Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science

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What is This?

Optimum design of a cutting tool for manufacturing rotary knives

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Abstract: In this article, the design parameters of a cutting tool for manufacturing a rotary knife with multi-cutting angles have been studied. An objective function and constraints are defined, and optimized solutions are found by using two methods: global search and sequential quadratic programming. Hobbing and the concept of tooth undercutting were used to manufacture a rotary knife with multi-cutting angles. The proposed method not only improved the manufacturing efficiency over the traditional method (milling and grinding), but also significantly improved the strength of the rotary knife and the chip discharge ability, as shown by the finite-element analysis of the stress distribution and deformation of two different kinds of rotary knives.

Keywords: optimum design, rotary knife, hob cutter

1 INTRODUCTION

Rotary knives have been widely used in many industries. For example, the rotary knife used in a plastic granulator/pelletizer for cutting plastic into small pellets to facilitate the jobs of downstream equipments is shown in Fig. 1. These types of knives are usually designed with multi-cutting angles to enhance theccite actions. In Fig. 2, the 'multi-cutting angles' are demonstrated by the transverse view. However, the manufacturing process of this rotary knife, adopting a traditional method, which involves milling followed by successive grinding of the flutes, requiring specialized and expensive tools, is quite lengthy and complicated [1].

As hobbing is the most frequently used and effective manufacturing process of gears in the gear industry [2], several novel hob cutter designs have been reported [3, 4]. A novel straight-sided hob cutter for manufacturing rotary knives with multi-cutting angles was proposed in reference [5]. The hob is designed from a standpoint opposite to the way that it would be for a gear. In particular, the hob

is designed to maximize undercutting [6–8] and to eliminate any part of the involute on the cutter surface. Different levels of undercutting with unsymmetrical tooth profile [9] were applied to generate the cutting angles simultaneously, the main body of the rotary knife, and the chip flutes in a single hobbing process.

Optimum design methods were successfully used in many fields to improve the design and manufacturing quality of a work piece. The application of computer-aided engineering and optimum design methods is feasible, as the computational prediction approaches the actual conditions. Chang *et al.* [10] used the optimum design method to reduce the kinematic errors of the modified helical involute gear train and determined the tooth profile of sprockets with the optimized results [11].

Despite the popularity of the finite-element method (FEM) for the stress analysis of the mechanical component, there has been little systematic and in-depth application of the method to the optimum design of a cutting tool for manufacturing a rotary knife, particularly, a hob cutter capable of generating a cutting tool with multi-cutting angles. The proposed formulation of an optimum design problem has provided practical and useful information for designing the rotary knives including design criteria and engineering experiences. After optimization, the generated rotary knife can be

Corresponding author: Department of Power Mechanical Engineering, National Formosa University, 64 Wnu-Hua Road, Huwei, Yunlin 632, Republic of China. email: changsl@nfu.edu.tw
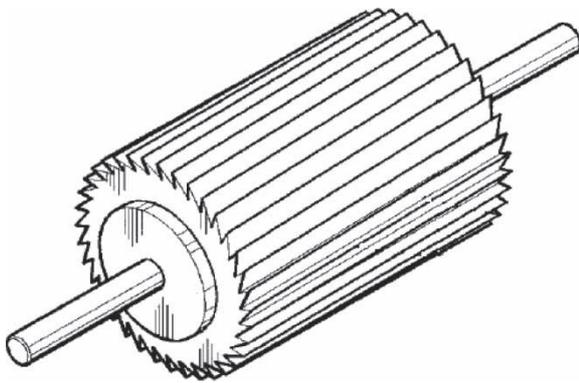
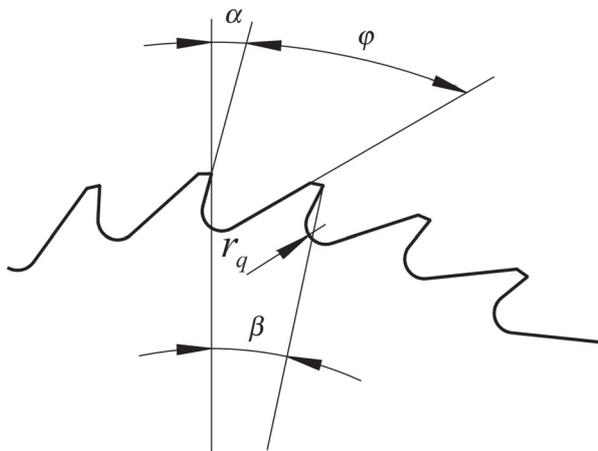**Fig. 1**   Rotary knife used in plastic granulator/pelletizer

Fig. 2   Transverse shape of the rotary knife manufactured by the traditional method

expected to obtain better characteristics than the one manufactured by the traditional methods (milling and grinding).

