tse Chang, and Sheng-Dong Xu

Abstract: Issues regarding the design of midcourse guidance laws for antimissiles are addressed. The antimissile is expected to be guided to a place with a desired direction, where a ballistic missile is predicted to pass in the reverse direction, so that the target can be easily found and locked for terminal interception. The predicted location and direction of a ballistic missile may vary with time, due to information update using a trajectory prediction algorithm. To fulfill the guidance performance, the guidance laws are designed by combining the Takagi-Sugeno (T-S) fuzzy approach and the Sliding Mode Control (SMC) technique. Under the designed guidance law, it is shown that the antimissile is able to be efficiently guided to a specified location and direction, even when the existence of uncertainties and disturbances.

Keywords: Guidance law, sliding mode control, Takagi-Sugeno (T-S) fuzzy systems.

1. INTRODUCTION

In the recent years, the study of the design of guidance laws for interception has attracted considerable attention (see e.g., [1-11] and the references therein). The approaches to this issue include exact feedback linearization [1], sliding mode control [2-5], fuzzy control [6], adaptive control [7], LQ-based control [9,10] and relative circular navigation guidance (RCNG) [11]. Among these approaches, the exact feedback linearization one requires perfect knowledge of system dynamics and uses this knowledge to exactly cancel the system nonlinearities; however, perfect knowledge of a system dynamics is usually hard to achieve because of the existence of uncertainties and disturbances. As a result, the performance of exact feedback linearization for uncertain systems is in general not satisfied [11]. The LQ-based approach [9,10] employed the linear quadratic optimization technique to study the impact angle constraint problem and derived an optimal solution for the linearized dynamics in the sense of a quadatic performace; however, they assume that the missile has constant speed and only consider the geometry of the pursuer and the target on a plane without presenting the three dimensional results. Although the RCNG approach [11] presented the impact angle error in a closed form without linearizing the pursuit dynamics, it only considers the ideal circumstance for planar engagement.

On the other hand, fuzzy technique has been widely used to model complex nonlinear plants. Theoretical justification of fuzzy model as a universal approximaitor has been given in the last decade (see e.g., [13-15]). An important class of these fuzzy systems is the so-called Takagi-Sugeno (T-S) fuzzy system [15-21]. The basic idea of the T-S approach is first to decompose the nonlinear model into several linear systems according to different cases in which the linear models best fit the nonlinear system, and then aggregate each individual linear model into a single nonlinear one in terms of their membership functions. In many applications, the relatively complex consequence part enables the number of fuzzy rules (local models) to be quite small. As a result, the T-S approach is able to relax computational burden for many applications. Likewise, the Sliding Mode Control (SMC) schemes are known to possess the advantages of rapid response and robustness to model uncertainties and/or disturbances (see e.g., [3, 22-23] and the references therein). Thus, it has been widely applied to control a variety of systems [3,22-26]. Due to the advantages of T-S and SMC approaches, we will combine them to design a midcourse guidance law for an antimissile to a location with a desired direction, where a ballistic missile is predicted to pass in the reverse direction. Once the antimissile moves toward the ballistic missile along its predicted trajectory, the target can be easily found and locked for terminal interception.

The rest of this paper is organized as follows. Section 2 describes the kinematics of relative motion for missiles and the main goal of this paper. Section 3 presents the construction of the T-S fuzzy model and the design of guidance law for midcourse guidance. An example is

Manuscript received October 4, 2007; revised March 15, 2009; accepted September 14, 2009. Recommended by Editor-in-Chief Jin Bae Park. This research was supported by the Chung-San Institute of Science and Technology, National Science Council, Taiwan, R.O.C., under Grants NSC 94-2623-7-009-005, NSC 95-2623-7-009-007-D and NSC 96-2623-7-009-013-D.

Chun-Hone Chen is with the Missile and Rocket Systems Research Division, Chung-Shan Institute of Science and Technology, Taoyuan, Taiwan (e-mail: cch123.cch123@msa.hinet.net).

Yew-Wen Liang, Der-Cherng Liaw, Shih-Tse Chang, and Sheng-Dong Xu are with the Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu 30010, Taiwan (e-mails: {ywliang, ldc}@cn.nctu.edu.tw, {stchang.ece94g, sdxu.ece91g}@nctu.edu.tw).

* Corresponding author.

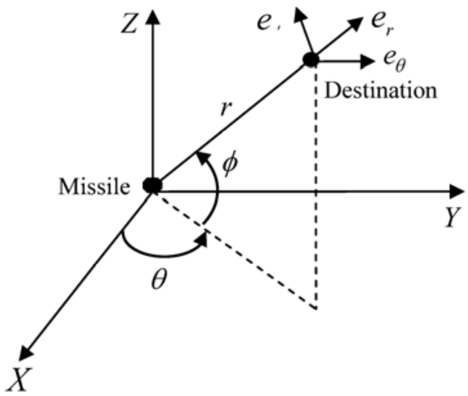

Fig. 1. Geometry of the guidance process.

also given there to demonstrate the use and benefits of the design. Finally, Section 4 gives the conclusions.

