Effective interactions and energy spectra for $N = 28$, 50, and 82 isotones*

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(Received 10 February 1978)

A tensor plus central potential is used to explore the universality of the effective interactions for $N = 28$, 50, and 82 isotones. It is found that the central components of the potential can be practically unified for the three mass regions. The tensor components are different for the three mass regions with the tendency of lesser importance for heavier nuclei. The energy spectra of the low-lying states of $N = 28$, 50, and 82 isotones are calculated and compared with experimental data.

[NUCLEAR STRUCTURE $N=28$, 50, and 82, calculated effective interactions and energy spectra.]

I. INTRODUCTION

In the calculations in seeking for effective interactions one usually chooses a model space and tries to determine the effective interactions by optimizing the agreement between the calculated results and experimental data. Therefore, the resulting effective interactions are strongly correlated with the choice of model space. Furthermore, the different forms of potentials used in theoretical calculations make the interpretation of theoretical calculations more obscure. It is, therefore, not easy to understand the physical contents of the numerous effective interactions from a fundamental point of view.

In past few years, the experimental data of the effective two-body interactions became more abundant and a systematic analysis of their properties became possible. Schiffer *et al.* made a rather extensive analysis on the experimental data of the "angular distribution" and the multipole coefficients of the effective two-body interactions of different multiplets.[1-3] The similarity of the "angular distribution" and the multipole coefficients between different multiplets suggests the universality of the two-body effective interactions. Since then, many efforts have been made to manifest the universal property of the two-body effective interactions by theoretical calculations.[4-7] In order to explore the universal property, we need a unified calculation in which many orbitals of different mass regions are involved. In the calculations of Schiffer *et al.*,[5] all orbitals with reliable experimental diagonal two-body effective interactions were included. Since the experimental information of the nondiagonal two-body effective interactions is quite meager, the nondiagonal matrix elements were not taken into account in their calculations. Therefore, configuration mixing effects were omitted implicitly. An alternative approach is to determine the effective interactions by the experimental energy spectra of the nuclei of several mass regions directly. This approach has the advantage that configuration mixing effects are taken into account automatically. However, relatively fewer orbitals can be included in one calculation, and the universality is explored only partially. In this work, we report some results in this approach.

From past calculations it is well known that shell model can be applied to $N=28$, 50, and 82 isotones with considerable success. Therefore, it may be possible to extract some information of universality of proton-proton effective interactions by a unified calculation on the $N=28$, 50, and 82 isotones. With this motivation in mind we have performed a shell model calculation on $N=28$, 50, and 82 isotones by using a two-range central plus tensor potential suggested by Schiffer *et al.* In Sec. II, we shall describe our model. The results of effective interactions and energy spectra are presented in Secs. III and IV, respectively. Finally we present our conclusions.

II. MODEL

In the calculation, ^{48}Ca, ^{88}Sr, and ^{132}Sn are assumed to be the inert cores for $N=28$, 50, and 82 isotones, respectively. For $N=28$ isotones, the Z-20 valence protons are assumed to occupy the $1f_{7/2}$ orbital only. There were many shell model calculations on $N=28$ isotones. In the earlier calculations only $1f_{7/2}$ orbit was included as the active orbit.[8-10] For low-lying energy spectra good

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agreement between the calculated results and experimental data was obtained except for very few states that did not have model space analogs. The model space was later on enlarged to include the $2p_{3/2}$ and $1f_{5/2}$ orbitals by numerous calculations in which the electromagnetic transition rates and spectroscopic factors are also analyzed in addition to the energy spectra. However, as far as the low-lying energy spectra are concerned the larger model spaces produce quite similar results. Therefore, the pure $1f_{7/2}$ configuration may serve as a good approximation for low-lying spectra of $N=28$ isotones. For $N=50$ isotones, we assume that the neutron shell is closed at $1g_{9/2}$ and the $Z-38$ valence protons are distributed in $2p_{1/2}$ and $1g_{9/2}$ orbitals. There were many shell model calculations on $N=50$ isotones with this choice of model space. It was shown that except for very few core excitation states the low-lying states can be described quite well by this choice of model space.[11-13] For $N=82$ isotones, the $Z-50$ valence protons are assumed to distribute in $1g_{7/2}$ and $2d_{5/2}$ orbitals. It was shown in previous calculations[14-16] that the configuration intensities of the wave functions for the low-lying states of $N=82$ isotones with proton number ≤ 7 are concentrated in this model space. With these prescriptions of the model spaces the complicated computation work in handling such a long string of nuclei is considerably reduced.

For the effective interactions, a two-range central plus tensor potential is assumed. The radial part of the potential is assumed to be of Yukawa type. The short range is chosen to be 1.415 fm which is about the value of the pion Compton wavelength. The long-range components of potential are included to mediate the core polarization effects. Previous calculation showed that the quality of fitting did not depend on the specific value of long range so we fix the value of long range at 2 fm tentatively. Harmonic oscillator wave functions are used for the single-particle wave functions. As a conventional choice, the mass dependent oscillation parameter is assumed to be $\nu = 0.96 \times A^{-1/3}$ fm^{-2}. In actual calculation the average masses were chosen to be 50, 90, and 140 for $N=28$, 50, 82 isotones, respectively.