2   PROFILE OF ROTARY KNIFE

2.1   Profile manufactured by the traditional method

Figure 2 shows the transverse shape of a rotary knife with multi-cutting angles manufactured by the traditional method, which uses a milling or grinding machine to manufacture the flute and then, by means of grinding, cut out the multi-cutting angles, which include the radial rake angle, the clearance angle, and the relief angle. The radial rake angle α, shown in Fig. 2, plays a significant role in cutting; the larger the α the sharper the knife. Therefore, when cutting a soft material such as plastic, the radial rake angle α is expected to be larger than that of the rotary knife for cutting metals. The performance of

the rotary knife could be improved by selecting suitable values for these cutting angles; however, the manufacturing process in terms of efficiency will be highly unsatisfactory if the traditional method is adopted.

2.2   Profile manufactured by the proposed novel method

Hob cutters with a straight-sided cutting face are commonly used in the manufacture of involute gears. In this article, the hob cutters can also be used to manufacture rotary knives with multi-cutting angles. Involute gears with a small number of teeth will have undercutting at the root of the gears. It reduces the strength of the gears, which should be carefully avoided in designing gears. However, for a rotary knife, it requires a large flute for the convenience of chip flow from cutting. Generally, this means digging out the width of the teeth to form larger fillets at the root, which is similar to undercutting a gear tooth. Specifically, undercutting can be utilized by using a straight-sided hob cutter to generate the required profile of a rotary knife, if the hob cutter parameters are appropriately selected. Usually, a rack cutter with one-parameter enveloping is used to simulate the process of gear hobbing instead of a two-parameter model [**12**, **13**]. Therefore, the most important step in manufacturing a hob cutter is the design of the rack cutter whose profile matches the normal section tooth profile of the hob cutter with a small lead angle.

Figure 3 shows the profile of the rack cutter, which is also the normal section profile of the hob cutter. The cutting face shown in Fig. 3 can be divided into five regions, i.e. left cutting face I, right cutting face II, fillet cutting face III and IV, and top land cutting face V. The profile of the cutter is similar to an ordinary hob cutter except for its large pressure angle (i.e. region I), and small pressure angle, i.e. region II. During generation, the rack cutter can be viewed as translating along the tangential direction of the cylindrical workpiece, which rotates about the Z axis to produce the desired shape. More specifically, region II will undercut the shape of the rotary knife and form the radial rake angle, whereas region I generates the involute shape of the main body of the rotary knife, as shown in Fig. 4. By applying the equations of the designed profiles of the hob cutter, $\mathbf{r}_a^{(n)}$, the equations of meshing, the principle of coordinate transformation, the theory of differential geometry gearing, the mathematical models of the rotary knife can be derived.

The equation of the five regions shown in the $S_a(X_a, Y_a, Z_a)$ coordinate system that is rigidly connected to the normal section profile of the hob cutter can be expressed as equation (1), where $n = 1$–5 represents the left cutting face (I), right cutting face (II), fillet cutting faces (III and IV), and top land cutting
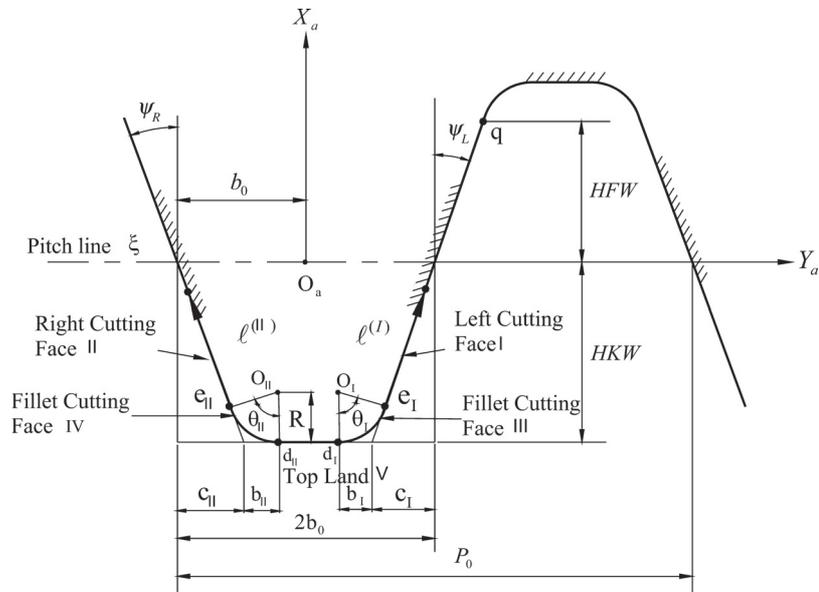
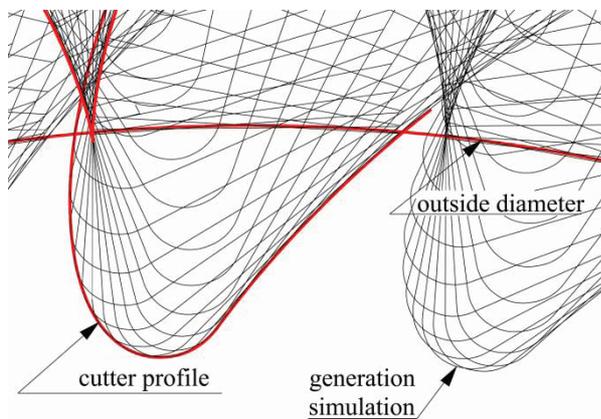**Fig. 3** Normal section tooth profile of the hob cutter