2. PROBLEM STATEMENT

This paper investigates the design of midcourse guidance laws for an antimissile to a predicted place with a specified direction. The relative motion between the antimissile and the predicted location is described by the spherical coordinate system with the origin fixed at the center of the antimissile as depicted in Fig. 1, where $\mathbf{e}_r$, $\mathbf{e}_\theta$, and $\mathbf{e}_\phi$ denote the three unit coordinate vectors in the spherical coordinate system, respectively.

For the design of the guidance law, we assume that the missile is point mass and only system kinematics is considered. The governing equation for the relative motion is given by [3]

$$\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\cos^2\phi = d_r - a_r, \tag{1}$$

$$r\ddot{\theta}\cos\phi + 2\dot{r}\dot{\theta}\cos\phi - 2r\dot{\phi}\dot{\theta}\sin\phi = d_\theta - a_\theta, \tag{2}$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} + r\dot{\theta}^2\cos\phi\sin\phi = d_\phi - a_\phi. \tag{3}$$

Here, r is the relative distance between the missile and the destination, θ and ϕ are the azimuth and pitch angles, respectively, a_r, $a_{M\theta}$, and a_ϕ denote the three command acceleration components of the missile in the spherical coordinates, which are designed to achieve the guidance mission. In addition, d_r, d_θ and d_ϕ denote possible uncertainties and disturbances.

The goal of this paper is to organize a guidance law to fulfill the guidance performance. That is, to realize the performance $r \to 0$, $\theta \to \theta_f$ and $\phi \to \phi_f$, where θ_f and ϕ_f denote the desired final azimuth and pitch angles.

3. GUIDANCE LAW ORGANTIZATION

To achieve the main goal of the paper as stated in Section 2, we will employ the Takagi-Sugeno (T-S) fuzzy approach and the Sliding Mode Control (SMC) technique to fulfill the design task.

Let $\mathbf{x} = (\mathbf{x}_1^T, \mathbf{x}_2^T)^T$, $\mathbf{x}_1 = (x_1, x_2, x_3)^T = (r, \theta, \phi)^T$ and $\mathbf{x}_2 = (x_4, x_5, x_6)^T = (\dot{r}, \dot{\theta}, \dot{\phi})^T$. Then System (1)-(3) can be written as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \tag{4}$$

$$\dot{\mathbf{x}}_2 = f(\mathbf{x}) = G(\mathbf{x})(\mathbf{u} + \mathbf{d}), \tag{5}$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{pmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ f_3(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} x_1x_6^2 + x_1x_5^2\cos^2 x_3 \\ 2x_5x_6\tan x_3 - \frac{2x_4x_5}{x_1} \\ -\frac{2x_4x_6}{x_1} - x_5^2\sin x_3\cos x_3 \end{pmatrix}, \tag{6}$$

$$G(\mathbf{x}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{1}{x_1\cos x_3} & 0 \\ 0 & 0 & \dfrac{1}{x_1} \end{pmatrix}, \tag{7}$$

$$\mathbf{u} = (u_1, u_2, u_3)^T = (-a_r, -a_\theta, -a_\phi)^T, \tag{8}$$

$$\mathbf{d} = (d_1, d_2, d_3)^T = (d_r, d_\theta, d_\phi)^T. \tag{9}$$

3.1. T-S fuzzy model description

It is known that a smooth nonlinear model can be accurately approximated by a T-S fuzzy model if enough fuzzy rules are used (see e.g., [15]). A T-S fuzzy model is described by a weighted combination of several linear models. Each of the linear models associates with a fuzzy implication (rule). Suppose that there are p rules with linear models described by

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \tag{10}$$

$$\dot{\mathbf{x}}_2 = A_i\mathbf{x} + B_i\mathbf{u}, \quad i = 1, \ldots, p. \tag{11}$$

Then the T-S model is constructed in the form of (12)-(13) below to approximate the original nonlinear system:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \tag{12}$$

$$\dot{\mathbf{x}}_2 = \sum_{i=1}^{p} \alpha_i(\mathbf{x})[A_i\mathbf{x} + B_i\mathbf{u}], \tag{13}$$

where $\alpha_i(\mathbf{x}) \geq 0$ for all i and $\sum_{i=1}^{p} \alpha_i(x) = 1$.