With these prescriptions we performed a least-squares-fit calculation on the low-lying energy levels of $N=28$, 50, and 82 isotones. For $N=28$ and 50 mass regions, nuclei with proton number up to 6, and for $N=82$ mass region, nuclei with proton number up to 7 are considered in the calculation. In selecting the input experimental data, those levels that are apparently beyond the model space analogs are excluded. Also excluded are those levels that can not be reproduced in previous shell model calculations with more adjustable parameters or even with larger model spaces. With these criteria, 23, 51, and 14 low-lying levels were selected, respectively, for $N=28$, 50, and 82 isotones.

In order to avoid Coulomb displacement energy correlation we tried to minimize the excited energies of those levels relative to the corresponding ground states of the isotones. Since we have only proton-proton interactions, we do not have to consider $T=0$ components. Therefore, there are only six parameters for the residual interactions, i.e., the central singlet-even, triplet-odd, and tensor-odd components of both ranges. In order to reduce the number of parameters further, we noticed that the long-range tensor-odd component was not as important as other components.[5,6] We therefore, omitted this component and left only five parameters for residual interactions. The $1g_{9/2}$-$2p_{1/2}$ and $2d_{5/2}$-$1g_{7/2}$ relative level spacings in $N=50$ and 82 isotones are also left as free parameters. The former 7 parameters were varied until a best agreement between the calculated and experimental energy levels was reached. In seeking for the minimum we performed least-squares-fit calculations for each mass region, separately, first. The universality of the effective interactions, if it exists, will manifest in the overlapping of the three sets of parameters for each mass region. We then performed a unified least-squares-fit calculation for the three mass regions together. The comparison of this set of parameters with the former three also gives us information about the universality of the effective interactions.

III. RESIDUAL INTERACTIONS

The best fit parameters determined in the least-squares calculations are summarized in Table I. In seeking for the best fit parameters for $N=28$ isotopes, two sets of parameters with nearly equal quality of fittings were found. They are denoted as set 1 and set 2 in Table I. The ambiguity in the $N=28$ parameters is, of course, closely related to the fact that our number of free parameters is more than the number of independent two-body matrix elements. However, the ambiguity should be removed when the larger calculation involving three mass regions is carried out. In the vast point of view, the number of free parameters is much smaller than the number of adjustable two-body matrix elements. The rms deviations corresponding to the two sets of parameters are of the same value, 0.16 MeV. Furthermore, they produce very similar two-body matrix elements and energy spectra. The resulting two-body matrix elements and energy spectra are displayed in Ta-

TABLE I. Strengths (in MeV) and relative ratios of the best fit potential parameters.

		$N=28$					
	Range	set 1	set 2	$N=50$	$N=82$	Overall	ST
CSE	short	−62.7	−99.14	−99.86	−101.12	−100.33	−49.32
	long	11.21	28.17	28.56	29.02	28.75	15.47
long/short	ratio	0.18	0.28	0.29	0.29	0.29	0.31
CTO	short	−250.15	−221.37	−217.32	−220.65	−216.23	−155.82
	long	105.76	92.56	91.36	92.13	90.68	62.06
long/short	ratio	0.42	0.42	0.42	0.42	0.42	0.40
TTO	short	15.97	37.0 ±3.0	19.0 ±1.0	6.0 ±4.0	22.16±3.0	−6.10
CSE/CTO	short ratio	0.25	0.45	0.46	0.45	0.46	0.32
CSE/CTO	long ratio	0.11	0.30	0.31	0.31	0.32	0.25

ble II and Table III for comparison. In Table II the two-body matrix elements calculated by McGrory *et al.* are also shown for comparison.[17] The similarity of the two sets of values is quite striking. This means that the two sets of parameters are equally acceptable. The strengths and relative ratio of short and long range components of the central triplet-odd components are quite similar for both sets. The major differences are in the central singlet-even and tensor components. The similarity between the two-body matrix elements corresponding to the two sets of parameters suggests that there are some correlations between the central singlet-even and tensor components. However, when the two sets of parameters were used as initial values in seeking for the best fit parameters for other mass regions, it was found that set 2 is preferred. The best fit parameters for $N=50$ and $N=82$ isotones are displayed in the fifth and sixth columns of Table I. The rms deviations for $N=50$ and $N=82$ isotones are 0.20 and 0.19 MeV, respectively. The best fit $1f_{9/2}$-$2p_{1/2}$ and $2d_{5/2}$-$1g_{7/2}$ single particle level spacings are found to be 1.23 and 1.10 MeV, respectively. These values are slightly larger than the corresponding experimental values 0.906 and 0.963 MeV deduced from the first excited $9/2^+$ and $5/2^+$ levels of ^{89}Y and ^{133}Sb. The central singlet-even and triplet-odd components for $N=50$ and $N=82$ isotones are nearly the same as those of the set 2 parameters for $N=28$ isotones. The tensor-odd components are, however, quite different from each other and also different from that of $N=28$ isotones.

