Adaptive VSS Blind Equalizers

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***Abstract*— It is well known that an adaptive filter with a large step size in the transient period and a small one in the convergence period gives small mean squared steady state error while achieving a fast convergence rate. Based upon this idea, we present two variable step size (VSS) blind equalizers. The first one employs an intersymbol interference (ISI) estimator to control the step size, the second algorithm uses a mean squared error (MSE) estimator to adjust the step size. Both accomplish what has been expected.**

I. Introduction

DUE TO ITS simplicity and robustness, the least-mean-square (LMS) type algorithms have been widely used in many applications. However, LMS-type algorithms are unable to satisfy the opposing requirements of fast convergence and small mean squared error (MSE). To solve this problem and meet both specifications, an approach frequently used is to employ a time varying step size in these LMS-type algorithms. Kwong and Johnson [1] suggested that the step size of an LMS-type adaptive algorithm be controlled by

$$\mu(n+1) = \alpha\mu(n) + \gamma\epsilon^2(n) \tag{1}$$

where $0 < \alpha < 1, \gamma > 0$, and $\mu(n)$, ϵ_n are the step size and the error signal at the nth iteration. Mathew and Xie [2] proposed a stochastic gradient algorithm that updates the step size via

$$\mu(n) = \mu(n-1) + \rho\epsilon(n)\epsilon(n-1)Y^T(n-1)Y(n) \tag{2}$$

where ρ is a small positive constant that controls the adaptive behavior of the step size sequence $\mu(n)$. $Y(n) = [y(n)\, y(n-1) \cdots y(n-N+1)]^T$, where $\{y(n)\}$ is the channel output (or filter input) sequence. Mayyas and Aboulnasr [3] noticed that both the energy of the error signal and the correlation between successive samples are small when the adaptive filter is near its optimum. They then employed an estimate of the autocorrelation between $\epsilon(n)$ and $\epsilon(n-1)$ to control the step size, i.e.,

$$p(n) = \beta p(n-1) + (1-\beta)\epsilon(n)\epsilon(n-1) \tag{3}$$

$$\mu(n+1) = \alpha\mu(n) + \gamma p^2(n). \tag{4}$$

These VSS algorithms all have an error signal $\epsilon(n)$, which is the difference between the desired signal and the LMS filter output, available. But in a blind environment, an exact error signal can not be obtained directly. To achieve the same goal in a blind environment we have to design a suitable measure to control the step size. ISI- and MSE-based VSS blind algorithms are suggested below. We will show that they do perform up to expectation.

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II. The VSS Blind Algorithms

A. ISI-Based Blind Algorithm

Denote the total impulse response of the channel and the equalizer by

$$\begin{aligned} s_i &= h_i \circ c_i \\ &= \sum_l h_{i-l}c_l \end{aligned} \tag{5}$$

where "$\circ$" represents the convolution operation, $H \stackrel{\text{def}}{=} [\cdots, h_{-1}, h_0, h_1, \cdots]$ is the channel impulse response, and $C \stackrel{\text{def}}{=} [\cdots, c_{-1}, c_0, c_1, \cdots]$ is the impulse response of the equalizer. The filter output $z(n)$ can be expressed as $z(n) = \sum_l a(n-l)s_l$, where $\{a(n)\}$ is the data sequence. In order to achieve zero ISI, $\mathbf{s} \stackrel{\text{def}}{=} [\cdots, s_{-1}, s_0, s_1, \cdots]$ must have the following form:

$$\mathbf{s} = [0, \ \cdots, \ 0, \ 1, \ 0, \ \cdots, \ 0]. \tag{6}$$

Let N be the filter length and define $K[a(n)] = E[a^4(n)] - 3E^2[a^2(n)]$, where $E[\cdot]$ is the expectation operator, μ the step size, $\gamma_1 = -2 - (1+\alpha)K[a(n)]/E^2[a^2(n)]$, and $\gamma_2 = \alpha K[a(n)]/E[a^2(n)]$, α being a positive number. To find a proper ISI indicator so as to adjust the step size $\mu(n)$ at the nth iteration, let us consider the following equations [4]:

$$E[z^2(n)] = E[a^2(n)] \sum_l |s_l|^2 \tag{7}$$

$$K[z(n)] = K[a(n)] \sum_l |s_l|^4. \tag{8}$$

Since $\sum_l |s_l|^4 \leq (\sum_l |s_l|^2)^2$ with equality holds if and only if $\{s_l\}$ is of the form (6), perfect equalization implies $\sum_l |s_l|^2 = 1$ and $\sum_l |s_l|^4 = 1$. The quantity

$$\begin{aligned} Q_{\text{ISI}}(n) = \{E[z^2(n)] - E[a^2(n)]\}^2 \\ + \{E[z^4(n)] - E[a^4(n)]\}^2 \end{aligned} \tag{9}$$

is minimized when zero ISI is achieved. Note that for a stationary data source $E[a^4(n)]$ and $E[a^2(n)]$ are known constants, but the filter output $z(n)$ is not a stationary process, not at least before it reaches a steady state. The required time-varying statistics, $\alpha_n \stackrel{\text{def}}{=} E[z^2(n)]$ and $\beta_n \stackrel{\text{def}}{=} E[z^4(n)]$ can be