Fig. 4 Generation of rotary knife by the hob cutter

face (V), respectively

$$\mathbf{r}_a^{(n)} = \begin{bmatrix} x_a^{(n)} & y_a^{(n)} & z_a^{(n)} \end{bmatrix}^{\mathrm{T}} \tag{1}$$

When a helical rotary knife is to be manufactured, it can be viewed as the normal section of the hob cutter that translates along the direction of the lead $\overline{O_w O_a}$ shown in Fig. 5, and the corresponding position of the cutting face can be derived. The equation can be obtained by transforming the equations of the normal section of the hob cutter in the S_a coordinate system to S_w coordinate system, as indicated by the transformation matrix $[\mathbf{M}_{wa}]$ shown in equation (2). The coordinate system S_w is rigidly connected to the cutting face to generate the helical rotary knife. The equation of the unit normal vector $\boldsymbol{n}_w$ of the cutting face represented as S_w can also be determined. The upper sign of the matrix $[\mathbf{M}_{wa}]$ corresponds to a right-hand helix of the cutting face, whereas the lower sign
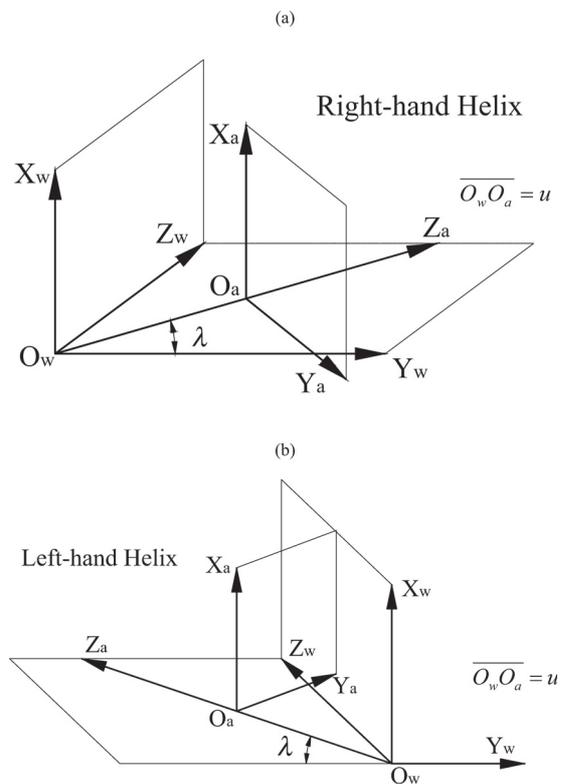

Fig. 5 Translation of the normal section profile of the hob cutter along the lead direction

indicates the left-hand helix

$$[\mathbf{M}_{wa}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sin\lambda & \pm\cos\lambda & \pm u\cos\lambda \\ 0 & \mp\cos\lambda & \sin\lambda & u\sin\lambda \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$
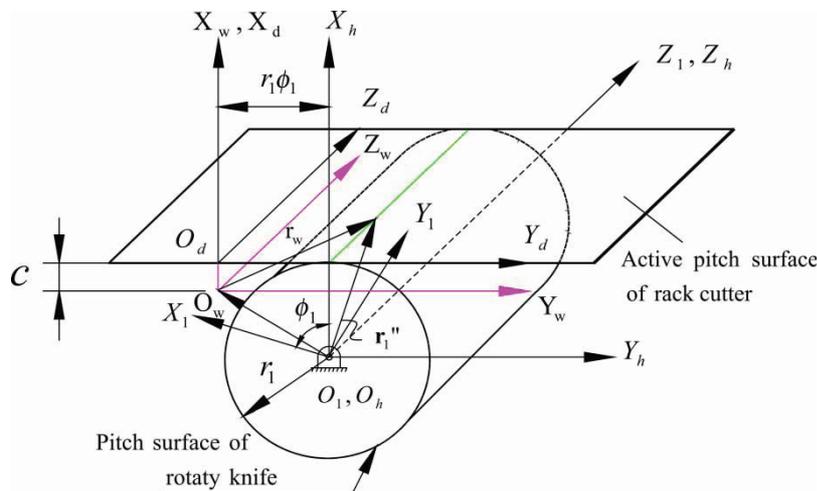
Fig. 6 Relationship of the coordinate system of the rack cutter and the generated rotary knife

When a gear is generated by a hob cutter, the generation mechanism can be simplified as shown in Fig. 6. As the cutting face translates left, the work piece rotates counterclockwise. The locus equation of the five regions shown in the $S_1(X_1, Y_1, Z_1)$ coordinate system, which is attached to the rotary knife, can be represented by equation (3), which is derived by the homogeneous matrix transformation, i.e. transforming from the S_w coordinate system to the S_1 coordinate system

$$\mathbf{r}_1^{(n)} = \begin{bmatrix} x_1^{(n)} & y_1^{(n)} & z_1^{(n)} \end{bmatrix}^{\mathrm{T}} \qquad (3)$$

In other words, equation (3) represents the locus of the hob cutter in the coordinate system of the rotary knife. The envelope of the locus can be solved by considering simultaneously the locus equation and the equation of meshing, which indicates that the relative velocity between two surfaces is perpendicular to the common normal of these two surfaces [**14**].