TABLE II. Calculated $\langle f_{7/2}^2|V|f_{7/2}^2\rangle_J$ matrix elements. Matrix elements calculated by McGrory *et al.* are also shown for comparison.

$2j_1$	$2j_2$	$2j_3$	$2j_4$	J	set 1	set 2	MKB
7	7	7	7	0	−2.558	−2.591	−2.11
7	7	7	7	2	−0.985	−0.984	−1.11
7	7	7	7	4	−0.114	−0.149	−0.10
7	7	7	7	6	0.415	0.341	0.23

In seeking for the minimums we found that the minimum corresponding to the tensor parameters are quite flat. For a considerable range of values the rms deviations will not be changed much. The ranges in which the values of rms deviations are not changed in the second decimal value are shown in Table I. The best fit parameters for the three mass regions together are displayed in the seventh column of Table I. When the central parameters for each mass region are replaced by the overall set and the tensor parameters are fixed at the corresponding best fit values, the changes in rms deviations are only in the third decimal place. Therefore, the calculations indicate that the central components can be practically unified. However, the tensor parameters for the three mass regions are quite different. The strength becomes weaker as the mass increases. An interesting question is does this also happen if we try to impose the universal condition on the tensor component. It was found that the tensor component is compromised to a value of 22.16 MeV as indicated in Table I. The overall rms deviation becomes 0.23 MeV. The partial rms deviation for $N=28$, 50, and 82 mass regions become 0.26, 0.21, and 0.23 MeV, respectively. Therefore, the fittings for $N=28$ and 82 isotones are worsened. This is because the low-lying levels in the $N=50$ mass region are the major input data (51 levels out of overall 88 levels), and the resulting value of the tensor component is compromised to a value near to the $N=50$ isotones value. Another comment we would like to make is that very large rms deviations can not be avoided if the set 1 central param-

TABLE III. Calculated energy spectra of $N=28$ isotones by the two different sets of potential parameters. Those levels marked with an asterisk are excluded from the least-squares fit.

Nucleus		E_{exp} (MeV)	E_{cal} (MeV) set 1	E_{cal} (MeV) set 2
$^{50}_{22}Ti_{28}$	0^+	0.00*	0.00*	0.00*
	2^+	1.56	1.57	1.61
	4^+	2.68	2.44	2.44
	6^+	3.20	2.97	2.93
$^{51}_{23}V_{28}$	$\frac{7}{2}^-$	0.00*	0.00*	0.00*
	$\frac{3}{2}^-$	0.93	1.06	1.11
	$\frac{5}{2}^-$	0.32	0.55	0.60
	$\frac{9}{2}^-$	1.81	1.75	1.75
	$\frac{11}{2}^-$	1.61	1.73	1.73
	$\frac{15}{2}^-$	2.70	2.85	2.78
$^{52}_{24}Cr_{28}$	0^+	0.00*	0.00*	0.00*
	$(2^+)_1$	1.43	1.58	1.61
	$(2^+)_2$	2.97	3.00	3.04
	$(4^+)_1$	2.37	2.45	2.45
	$(4^+)_2$	2.77	2.62	2.65
	5^+	3.62	3.50	3.50
	6^+	3.11	2.97	2.93
	8^+	4.75	4.87	4.79
$^{53}_{25}Mn_{28}$	$\frac{7}{2}^-$	0.00*	0.00*	0.00*
	$\frac{3}{2}^-$	1.29	1.06	1.11
	$\frac{5}{2}^-$	0.38	0.55	0.60
	$\frac{9}{2}^-$	1.62	1.75	1.75
	$\frac{11}{2}^-$	1.44	1.73	1.73
	$\frac{15}{2}^-$	2.69	2.85	2.78
$^{54}_{26}Fe_{28}$	0^+	0.00*	0.00*	0.00*
	2^+	1.41	1.57	1.61
	4^+	2.54	2.44	2.44
	6^+	2.95	2.97	2.93
rms			0.16	0.16

eters is used in calculating the energy spectra of $N=50$ and 82 isotones. Hence, in view of universality, the set 2 parameters is much more preferred.

In Table I, the last column shows the parameters found by Schiffer and True.[5] The magnitudes of the long range to short range ratio of the central force components are also displayed in Table I for comparison. They are all quite similar except for the singlet-even ratio of the set 1 parameters for $N=28$ isotones. Also presented in Table I are the central singlet-even to the triplet-odd ratios. Again the values found by Schiffer and True are deviated away from those of the set 1 parameters of $N=28$ isotones. The two-body matrix elements obtained from the best fit parameters for $N=50$ isotones are presented in Table IV. Some results of previous calculation are also displayed for comparison.[17,18] In Table V the two-body matrix elements obtained from the best fit parameters of $N=82$ isotones are listed for reference.