With the help of the T-S model, we may rewrite the original System (4)-(5) as follows:

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2, \tag{14}$$

$$\dot{\mathbf{x}}_2 = \sum_{i=1}^{p} \alpha_i(\mathbf{x})A_i\mathbf{x} + \Delta\mathbf{f} + \left(\sum_{i=1}^{p} \alpha_i(\mathbf{x})B_i + \overline{\Delta G}\right)(\mathbf{u} + \mathbf{d}), \tag{15}$$

where $\Delta\mathbf{f} = \Delta\mathbf{f}(\mathbf{x}) := \mathbf{f}(\mathbf{x}) - \sum_{i=1}^{p} \alpha_i(\mathbf{x})A_i\mathbf{x}$ and $\overline{\Delta G} = \overline{\Delta G}(\mathbf{x}) := G(\mathbf{x}) - \sum_{i=1}^{p} \alpha_i(\mathbf{x})B_i$. During the guidance process, we assume that $r > 0$ and $0 < \phi < \pi/2$. It implies that $G(\mathbf{x})$, as given by (7), and B_i are diagonal and positive definite matrices for all

$i=1,\ldots,p.$ It follows that $\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i>0$ during the guidance process. Thus, we may further rewrite System (14)-(15) in the form of (16)-(17) below:

$$\dot{\mathbf{x}}_1=\mathbf{x}_2 \tag{16}$$

$$\dot{\mathbf{x}}_2=\sum_{i=1}^{p}\alpha_i(\mathbf{x})A_i\mathbf{x}+\Delta\mathbf{f}+\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i\right)(I+\Delta G)(\mathbf{u}+\mathbf{d}), \tag{17}$$

where $\Delta G=(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i)^{-1}\cdot\overline{\Delta G}.$

3.2. SMC controller design

As is well known, the Sliding Mode Control (SMC) scheme has the advantages of rapid response and robustness to model uncertainties and/or disturbances [22,25,26]. Thus, it has been widely applied to many control problems. In the following, we will employ the SMC scheme to realize the guidance performance based on the T-S model expression (16)-(17).

It is known that the SMC design consists of two main steps [25,26]. The first step is to select an appropriate sliding surface, which should have the property that the desired performance can be achieved if the system state maintains itself on the selected sliding surface. The next step is to organize a control law that forces the system state to reach the sliding surface in a finite amount of time and make the sliding surface an invariant manifold. For the first step, we let

$$\mathbf{e}=\mathbf{x}_1-\mathbf{x}_{1f} \quad\text{and}\quad \mathbf{x}_{1f}=(0,\theta_f,\phi_f)^T. \tag{18}$$

Select the sliding surface to be

$$\mathbf{s}=(s_1,s_2,s_3)^T=\dot{\mathbf{e}}+M\mathbf{e}=0, \tag{19}$$

where $M=\mathrm{diag}\{m_1,m_2,m_3\}>0.$ Clearly, the objective is achieved, i.e., $\mathbf{e}\to 0,$ if the state stays on the sliding surface, and the convergence rate depends on the choice of the eigenvalues of M.

For the second step, it is noted from (17)-(19) that

$$\begin{aligned}\dot{\mathbf{s}}&=\ddot{\mathbf{e}}+M\dot{\mathbf{e}}\\&=\ddot{\mathbf{x}}_1-\ddot{\mathbf{x}}_{1f}+M\dot{\mathbf{e}}\\&=-\ddot{\mathbf{x}}_{1f}+M\dot{\mathbf{e}}+\sum_{i=1}^{p}\alpha_i(\mathbf{x})A_i\mathbf{x}+\Delta\mathbf{f}\\&\quad+\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i\right)(I+\Delta G)(\mathbf{u}+\mathbf{d}).\end{aligned} \tag{20}$$

According to the SMC design procedure [25,26], we choose

$$\mathbf{u}=\mathbf{u}^{eq}+\mathbf{u}^{re}, \tag{21}$$

$$\mathbf{u}^{eq}=\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i\right)^{-1}\left[-\sum_{i=1}^{p}\alpha_i(\mathbf{x})A_i\mathbf{x}+\ddot{\mathbf{x}}_{1f}-M\dot{\mathbf{e}}\right]. \tag{22}$$

It follows that

$$\begin{aligned}\dot{\mathbf{s}}&=\Delta\mathbf{f}+\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i\right)(I+\Delta G)(\mathbf{u}^{re}+\mathbf{d})\\&\quad+\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i\right)\Delta G\cdot\mathbf{u}^{eq}.\end{aligned} \tag{23}$$

In order to force the system state to reach the sliding surface in a finite amount of time, we impose the following assumption, where $(\cdot)_j$ and $(\cdot)_{jj}$ denote the jth and the (j,j)th entries of a vector and a matrix, respectively.

Assumption 1: There exists nonnegative functions $\rho_j(\mathbf{x},t)$ and $\sigma_j(\mathbf{x},t),$ $j=1,2,3,$ such that

$$\begin{aligned}&|(\Delta\mathbf{f})_j+\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})(B_i)_{jj}\right)\\&\quad\times[d_j+(\Delta G)_{jj}\cdot(d_j+(\mathbf{u}^{eq})_j)]|\le\rho_j(\mathbf{x},t),\end{aligned} \tag{24}$$

$$(\Delta G)_{jj}\le\sigma_j(\mathbf{x},t)<1. \tag{25}$$

Under Assumption 1, we propose

$$\begin{aligned}\mathbf{u}^{re}&=-\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})B_i\right)^{-1}\\&\quad\times\left(\frac{\rho_1(\mathbf{x},t)+\eta_1}{1-\sigma_1(\mathbf{x},t)}\mathrm{sgn}(s_1),\cdots,\frac{\rho_3(\mathbf{x},t)+\eta_3}{1-\sigma_3(\mathbf{x},t)}\mathrm{sgn}(s_3)\right)^T,\end{aligned} \tag{26}$$