3   OPTIMUM DESIGN OF THE HOB CUTTER

As mentioned earlier, the generation of gears by a hob cutter can be simulated by the motion of a rack cutter. Therefore, the most important step in manufacturing a hob cutter is to design a rack cutter. In this section, different optimum methods are applied to determine the parameters of the rack cutter for generating the rotary knife.

3.1   Global search method

Global search is a simple and straightforward optimization method. As the name implies, the full range of parameters is entered as discrete values into a pre-developed programme, and the parameter values

satisfying the specified constraints are identified, from which the optimum values can be chosen by comparing with the objective function. In this article, considering the rotary knife used in the plastic industry as an example, the objective function is defined as the maximum radial rake angle and is shown in equation (4) to facilitate cutting for such a soft material.

3.1.1   Objective function

$$f(x) = \mathrm{Max}\,(\alpha) \qquad (4)$$

where α is the radial rake angle shown in Fig. 2.

The equations of constraint are specified by considering the geometric constraints of the rack cutter shown in Fig. 3 and the generated rotary knife. There are five constraints as listed below.

Constraint 1: The point of intersection of two straight-sides is located at the outside diameter of a work piece. This can be expressed as

$$[\mathrm{HFW} \cdot \tan\psi_{\mathrm{L}} + \mathrm{HFW} \cdot \tan\psi_{\mathrm{R}}] < P_0 - 2b_0 \qquad (5)$$

where HFW, HKW, $2b_0$, ψ_L, ψ_R, and the P_0 are the dedendum, addendum, tooth thickness, left pressure angle, right pressure angle, and pitch of the rack cutter, respectively.

Constraint 2: The minimum top land width of the rack cutter is zero. The constraint is expressed as

$$2b_0 - \mathrm{HKW} \cdot \tan\psi_{\mathrm{L}} - \mathrm{HKW} \cdot \tan\psi_{\mathrm{R}}$$
$$> R\left[\cot\left(\frac{(90° + \psi_{\mathrm{L}})}{2}\right) + \cot\left(\frac{(90° + \psi_{\mathrm{R}})}{2}\right)\right] \qquad (6)$$

Constraint 3: According to Tseng [**4**], it is required that the parameters of a rack cutter satisfy the full under-cutting condition, i.e. there is no involute curve on the cutting edge of the rotary knife as shown in Fig. 3.

Constraint 4: In the proposed design, the smallest pressure angle of the rack cutter (right cutting face shown in Fig. 2) generates the shape of the rotary knife using the undercutting phenomenon. However, a small pressure angle of the basic rack cutter renders the hob machining process difficult. In this article, the lower limit for the pressure angle is set to 3°.

Constraint 5: The top land width will influence the shape strength of the cutter. The range of this parameter is constrained between 0.3 and 0.5 mm. It can be adjusted according to different types of cutters.

3.1.2 Design variables

From the developed mathematical model of the rotary knife, it is clear that the key parameters affecting the properties of the knife are the pressure angle of the cutting left face ψ_L, the pressure angle of the cutting right face ψ_R, and the tooth thickness $2b_0$. These parameters are therefore set as the design variables in the optimization problem. Considering the above-mentioned constraints, the mathematical model of the optimum design can thus be represented by equation (7), in which $g_1(x)-g_5(x)$ represent constraints 1–5. Parameters x_1, x_2, and x_3 denote ψ_L, ψ_R, and $2b_0$, respectively

$$f(\mathbf{x}) = \max(\alpha)$$
$$\text{subject to} \quad g_1(x); g_2(x); g_3(x); g_4(x); g_5(x) \quad (7)$$
$$\mathbf{x} = [x_1, x_2, x_3]^T$$

where α is the radial rake angle and can be obtained by the vector dot operation and shown as $\cos\alpha = [(x_a \cdot t_x) + (y_a \cdot t_y)]/(r_a \cdot r_t)$; (x_a, y_a) is the coordinate of the tip of the cutter and $(\boldsymbol{t}_x, \boldsymbol{t}_y)$ is the unit tangential vector of the undercutting curve.

3.2 Sequential quadratic programming method

Despite the simplicity of the global searching method, it is inefficient and the optimum solution is influenced by the iteration interval. An alternative method uses the subroutine 'fgoalattain' in MATLAB [**15**, **16**] to determine the optimum parameters based on the objective function, the equations of constraint, and the upper and lower bounds of the design variables. The subroutine uses the sequential quadratic programming (SQP) method to precede the iteration calculation.

Example: As a numerical example, the design parameters of a rotary knife satisfying the manufacturing specification of the traditional method are determined by using the two optimum methods described earlier. The specifications of the cutters manufactured by the

two methods satisfy the following conditions.

1. The outside diameters are the same.
2. The root diameters are the same.
3. The number of teeth and helix angle are the same.
4. The top land widths of the rotary knives are close to each other.
5. The cutting angles of the rotary knives are close to each other.