TABLE IV. Calculated two-body matrix elements for the $N=50$ mass region. The results of Ball *et al.* (BML) and Kuo-Brown (KB) are also displayed for comparison.

$2j_1$	$2j_2$	$2j_3$	$2j_4$	j	This work	BML	KB
1	1	1	1	0	−0.497	−0.484	−0.249
1	1	9	9	0	0.630	0.901	0.650
1	9	1	9	4	0.381	0.690	···
1	9	1	9	5	−0.023	0.175	···
9	9	9	9	0	−2.077	−1.719	−1.147
9	9	9	9	2	−0.735	−0.603	−0.852
9	9	9	9	4	−0.156	0.164	−0.259
9	9	9	9	6	0.084	0.508	−0.037
9	9	9	9	8	0.336	0.570	0.094

IV. ENERGY SPECTRA

The calculated energy spectra produced by the best fit parameters together with the experimental data are shown in Figs. 1–3. Those levels that are

TABLE V. Calculated two-body matrix elements for the $N=82$ region.

$2j_1$	$2j_2$	$2j_3$	$2j_4$	j	This work
7	7	7	7	0	−1.490
7	7	7	7	2	−0.294
7	7	7	7	4	0.049
7	7	7	7	6	0.283
5	5	5	5	0	−1.213
5	5	5	5	2	−0.200
5	5	5	5	4	0.176
7	5	7	5	1	0.161
7	5	7	5	2	0.313
7	5	7	5	3	0.361
7	5	7	5	4	0.233
7	5	7	5	5	0.315
7	5	7	5	6	−0.208
7	7	5	5	0	−0.596
7	7	5	5	2	−0.127
7	7	5	5	4	−0.067
7	7	7	5	2	−0.063
7	7	7	5	4	−0.111
7	7	7	5	6	−0.119
7	5	5	5	2	−0.019
7	5	5	5	4	−0.063

excluded from the least-squares fit are marked with an asterisk. In general, the low-lying energy spectra of $N=28$ isotones are reproduced quite well as indicated in Fig. 1. The quality of fittings is very similar to other calculations with the same choice of model space. It is interesting to notice that the experimental energy spectra for ^{50}Ti and ^{54}Fe are quite similar. This similarity is also manifested in the ^{51}V and ^{53}Mn spectra. This suggests that $(1f_{7/2})^n$ configuration is a good approximation for those levels selected in Fig. 1 and there is a particle-hole conjugation between ^{50}Ti, ^{54}Fe and ^{51}V, ^{53}Mn nuclei. In fact, the $(1f_{7/2})^n$ configuration approximation was studied quite extensively by theoretical calculations including larger model spaces.[19-23] It was concluded by Lips *et al.*[20] that for the lowest level of each J^π, the intensity of the $(1f_{7/2})^n$ configuration is $\geq 80\%$.

The calculated and experimental energy spectra for $N=50$ isotones are shown in Fig. 2. The quality of fitting is slightly worse than that of $N=28$ isotones. This is probably due to the reason that ^{88}Sr is not doubly closed. In the energy spectra of ^{90}Zr, the 3_1^- state at 2.75 MeV and the 2_2^+ state at 3.33 MeV are not shown since they do not have model space analogs and are not expected to be reproduced in our calculation. Other levels marked with an asterisk such as the $3/2_1^+$ state of ^{91}Nb, the 3_1^- state of ^{92}Mo, the $3/2_1^-$ and $5/2_1^+$ states of ^{93}Tc, and the 0_2^+ and 0_3^+ states of ^{94}Ru were not included in the least-squares fit. Except for 0^+ states, these levels could not be reproduced well in previous calculations in which the interaction matrix elements were used as adjustable parameters. Our calculated 0^+ states for the even-mass isotones are too low as compared with experimental levels. These states are constructed mostly from seniority-zero components and probably are of two-particle two-hole admixtures. Therefore they are difficult to reproduce with the effective interactions. However, in previous calculations it was shown that these states can be reproduced quite well using two-body matrix elements as free parameters. In Ref. 12 it was also found that there was some incompatibility between these 0^+ states and the high-spin negative parity states. If the