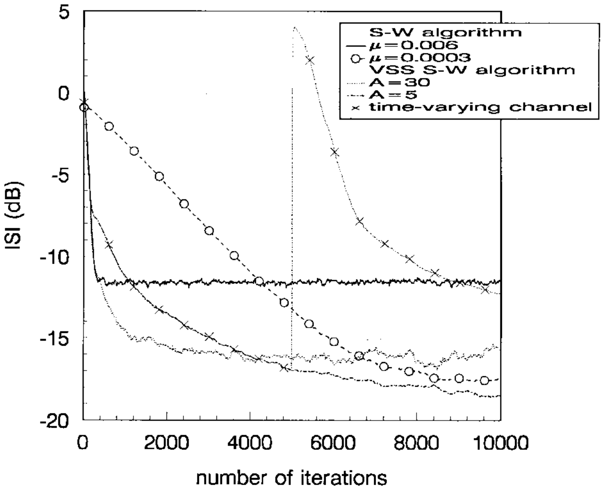

Fig. 1. ISI performance comparison of FSS and VSS algorithms that operate in Channel 1. The time-varying channel changes its characteristic from Channel 1 to Channel 3 at the 5000th iteration.

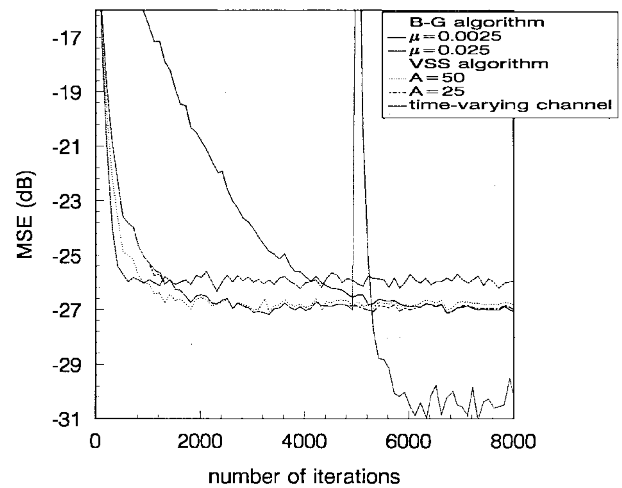

Fig. 2. MSE performance comparison of FSS and VSS algorithms that operate in Channel 2. The time-varying channel changes its characteristic from Channel 2 to Channel 1 at the 5000th iteration.

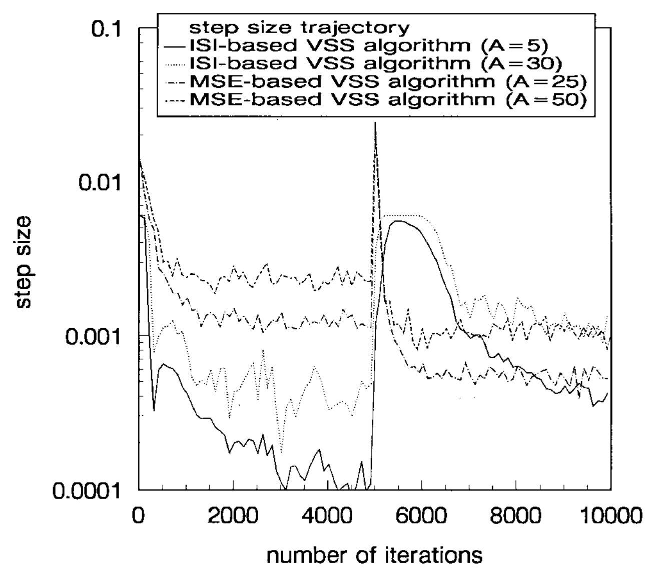

Fig. 3. Step size trajectories of VSS blind algorithms that operate in the time-varying channels defined the previous two figures.

estimated recursively as

$$\alpha_{n+1} = \lambda_1\alpha_n + (1-\lambda_1)z^2(n+1) \tag{10}$$

$$\beta_{n+1} = \lambda_2\beta_n + (1-\lambda_2)z^4(n+1) \tag{11}$$

where $0 < \lambda_1,\ \lambda_2 < 1$ are forgetting factors. Substituting (10) and (11) into (9) we then obtain a real-time estimate of the ISI measure $Q_{\text{ISI}}(n)$. The step size can therefore be controlled by $Q_{\text{ISI}}(n)$ through

$$\mu(n) = \mu_{\max}[1 - e^{-AQ_{\text{ISI}}(n)}] \tag{12}$$

where $\mu_{\max}$ is the initial step size and A is a positive constant. As can be seen, when $Q_{\text{ISI}}(n)$ is large, the step size $\mu(n)$ is large; otherwise, $\mu(n)$ becomes small. This variable step size can be used in any LMS-like blind equalizer, e.g., the unconstrained blind algorithm proposed by Shalvi and Weinstein [4] that updates its tap-weight vector $C_{\text{ISI}}(n)$ by

$$\begin{aligned} C_{\text{ISI}}(n) = C_{\text{ISI}}(n-1) + \mu\,\text{sgn}\,(K[a(n)])\{z^2(n) \\ + (\gamma_1 - 1)E[z^2(n)] + \gamma_2\}z(n)Y(n). \end{aligned} \tag{13}$$