By substituting the starting values listed in Table 1 into the programme developed for the SQP method, the parameters of the hob cutter used to generate the rotary knife with maximum radial rake angle are obtained. Table 2 and Fig. 2 show the definition of the parameters of the rotary knife manufactured by milling. As shown in Table 3, the value of the objective function has been improved from 15° (the same as that by milling) to 22° by 46.6 per cent. In other words, the performance of the rotary knife in cutting plastic materials can be greatly improved under the constraints of the rack cutter and the generated rotary knife. Note

Table 1 Parameters of a rotary knife and the rack cutter

Parameters of rotary knife		Parameters of rack cutter	
Circular pitch (cp)	16.9646	Addendum (HKW)	8
Pitch diameter (mm)	162	Dedendum (HFW)	0.75
Number of teeth (T)	30	Tooth thickness of rack cutter ($2b_0$)	14
Helix angle (°)	2°	Tip radius of rack cutter (r)	2.0
Outside diameter (D)	163.5	Pressure angle of the left cutting face (ψ_L)	48.72
Root diameter (d)	146	Pressure angle of the right cutting face (ψ_R)	5
Length of cutter (mm)	250	Shifted amount (c)	0.0

Table 2 Parameters of a rotary knife by milling

Number of teeth (T)	30
Outside diameter (D)	163.5
Root diameter (d)	146
Topland width	2 mm
Helix angle (°)	2°
Root radius of cutter (r_q)	3 mm
α	15°
β	12°
φ	45°

Table 3 Result of optimization

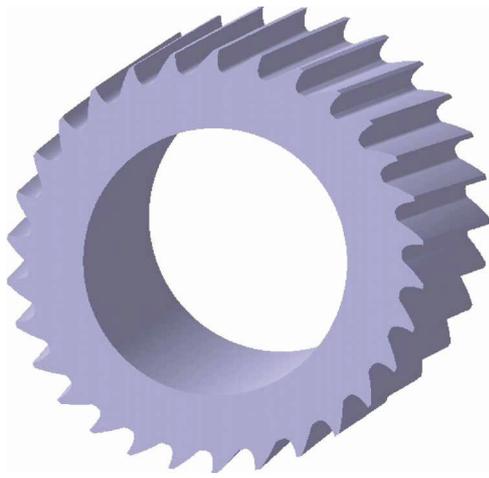| Optimization terminated successfully			
Optimum parameter	$\psi_L = 49.2269$	$\psi_R = 3.9207$	$b_0 = 1.0067$
Radial rake angle	$\alpha = -22.0661$		
	elapsed_time = 43.1720		

Fig. 7   Solid modelling of a rotary knife by hobbing

that the results from the two optimization methods are close to each other (global search: 22.0636°; SQP: 22.0661°). However, the elapsed time for the computation of a global search method is five times that of the SQP method. This is because the global search method currently used is a zeroth-order optimization method, in which no instructions for searching direction could be obtained, unlike the SQP method. Nevertheless, for practical aspects, the global search method is still proved to be useful. By appropriately specifying the boundaries and searching step sizes of the design variables, a list of feasible designs could be generated and compared with the existing tool stock database to choose a suitable tool without redesigning and reproducing a new one. Figure 7 shows the solid model of the optimized rotary knife.

For further comparison, with the same values of radial rake angle, Fig. 8 shows two transverse profiles
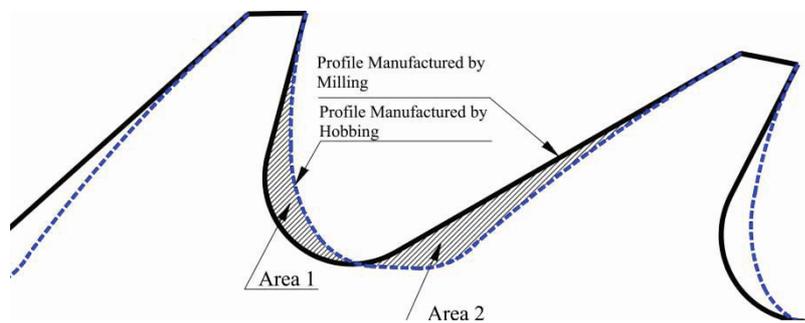

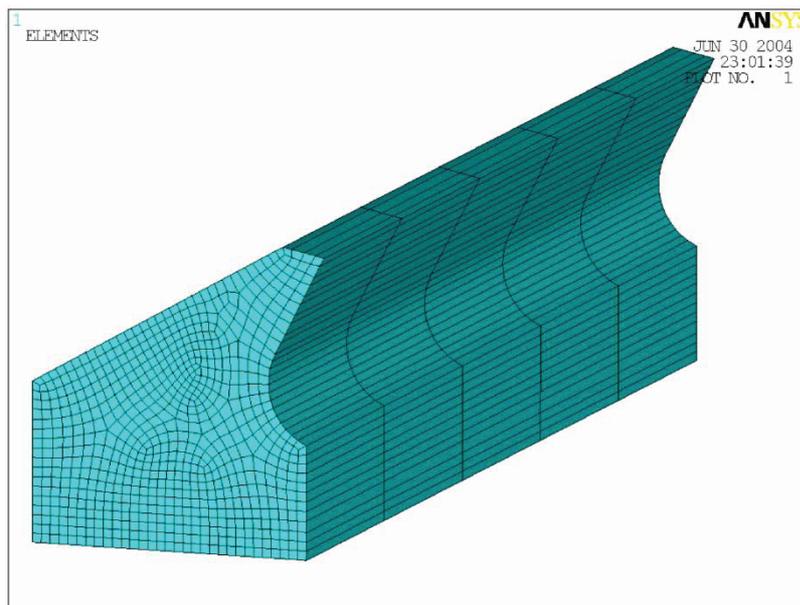**Fig. 8**   Compare the area of chip flute with different design method