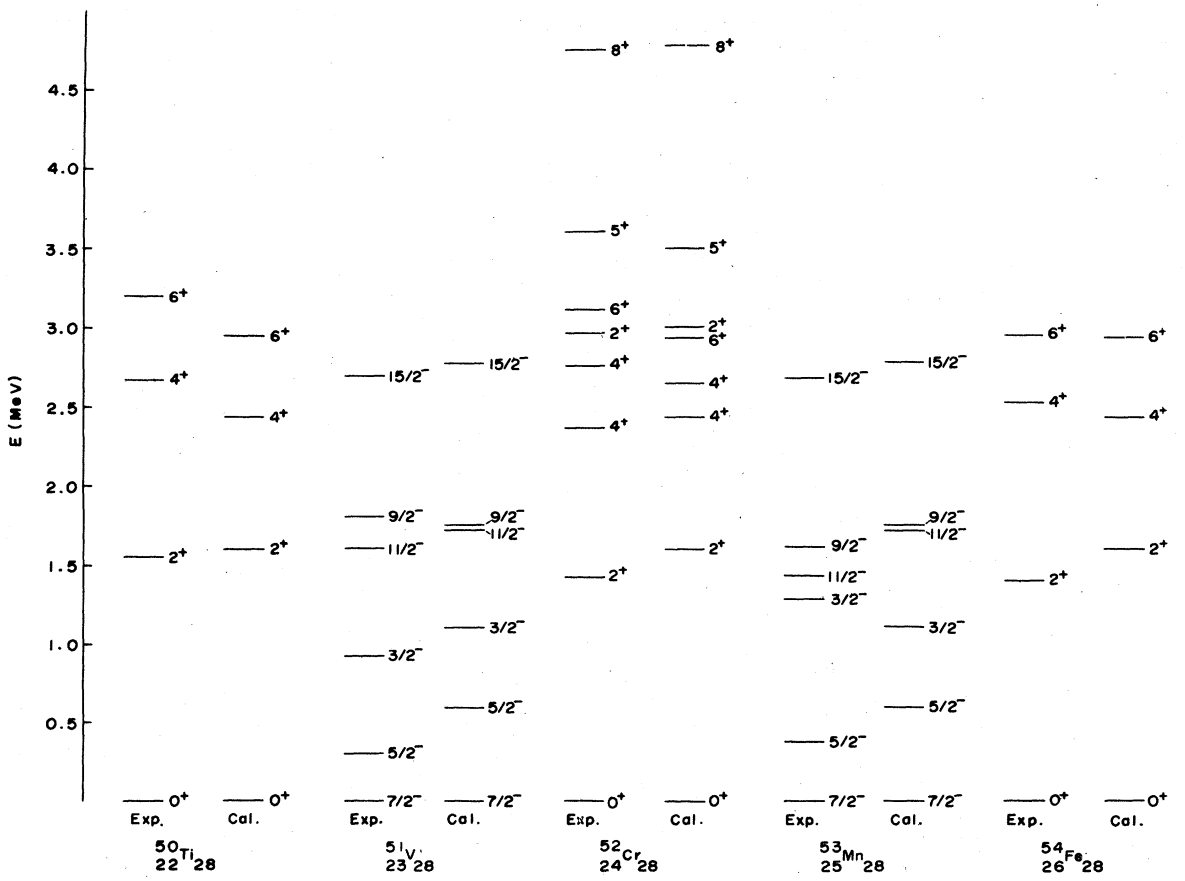

FIG. 1. Calculated and experimental energy spectra for $N=28$ isotones.

higher seniority levels were included in the least-squares fit, the 0^+ states would be displaced away from the experimental levels. The calculated 0^+ states were then displaced lower than the experimental values in consistency with the general features in our calculation. In the calculated ^{90}Zr spectra, the order of 2^+ and 5^- states is inverted. This inversion also happened in previous calculations. The calculated $1/2^-$ state in ^{91}Nb is lower than the ground state as shown in Fig. 2. This inversion did not happen in other similar calculations. An interesting feature of ^{91}Nb and ^{93}Tc energy spectra is that high spin states of both parities show up in a rather low energy region. These states can be reproduced quite well in general. This suggests that these states can be accommodated in our model space. However, some previous analyses on the $E2$ transition rates yield different values of effective charges. For example, Brown *et al.* calculated the $B(E2)$ for the $17/2^- \rightarrow 13^-/2$ transition of ^{93}Tc and found that an effective charge $e_p = 2.49 \pm 0.1$ is needed.[24] This value is much larger than the average value of even-mass $N = 50$ isotones calculated by Gloeckner *et al.*[13] It is also much larger than the corresponding value of the $17/2^- \rightarrow 13^-/2$ transition in ^{91}Nb.[25] These inconsistencies suggest that the model space expansion is needed for refined description for these states. The configuration distributions of the calculated wave functions for some low-lying states are presented in Table VI. Previous calculated results of Auerbach and Talmi[26] (denoted as AT) and Talmi and Unna[27] (denoted as TU) are also displayed for comparison. Except for the second 0^+ state of ^{90}Zr our results are in good agreement with previous calculations in which two-body matrix elements are treated as interaction parameters. For the ground state of ^{90}Zr, our result is in agreement with the TU value but not with the AT value. The TU value and our result are also in agreement with previous experimental analysis[28] in which the contributions of the $p^2_{1/2}$ and $g^2_{9/2}$ configurations were estimated to be 78% and 22%, respectively.

The low-lying experimental and calculated energy spectra for $N = 82$ isotones are shown in Fig. 3.