where $\eta_1,$ η_2 and η_3 are three positive constants. It follows from (23), (26) and Assumption 1 that

$$\begin{aligned}s_j\dot{s}_j&\le\rho_j(\mathbf{x},t)|s_j|+s_j\left(\sum_{i=1}^{p}\alpha_i(\mathbf{x})(B_i)_{jj}\right)\\&\quad\times(1+(\Delta G)_{jj})(\mathbf{u}^{re})_j\\&\le\rho_j(\mathbf{x},t)|s_j|-\tfrac{\rho_j(\mathbf{x},t)+\eta_j}{1-\sigma_j(\mathbf{x},t)}(1+(\Delta G)_{jj})|s_j|\\&\le\rho_j(\mathbf{x},t)|s_j|-\tfrac{\rho_j(\mathbf{x},t)+\eta_j}{1-\sigma_j(\mathbf{x},t)}(1-\sigma_j(\mathbf{x},t))|s_j|\\&\le-\eta_j\cdot|s_j|.\end{aligned} \tag{27}$$

That is, the system state will reach the sliding surface in a finite amount of time and remain there hereafter. According to the selected sliding surface (19), the guidance performance is then achieved. From the discussions above, we hence have the following result:

Theorem 1: Suppose that $G(\mathbf{x})$ is a diagonal matrix and Assumption 1 holds. Then the guidance performance for System (4)-(5) can be achieved by the control law (21), (22) and (26).

To compare the structure of the T-S type SMC controller presented in this paper with that of the classic SMC controller given in [3], it is noted that the latter controller needs to compute the nonlinear term $\mathbf{f}(\mathbf{x})$ and the inverse dynamics $G^{-1}(\mathbf{x})$ on-line, which might be involved and consume significant computing effort and time. On the contrary, the terms $A_i,$ $B_i^{-1},$ $\sigma_i(\mathbf{x},t)$ and

$\rho_i(\mathbf{x},t)$ in the T-S type SMC controller can be computed off-line. Thus, the T-S type approach may alleviate significant on-line computational burden, especially when the system dynamics is complicated. Although the controller design, as stated above, is mainly for the case of $n=3$, i.e., three 2^{nd}-order differential equations, it is easy to extend to the case of general n.

3.3. An illustrative example

Here, we present an example to demonstrate the use and benefits of the approach. To determine the T-S model, it is observed that the azimuth angle θ does not appear in the governing equations (1)-(3), thus we only select r and ϕ as premise variables. The associated operating points for deriving the linear models are selected from the possible workspace, so that the motion of the anti-missile can be well approximated using a convex combination of these linear models. In this example, we select the operating points to be in the form $(r,\theta,\phi,\dot{r},\dot{\theta},\dot{\phi})$ $=(r_i,\ 0,\ \phi_j,\ -2.5,\ 0,\ 0)$ where r_i and ϕ_j are described in Fig. 2 with triangular membership functions, and $\dot{r}=-2.5$ km/sec ≈ 7.4 Mach for each selected operating point. Since the T-S type controller uses only two premise variables r and ϕ, it therefore triggers at most four rules (i.e., at most four linear models) at each time instant, as shown in Fig. 3. Thus, it does not create extra on-line computational burden if the partition for the regions of r and ϕ are made finer. However, since the maximum of a function over the whole region is greater than or equal to that of the same function over a smaller sub-region, it follows that a finer partition for the region of r and ϕ will result in a smaller magnitude of $\sigma_k(\mathbf{x},t)$, $\rho_k(\mathbf{x},t)$, as stated in Assumption 1. As a result, the control magnitude will be smaller so that the physical control magnitude constraint is easier to be fulfilled for practical applications if the partition of r and ϕ are made finer. According to the selected operating points, the associated linear models can be easily determined. Three of them are listed as below, while the others are omitted:

$$A_{11}=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix},\ B_{11}=\begin{pmatrix}1&0&0\\0&0.2611&0\\0&0&0.2\end{pmatrix},$$

$$A_{12}=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{pmatrix},\ B_{12}=\begin{pmatrix}1&0&0\\0&0.311&0\\0&0&0.2\end{pmatrix},$$

$$A_{21}=\begin{pmatrix}0&0&0&0&0&0\\0&0&0&0&0.5&0\\0&0&0&0&0&0.5\end{pmatrix},\ B_{21}=\begin{pmatrix}1&0&0\\0&0.1305&0\\0&0&0.1\end{pmatrix},$$

where (A_{ij},B_{ij}) is the linear model associated with the operating point $(r_i,\ 0,\ \phi_j,\ -2.5,\ 0,\ 0)$. To verify Assumption 1, we define D_{ij} the region $D_{ij}=\{\mathbf{x}\,|\,r_i\le r\le r_{i+1},\ \phi_j\le\phi\le\phi_{j+1}\}$. Then the upper bounds for $|(\Delta\mathbf{f})_k|$ and $\sigma_k(\mathbf{x},t)$, as stated in Assumption 1, in the region D_{ij} can be off-line computed, which are shown in Table 1. In addition, we assume that the disturbance with the wind and aerodynamic effects adopted from [10] has the following form:

$$d_i(t)=0.1\sin(t)+u_s(t-3)-u_s(t-4) \tag{28}$$

for all i, where $u_s(t)$ denotes the unit step function. We also use the saturation function $\mathrm{sat}(s_i/0.05)$ instead of $\mathrm{sgn}(s_i)$ for both the T-S type and the classic SMC schemes, and the control parameters for both schemes are selected to be $\eta_1=\eta_2=2$, $\eta_3=1$ and $m_i=1$ for $i=1, 2, 3$. The initial states and the desired final azimuth and pitch angles are chosen as $r_0=25$ km, $\dot{r}_0=-1$ km/sec, $\theta_0=25^0$, $\dot{\theta}_0=-0.1^0$/sec, $\phi_0=75^0$, $\dot{\phi}_0=-0.1^0$/sec, $\theta_f=15^0$ and $\phi_f=80^0$, respectively. It is noted that the planar engagement algorithms may not be directly applied since the initial velocity and the final velocity with the specified direction are generally not in a plane that contains the initial locations of the missile and the target.

Numerical simulations are summarized in Figs. 4-6 and Table 2. Among these, Figs. 4 and 5 display the time history of the tracking errors and controls, respectively, where $e_1:=r$, $e_2:=\theta-\theta_f$ and $e_3:=\phi-\phi_f$. The associated space trajectories are depicted in Fig. 6, and the performances of the two SMC designs are shown in Table 2. It is observed from Figs. 4 and 6 that the guidance mission is successfully achieved by both the T-S type and

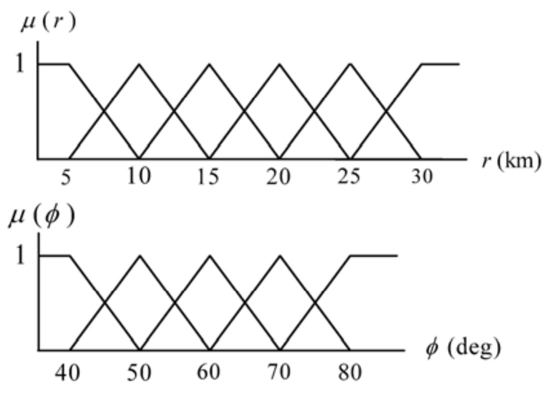

Fig. 2. The membership functions for r and ϕ.

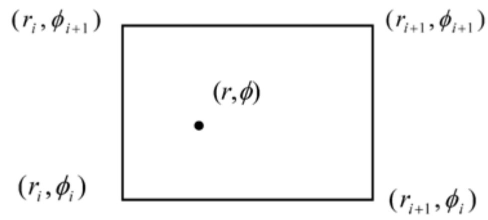

Fig. 3. The four adjacent operating points being triggered at each time instant.

Table 1. Estimated upper bounds for $\sigma_k(\mathbf{x},t)$ and $|(\Delta\mathbf{f})_k|$ in the region D_{ij}.

	$\sup_{\mathbf{x}\in D_{ij}} \sigma_k(\mathbf{x},t)$, $k=1, 2, 3$	$\sup_{\mathbf{x}\in D_{ij}} \|(\Delta\mathbf{f})_k\|$, $k=1, 2, 3$
D_{11}	{0, 0.1806, 0.1}	{0.1587,0.1748,0.1616}
D_{12}	{0, 0.2444, 0.1}	{0.1413, 0.1837,0.1616}
D_{13}	{0, 0.3848, 0.1}	{0.125,0.1913,0.161}
D_{14}	{0, 0.8594, 0.1}	{0.1117, 0.2701,0.1599}
D_{21}	{0, 0.0685, 0.0333}	{0.238, 0.1048, 0.0916}
D_{22}	{0, 0.0963, 0.0333}	{0.212, 0.1135, 0.0916}
D_{23}	{0, 0.159, 0.0333}	{0.1875, 0.1318, 0.091}
D_{24}	{0, 0.381, 0.0333}	{0.1669,0.1416,0.0899}
D_{31}	{0, 0.0384, 0.0167}	{0.3174, 0.0787,0.0655}
D_{32}	{0, 0.0555, 0.0167}	{0.2826, 0.0875,0.0655}
D_{33}	{0, 0.949, 0.0167}	{0.25, 0.0952,0.0649}
D_{34}	{0, 0.2377, 0.0167}	{0.2234, 0.1736,0.0638}
D_{41}	{0, 0.0256, 0.01}	{0.3967, 0.0649,0.0516}
D_{42}	{0, 0.0378, 0.01}	{0.3533,0.0734, 0.0516}
D_{43}	{0, 0.0662, 0.01}	{0.3125, 0.0813,0.051}
D_{44}	{0, 0.1710, 0.01}	{0.2792, 0.0702,0.0499}
D_{51}	{0, 0.0179, 0.0067}	{0.476, 0.056,0.0429}
D_{52}	{0, 0.0281, 0.0067}	{0.427, 0.0649,0.0429}
D_{53}	{0, 0.0503, 0.0067}	{0.375, 0.0825,0.0423}
D_{54}	{0, 0.1329, 0.0067}	{0.3351, 0.0781,0.0412}