Fig. 9   Finite-element model of the rotary knife by milling

produced by the two methods (milling and hobbing). In Fig. 8, area 1 is 5.21 mm^2 and area 2 is 7.39 mm^2, indicating that, even for same values of α, the rotary knife manufactured by the hobbing method has a larger chip flute, which has the advantage of a better discharge of the chip during cutting. This will, therefore, result in a much improved surface quality of the work piece. Besides, it is obvious that the profile produced by hobbing has a stronger root portion, so that larger cutting forces could be sustained without tool breakage, which will be verified by finite-element analysis (FEA) in the next section.

4 FINITE-ELEMENT ANALYSIS

4.1 Stress analysis of the rotary knife manufactured by the traditional method

In this section, the stress distribution at the cutting tooth of a rotary knife is studied by using the

Table 4 FEM parameters of the rotary knife

Material	SHK2
Young's modulus E(MPa)	207 000
Poisson's ratio	0.3
Element type	SOLID45
Mesh form	Hexahedral element
Pressure (N)	1500

FEM. By applying the mathematical model of the rotary knife manufactured by milling, a solid model of the knife was constructed using the computer-aided design software, CATIA, and subsequently transformed into a finite-element model (ANSYS) for the three-dimensional stress analysis. Generally, the cutting force range is expected to be 500–1500 N [17–22]. Therefore, it was set as 1500 N to investigate the stress distribution of cutters of different shapes. The parameters are shown in Table 4. Figure 9 shows the finite-element model of the rotary knife
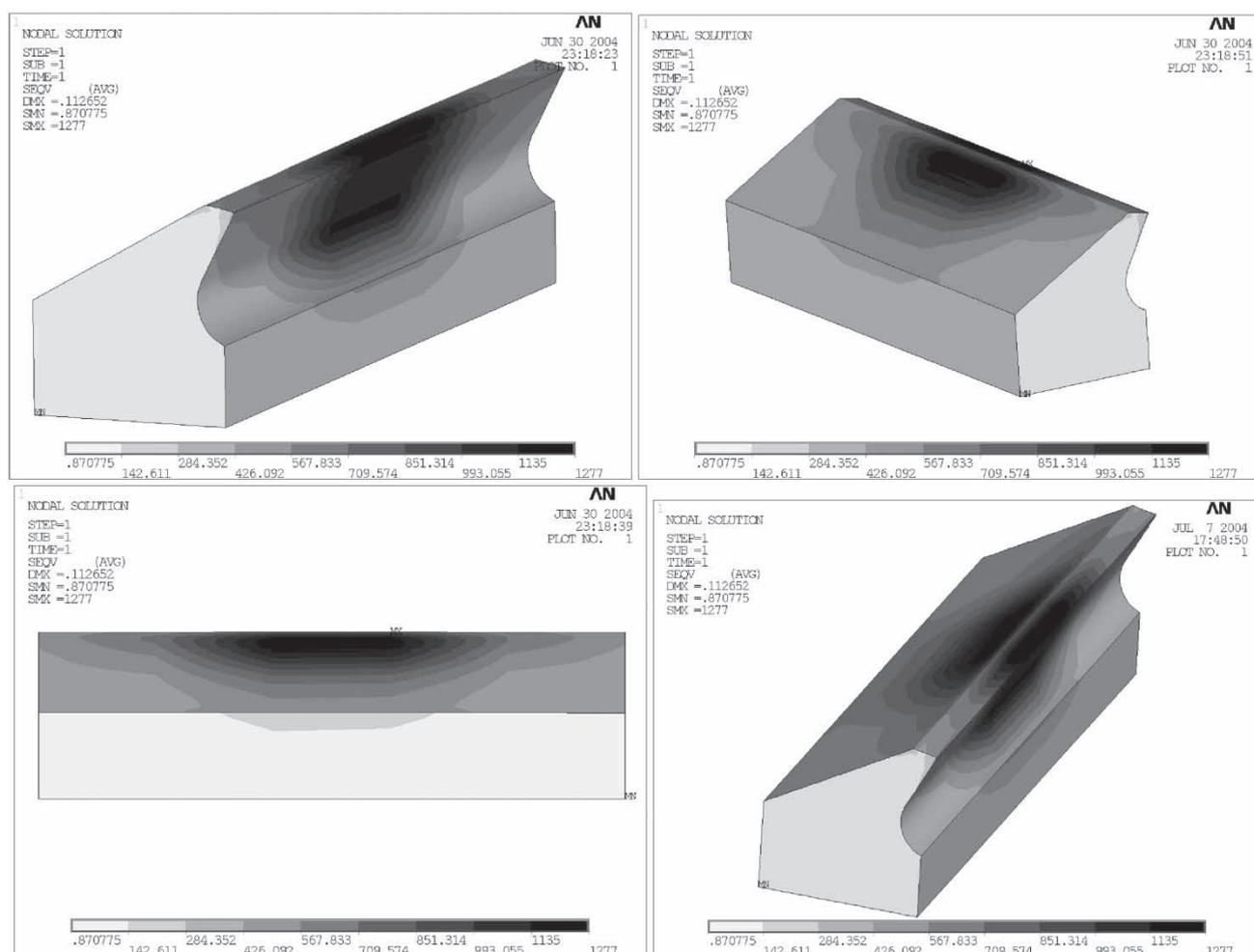