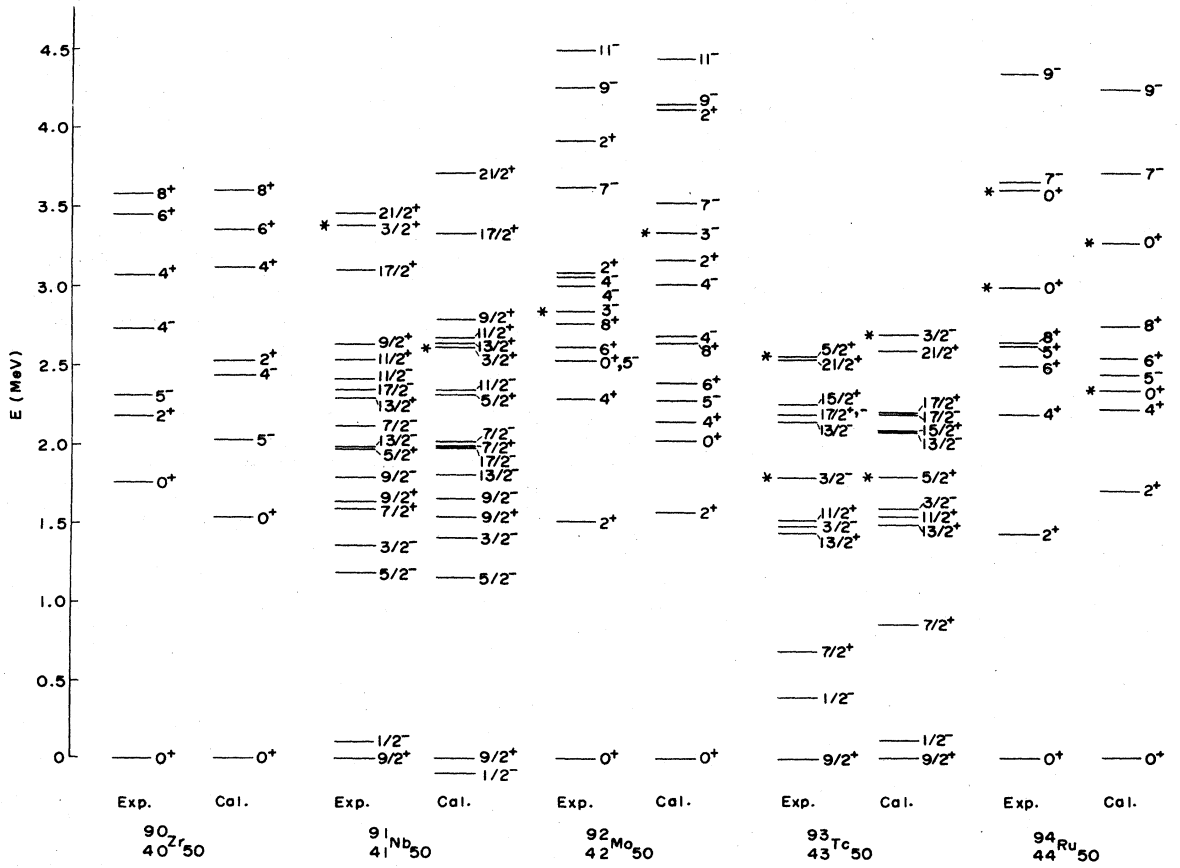

FIG. 2. Calculated and experimental energy spectra for $N = 50$ isotones.