the classic SMC schemes. The response curves, including the control curves in Fig. 5, of the two schemes are seen close to each other. However, as seen from Table 2 of this example, the T-S type scheme requires less total energy $\int \mathbf{u}^T\mathbf{u}$ and smaller maximum control magnitude $\|\mathbf{u}\|_\infty := \max_t \|\mathbf{u}\|$ than the classic SMC scheme. It is also noted from Table 2 that the T-S type scheme consumes more energy in the radial direction than the classic scheme in the following relation: $(\int |u_1|^2)_{\text{T-S}} = 4.010 > (\int |u_1|^2)_{\text{classic}} = 3.258$. Thus, the time t_{reach} when the antimissile reaches the destination for the T-S type scheme is also faster than that of the classic SMC scheme. Both control efforts for the two schemes are observed to experience abrupt change during the time period of $t=3$ to $t=4$. These are clearly resulted from the square disturbance $u_s(t-3)-u_s(t-4)$. The T-S type scheme is also found from Fig. 5(a) to have several small jumps near $t=4.73$, 6.80 and 7.26. These jumps come from the variation of the upper bounds $\sigma_k(\mathbf{x},t)$ and $\rho_k(\mathbf{x},t)$, which affect the magnitude of $\mathbf{u}^{re}$, when the system state switches from one region D_{ij} to another. As for the CPU time for on-line implementing the controllers, the T-S type scheme (including the determination of the weightings α_i) is found to require

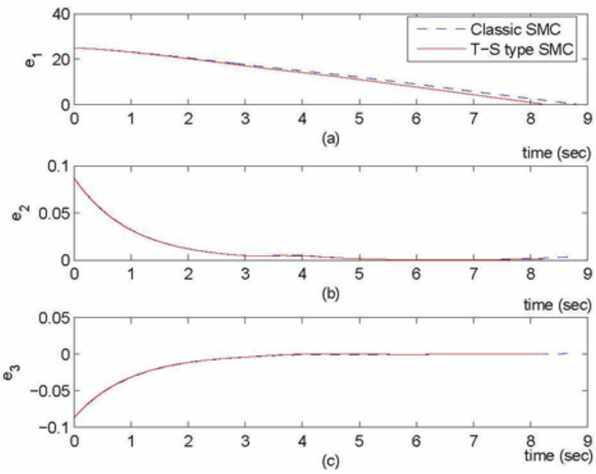

Fig. 4. Time history of the tracking errors by the T-S type and the classic SMC designs.

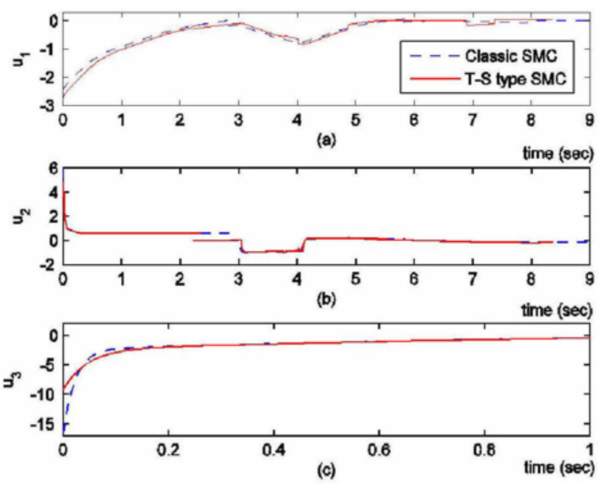

Fig. 5. Time history of the controls by the T-S type and the classic SMC designs.

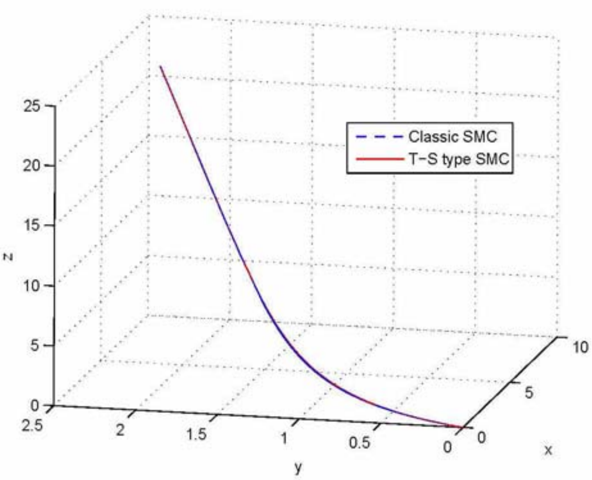

Fig. 6. The space trajectories by the T-S type and the classic SMC designs.