Fig. 10 Four views of the von Mises stress distribution for the milled geometry

manufactured by milling, and the von Mises stress distribution from a different perspective is shown in Fig. 10. Figure 11 shows the distribution of the von Mises stress at the tooth and the root sections.

4.2 Stress analysis of the rotary knife manufactured by the hobbing method

For comparison, the same stress analysis method was applied on the cutter manufactured by the hobbing method. Figure 12 shows the distribution of the von Mises stress.

4.3 Discussion

From the FEA, the maximum stress and deflection of the rotary knife are determined and listed in Table 5. It is obvious that although the maximum stress at tooth tip of the cutter manufactured by hobbing remains at the same level as that by milling, the maximum stress at the tooth root has indeed decreased. It reveals that the cutter manufactured by the proposed hobbing method has higher strength and longer life than that produced by the traditional method and hence the cutter is more cost-effective and useful in cutting.
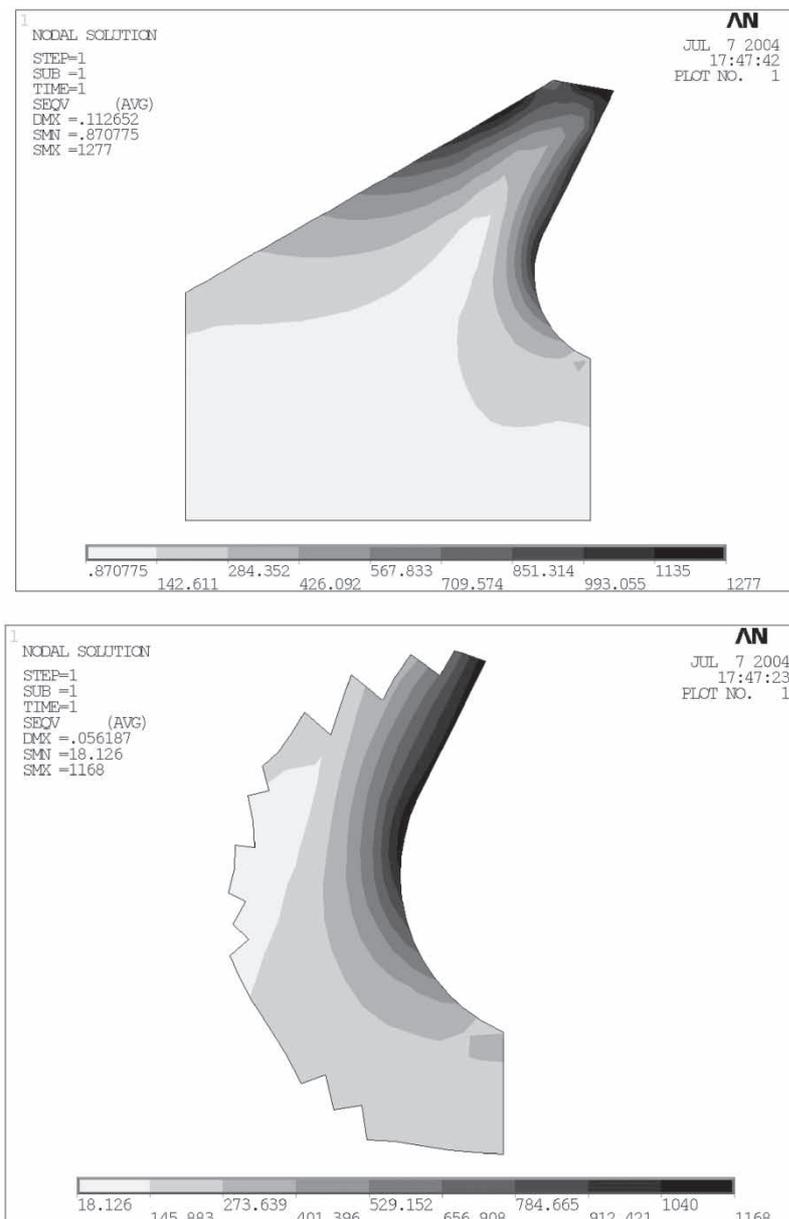