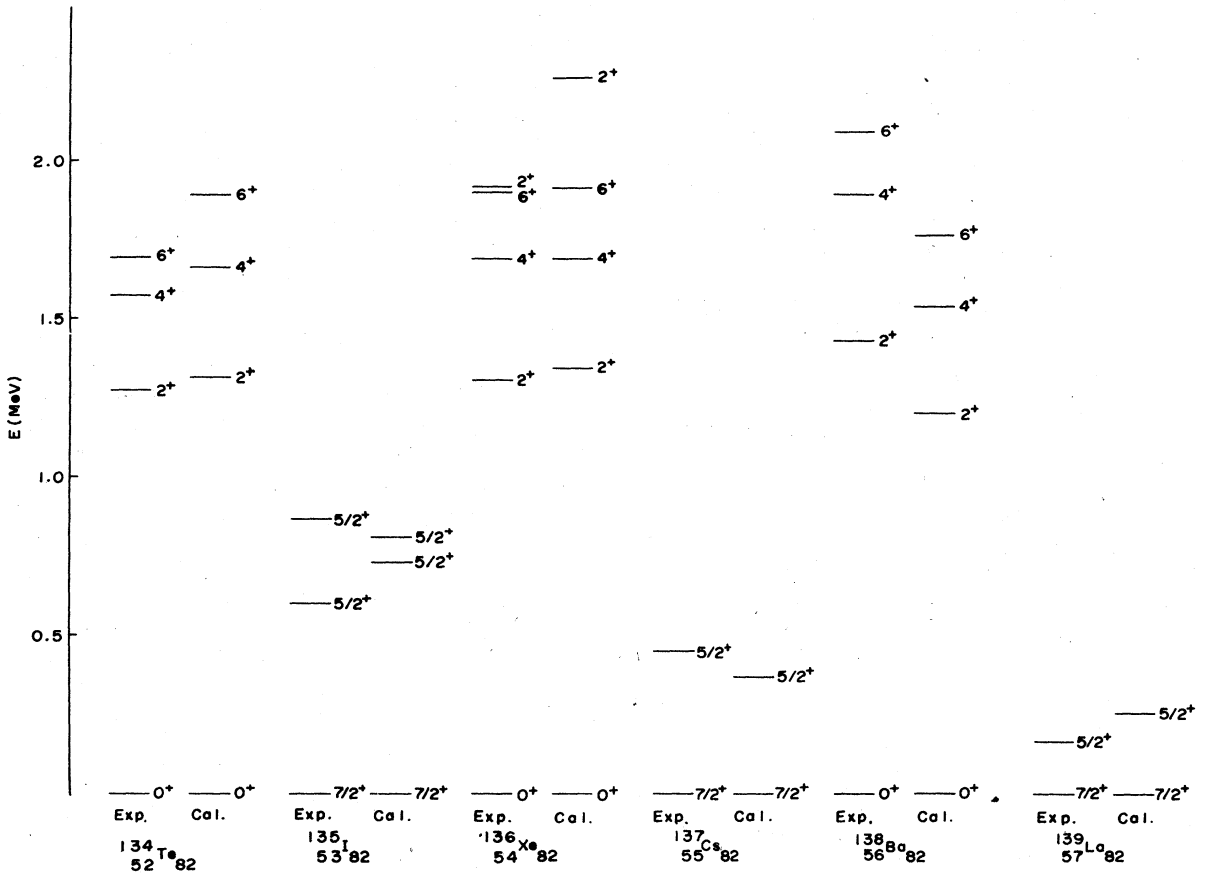

FIG. 3. Calculated and experimental energy spectra for $N=82$ isotones.

Higher excitation levels are omitted for the reasons of uncertainties in experimental data and our choice of limited model space. For these states shown in the figure, the calculated levels usually fit with the experimental data quite well. The produced energy spectra are very similar to those obtained in previous shell model calculations using surface delta potential and a larger model space in which one-particle excitation to $3S_{1/2}$ and $2d_{3/2}$ is included.[14-16] In fact, for low-lying states

TABLE VI. Calculated configuration admixtures for the ground states and the low-lying states for $N=50$ isotones. The results of Auerbach and Talmi (AT) and Talmi and Unna (TU) are also displayed for comparison.

Nucleus	Level	Configuration admixture AT	TU	This work
^{90}Zr	0^+(g.s.)	60% $p_{1/2}^2$ 40% $g_{9/2}^2$	75% $p_{1/2}^2$ 25% $g_{9/2}^2$	79% $p_{1/2}^2$ 21% $g_{9/2}^2$
	0^+(1.76)	40% $p_{1/2}^2$ 60% $g_{9/2}^2$	...	21% $p_{1/2}^2$ 79% $g_{9/2}^2$
	2^+(2.19)	$g_{9/2}^2$	...	$g_{9/2}^2$
^{91}Nb	$\frac{9}{2}^+$(g.s.)	71% $p_{1/2}^2 g_{9/2}$ 29% $g_{9/2}^3$	83% $p_{1/2}^2 g_{9/2}$ 17% $g_{9/2}^3$	84% $p_{1/2}^2 g_{9/2}$ 16% $g_{9/2}^3$
	$\frac{1}{2}^-$(0.11)	$p_{1/2} g_{9/2}^2$	...	$p_{1/2} g_{9/2}^2$
^{92}Mo	0^+(g.s.)	72% $p_{1/2}^2 g_{9/2}^2$ 28% $g_{9/2}^4$	...	81% $p_{1/2}^2 g_{9/2}^2$ 19% $g_{9/2}^4$
	2^+(1.51)	82% $p_{1/2}^2 g_{9/2}^2$ 18% $g_{9/2}^4$	...	90% $p_{1/2}^2 g_{9/2}^2$ 10% $g_{9/2}^4$
^{93}Tc	$\frac{9}{2}^+$(g.s.)	81% $p_{1/2}^2 g_{9/2}^3$ 19% $g_{9/2}^5$	88% $p_{1/2}^2 g_{9/2}^3$ 12% $g_{9/2}^5$	86% $p_{1/2}^2 g_{9/2}^3$ 14% $g_{9/2}^5$
	$\frac{1}{2}^-$(0.39)	$p_{1/2} g_{9/2}^4$		$p_{1/2} g_{9/2}^4$