Table 2. Performances of the two SMC schemes.

	Classic SMC design	T-S type SMC design
t_{reach} (time when $r = 1\text{km}$)	8.53	7.98
$\max_t \lvert u_i(t) \rvert$, $i = 1, 2, 3$	{2.467, 6.441, 16.648}	{2.802, 4.517, 8.840}
$\lVert \mathbf{u} \rVert_\infty$	18.020	10.315
$\int \lvert u_i \rvert^2$, $i = 1, 2, 3$	{3.258, 1.657, 8.137}	{4.010, 1.540, 6.680}
$\int \mathbf{u}^T \mathbf{u}$	13.052	12.23

less CPU time than that of the classic SMC scheme in the following relation (when compute both controllers a million times): $(\text{CPU})_{\text{T-S}} = 4.766 \sec < (\text{CPU})_{\text{classic}} = 7.625$ sec. From this example, it is concluded that the T-S type approach can not only alleviate the on-line computational burden, but can also efficiently guide the antimissile to perform the midcourse guidance mission as that of the classic SMC designs.

4. CONCLUSION

We have employed the Takagi-Sugeno (T-S) fuzzy approach and the Sliding Mode Control (SMC) technique to design the guidance law for an antimissile to a place with a specified direction. The desired location and direction may be time-varying due to the information update from a trajectory prediction algorithm. It was shown that the guidance law is able to guide the antimissile to achieve the guidance performance, even when the existence of disturbances. In addition, the control parameters of the T-S approach can be computed off-line so that the on-line computational burden of the classic SMC designs can be greatly alleviated, especially when the system dynamics is complicated. Simulation results demonstrate the benefits of the scheme.

REFERENCES

[1] B. T. Burchett, "Feedback linearization guidance for approach and landing of reusable Launch Vehicles," *Proc. of American Control Conference*, Portland, OR, USA, June 8-10, 2005.

[2] A. Das, R. Das, S. Mukhopadhyay, and A. Patra, "Sliding mode controller along with feedback linearization for a nonlinear missile model," *Proc. of the 1st International Symposium on Syst. and Control in Aerospace and Astronautics*, 2006.

[3] D.-C. Liaw, Y.-W. Liang, and C.-C. Cheng, "Nonlinear control for missile terminal guidance," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 122, pp. 663-668, Dec. 2000.

[4] F.-K. Yeh, K.-Y. Cheng, and L.-C. Fu, "Variable structure-based nonlinear missile guidance/ autopilot design with highly maneuverable actuators," *IEEE Trans. Control Systems Technology*, vol. 12, no. 6, pp. 944-949, Nov. 2004.

[5] F.-K. Yeh, H.-H. Chien, and L.-C Fu, "Design of optimal midcourse guidance sliding-mode control for missiles with TVC," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 39, no. 3, pp. 824-837, 2003.

[6] C.-L. Lin, H.-Z. Hung, Y.-Y. Chen, and B.-S. Chen, "Development of an integrated fuzzy-logic based missile guidance law against high speed target," *IEEE Trans. Fuzzy Syst.*, vol. 12, no. 2, pp. 157-169, Apr. 2004.

[7] C.-M. Lin and Y.-F. Peng, "Missile guidance law design using adaptive cerebellar model articulation controller," *IEEE Trans. Neural Netw.*, vol. 16, pp. 636-644, no. 3, May 2005.

[8] Y. Oshman and D. Arad, "Differential-game-based guidance law using target orientation observations," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 42, no. 1, pp. 316-326, Jan. 2006.

[9] C.-K. Ryoo, H. Cho, and M.-J. Tahk, "Optimal guidance laws with terminal impact angle constraint," *Journal of Guidance, Control, and Dynamics*, vol. 28, no. 4, pp. 724-732, 2005.

[10] J.-I. Lee, I.-S. Jeon, and M.-J. Tahk, "Guidance law to control impact time and angle," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 43, no. 1, pp. 301-310, 2007.

[11] M.-G. Yoon, "Relative circular navigation guidance for the impact angle control problem," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 44, no. 4, pp. 1449-1463, 2008.

[12] B.-S. Chen, C.-S. Tseng, and H.-J. Uang, "Robustness design of nonlinear dynamic system via fuzzy linear control," *IEEE Trans. on Fuzzy Systems*, vol. 7, pp. 571-585, Oct. 1999.

[13] B. Kosko, "Fuzzy systems as universal approximators," *IEEE Trans. on Computers*, vol. 43, pp. 1329-1333, 1994.

[14] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. on Systems, Man, and Cybernetics*, vol. SMC-15, pp. 116-132, Feb. 1985.

[15] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis - A Linear Matrix Inequality Approach,* John Wiley & Sons, Inc., 2001.

[16] D. W. Kim, J. B. Park, Y. H. Joo, and S. H. Kim, "Multirate digital control for fuzzy systems: LMI-based design and stability analysis," *International Journal of Control, Automation, and Systems*, vol. 4, no. 4, pp. 506-515, 2006.