B. MSE-Based Blind Algorithm

When an LMS algorithm converges, we expect the associated mean squared (real) error to be small. Hence, we can use

$$Q_{\text{MSE}}(n) = E[e_R^2(n)] \stackrel{\text{def}}{=} \gamma_n \tag{14}$$

where $e_R(n)$ is the real error signal as a mean squared convergence indicator. $E[e_R^2(n)]$ can be estimated recursively via

$$\gamma_{n+1} = (1-\lambda_3)\gamma_n + \lambda_3 e_R^2(n+1) \tag{15}$$

where $0 < \lambda_3 < 1$. In a blind equalization situation, $e_R(n)$ is not available, the following error signal can be chosen to replace $e_R(n)$

$$e_R(n) = e_{dd}(n) + U\{E[|e_{dd}(n)|] - v\}e_{sa}(n) \tag{16}$$

where v is a threshold, $U(\cdot)$ is the unit step function, $e_{dd}(n)$ is the decision-directed error signal and $e_{sa}(n)$ is a Sato-type error signal [6], e.g.,

$$e_{sa}(n) = \alpha\ \text{sgn}\ [z(n)] - z(n) \tag{17}$$

where $\alpha \stackrel{\text{def}}{=} E[a^2(n)]/E[|a(n)|]$. $\epsilon_n \stackrel{\text{def}}{=} E[|e_{dd}(n)|]$ can be recursively estimated by

$$\epsilon_{n+1} = (1-\lambda_4)\epsilon_n + \lambda_4|e_{dd}(n+1)| \tag{18}$$

with $0 < \lambda_4 < 1$. The above discussion indicates that a suitable control scheme for the step size is

$$\mu(n) = \mu_{\max}[1 - e^{-AQ_{\text{MSE}}(n)}] \tag{19}$$

where a real-time estimate for the MSE measure $Q_{\text{MSE}}(n)$ is obtained from substituting (16)–(18) into (15).

(12) and (19) can be replaced by $\mu(n) = \mu_{\max}G[Q(n)]$, where $Q(n) = Q_{\text{MSE}}(n)$, or $Q_{\text{ISI}}(n)$ and $G(x)$ is a soft-limiter

defined by $G(x) = x/\gamma$, if $x < \gamma$, and $= 1$, otherwise, γ being a positive real number representing the threshold (saturation level). Such an implementation is simpler than (12) and (19). We also find that the associated performance is similar and is insensitive to the choice of γ.

III. Simulation Results

This section presents some Monte Carlo simulation results of the two proposed VSS blind equalizers. Binary PSK $\{\pm 1\}$ is transmitted and the following three channels are used in our simulations:

Channel 1 $y(n) = -0.4a(n) + a(n-1) + 0.4a(n-2)$.
Channel 2 $y(n) = 0.2798a(n) + a(n-1) + 0.2798a(n-2)$.
Channel 3 $y(n) = a(n) - 0.6a(n-1) + 0.2a(n-2)$.

We choose to use $\mu_{\max} = 0.006$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.99$, $v = 0.5$ for our simulations. The equalizer length is 11. Fig. 1 shows the ISI performance comparison where ISI is defined as

$$\text{ISI} = \frac{\sum_i |s_i| - \max |s_i|}{\max |s_i|}. \tag{20}$$

We assume that the transmitted channel is switched from Channel 1 to Channel 3 at the 5000th iteration. Fig. 2 presents the MSE learning curves for the MSE-based VSS blind equalizer and its fixed step size (FSS) counterpart. We choose $\mu_{\max} = 0.025$. The transmitted channel is switched from Channel 2 to Channel 1, also at the 5000th iteration. Both figures demonstrate that, when compared with the FSS algorithms, our VSS algorithms not only have faster learning speeds, but also result in smaller steady state ISI or MSE. Furthermore, the learning behavior of both algorithms reveals another advantage of our VSS algorithms—their behavior is not very sensitive to the choice of the value for the constant A. This is because the product $AQ(n)$, instead of $Q(n)$ alone, controls the step size within the designed limit $\mu_{\max}$. The step size trajectories given in Fig. 3 show that the step size in the transient periods is almost independent of A for both algorithms. This fact indicates that during the transient periods where there are significant $z(n)$ variations, our algorithms have the self-adjusting capability of making the ISI or MSE measure smaller for a larger A.

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