of the isotones with proton number ≤ 7, configurations including $3s_{1/2}$ and $2d_{3/2}$ orbitals contribute very little to the wave functions.[15] This justifies our choice of model space for these states. The wave functions obtained in our calculations have dominant configurations similar to those obtained in Ref. 15. However, our wave functions are purer in the sense that the intensities are more concentrated in the dominant configurations. For the isotones with proton number ≥ 7, it was found that the configuration mixings with $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$ orbitals are large even for rather low-lying states. This is, in fact, the major reason that we omit these isotones in our calculation. The ground states and the first excited states for the odd-mass isotones are expected to be quite pure $1g_{7/2}$ and $2d_{5/2}$ states even if the proton number is >7. The well known ground state and first excited state $7/2^+ - 5/2^+$ order inversion in odd mass isotones can also be reproduced in our calculation. As in Ref. 14, in order to account for the energy spectra for heavier isotones a smaller $2d_{5/2}$-$1g_{7/2}$ single-particle level spacing is needed. It was found that if the single-particle level spacing for heavy isotones is chosen to be 0.7 MeV approximately, the agreement with experimental data is quite good. The calculated and experimental energy spectra for the ground states and the first excited states for odd-mass $N=82$ isotones are shown in Fig. 4. The single-particle level spacings are larger than those found by Wildenthal *et al.* However, it is interesting to notice that a nearly equal relative difference of the single-particle level spacings for light and heavy mass isotones is needed in both calculations.

V. CONCLUSIONS

The universality of the proton-proton effective interactions is explored for $N=28$, 50, and 82 isotones. A two-range central plus tensor potential is used to parametrize the effective interactions. It was found that the central components are much more important than the tensor component. The major effect produced by the tensor component is to adjust the relative spacings between the ground states and the excited states. The contributions of the tensor component decrease as the mass increases, and become almost negligible for $N=82$ isotones. This suggests that the effective potential is essentially central for heavy mass nuclei and may not be so complicated as commonly believed. The universal behavior manifested in central potential is very encouraging. Since the central potential is the dominant part in our calculation, the result supports the conjecture of the existence of universality of effective interactions. Another calculation which can correlate the nuclear properties for $N=28$ and 82 isotones with a unified set of interaction parameters seems to support this idea.[29] The universality is not only an attractive theoretical idea but also a useful working principle in searching for effective interactions. In this work, two sets of different potential parameters are obtained for $N=28$ isotones. Both sets are acceptable since they produce very similar two-body matrix ele-

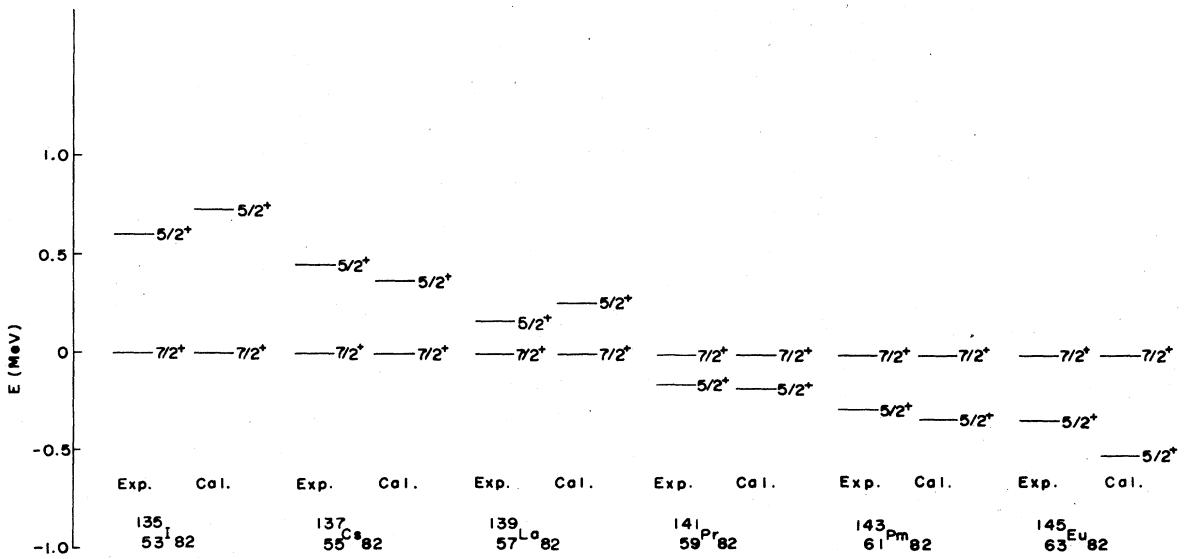

FIG. 4. The spins and energy spacings of the $5/2^+$ states relative to the $7/2^+$ states for odd mass $N=82$ isotones as a function of mass.

ments. However, when the universality is used as a guiding principle, it becomes quite easy to determine which set is more preferable. Furthermore, it also provides a good guideline in searching for effective interaction in other mass regions. The analyses of the electromagnetic transition rates of the energy levels in these mass regions suggest that extended calculations by expanding the model space will give us more information about the effective interactions. The calculation in this work, however, has provided rather encouraging preliminary results.

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*Work supported by the National Science Council, Republic of China.

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