[17] F.-H. Hsiao, J.-D. Hwang, C.-W. Chen, and Z.-R. Tsai, "Robust stabilization of nonlinear multiple time-delay large-scale systems via decentralized fuzzy control," *IEEE Trans. on Fuzzy Systems*, vol. 13, no. 1, pp. 152-163, 2005.

[18] J. Dong and G.-H. Yang, "Static output feedback control synthesis for discrete-time T-S fuzzy systems," *International Journal of Control, Automation, and Systems*, vol. 5, no. 3, pp. 349-354, 2007.

[19] C.-H. Sun and W.-J. Wang, "An improved stability

criterion for T-S fuzzy discrete systems via vertex expression," *IEEE Trans. on Systems, Man, and Cybernetics, Part B*, vol. 36, no. 3, pp. 672-678, 2006.

[20] C.-S. Tseng, B.-S. Chen, and H.-J. Uang, "Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model," *IEEE Trans. on Fuzzy Systems*, vol. 9, no. 3, pp. 381-392, 2001.

[21] J. H. Kim, J. B. Park, and H. S. Yang, "Implementation of the avoidance algorithm for autonomous mobile robots using fuzzy rules," *Fuzzy Systems and Knowledge Discovery, Lecture Notes in Computer Science*, vol. 4223, pp. 836-845, SpringerLink, 2006.

[22] R. A. Decarlo, S. H. Zak, and G. P. Mathews, "Variable structure control of nonlinear multivariable systems: a tutorial," *Proc. of the IEEE*, vol. 76, pp. 212-232, 1988.

[23] N. M. Dung, V. H. Duy, N. T. Phuong, S. B. Kim, and M. S. Oh, "Two-wheeled welding mobile robot for tracking a smooth curved welding path using adaptive sliding-mode control technique," *International Journal of Control, Automation, and Systems*, vol. 5, no. 3, pp. 284-294, 2007.

[24] H. K. Khalil, *Nonlinear Systems*, 3rd edition, Prentice-Hall, New Jersey, 2000.

[25] Y.-W. Liang and S.-D. Xu, "Reliable control of nonlinear systems via variable structure scheme," *IEEE Trans. on Automatic Control*, vol. 51, no. 10, pp. 1721-1726, 2006.

[26] Y.-W. Liang, S.-D. Xu, and C.-L. Tsai, "Study of VSC reliable designs with application to spacecraft attitude stabilization," *IEEE Trans. on Control Systems Technology*, vol. 15, no. 2, pp. 332-338, 2007.

Chun-Hone Chen received his Ph.D. degree in Institute of Electrical and Control Engineering from the National Chiao Tung University, Hsinchu, Taiwan, R.O.C. in 2002. Currently, he is an Associate Research Fellow in Missile and Rocket System Research Division of Chung-Shan Institute of Science and Technology, Taoyuan, Taiwan, R.O.C.

Yew-Wen Liang was born in Taiwan, Republic of China, in 1960. He received his B.S. degree in Mathematics from Tung Hai University, Taichung, Taiwan, R.O.C., in 1982, an M.S. degree in Applied Mathematics in 1984, and a Ph.D. degree in Electrical and Control Engineering in 1998 from National Chiao Tung University, Hshinchu, Taiwan, R.O.C. Since August 1987, he has been with the National Chiao Tung University, where he is currently an Associate Professor of Electrical and Control Engineering. His research interests include nonlinear control systems, reliable control, and fault detection and diagnosis issues.

Der-Cherng Liaw received his B.S. degree in Control Engineering from National Chiao Tung University, Hshinchu, Taiwan, Republic of China, in 1982, an M.S. degree in Electrical Engineering from the National Taiwan University in 1985 and a Ph.D. degree in Electrical Engineering from University of Maryland, College Park, in 1990. He is currently a senior member of IEEE. During1990-1991, he was a post-doctoral fellow with the Institute for Systems Research, University of Maryland for one year. Since August 1991, he has been with the National Chiao Tung University, where he is currently a Professor of Electrical and Control Engineering. During 1994-1995, he was the Director of the Institute of Control Engineering, National Chiao Tung University. His research interests include nonlinear control systems, spacecraft control, singular perturbation methods, bifurcation control, applications to jet engine control, flight control, electric power systems and power electronics, RFID, and implementation issues. During 1990-1992, he served as a Designated Assistant to the Associate Editor for the IEEE Transactions on Automatic Control. Dr. Liaw was also a member of Editorial Board of 1993 American Control Conference.

Shih-Tse Chang received his M.S. degree in Electronic Engineering from I-Shou University, Kaohsiung, Taiwan in 2005. He is currently working toward a Ph.D. degree in the Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu, Taiwan. His research interests include nonlinear control systems, bifurcation control, and system identification.

Sheng-Dong Xu received his Ph.D. degree from the Department of Electrical and Control Engineering at National Chiao Tung University, Taiwan, in 2009. He is currently a Faculty Member with the Department of Computer Science at National Chengchi University, Taiwan. His research interests include VLSI design, embedded systems, control engineering, and biomedical electronics.