Fig. 11   Distribution of the von Mises stress at the tooth and root section by milling
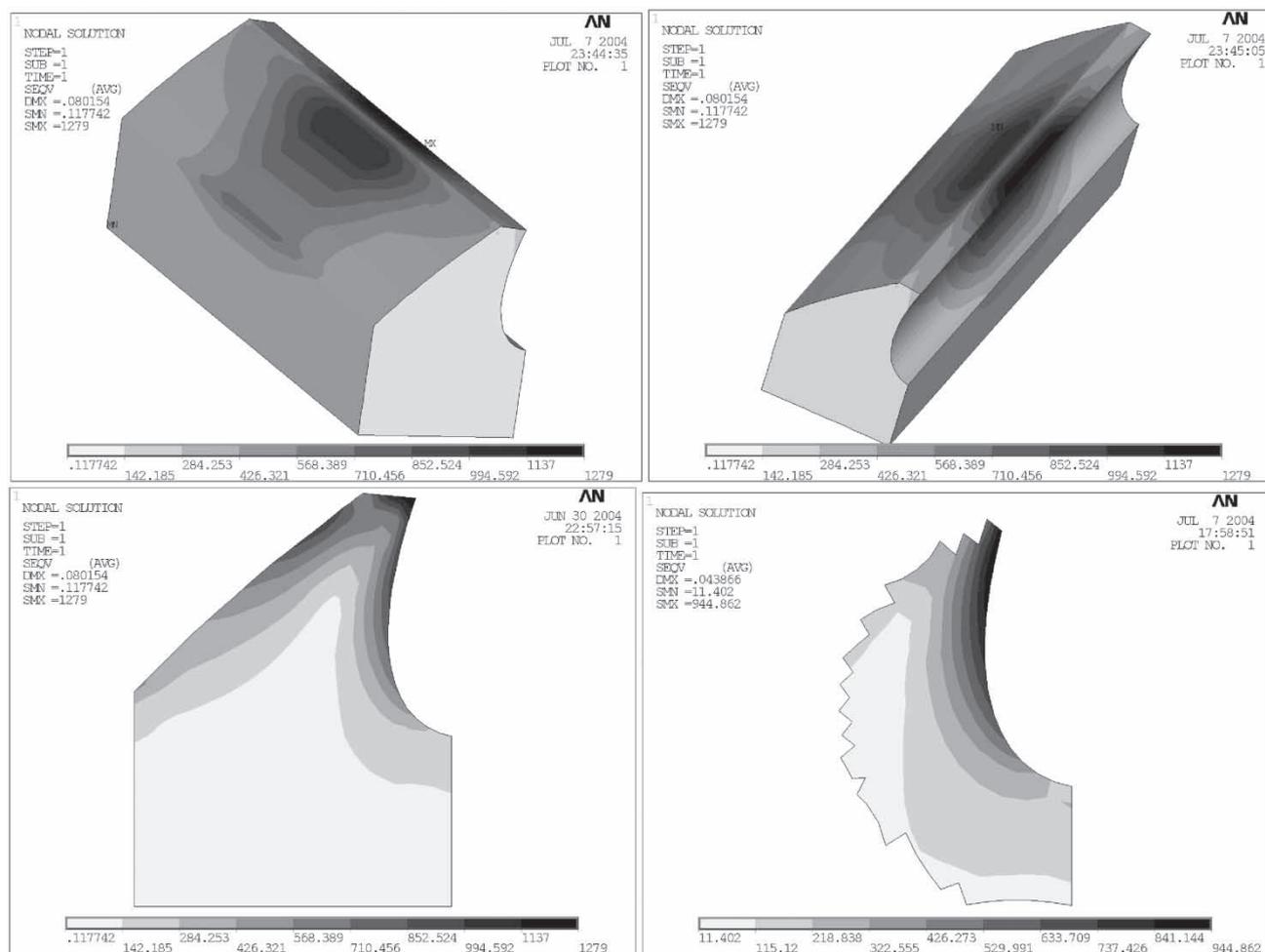
Fig. 12 Four views of the von Mises stress distribution for the hobbed geometry

Table 5 Maximum stress and displacement of the rotary knife with different design method

Manufacture process of rotary knife	Traditional method (milling)	Novel method (hobbing)
Maximum stress (MPa)	1277	1279
Maximum displacement (mm)	0.112 652	0.0801
Maximum stress at tooth root (MPa)	1168	944.86
Maximum displacement at tooth root (mm)	0.056 187	0.043 866

5 CONCLUSIONS

A novel hob cutter for the efficient manufacturing of the rotary knife with multi-cutting angles in one hobbing process was designed. Based on the developed mathematical models, defined objective function and constraints, and the desired characteristics of a rotary knife, two optimum methods were used to determine the parameters of a rack cutter. All solutions from the global search method satisfied the constraints, but showed low efficiency. In contrast, the SQP method resulted in one optimum solution that satisfied the definition of the optimization problem. The systematic approach in designing a hob cutter can be used as a reference for cutter designers and manufacturers.

Two profiles generated by hobbing and milling were compared. The results showed that the cutter manufactured by the proposed hobbing method had higher strength and larger chip flute for ease of chip discharge. The proposed technique can thus provide guidance for future designs of rotary knives or similar tools, particularly for cutters that generate multi-cutting angle tools. The results can also act as a basis for future research on optimal tool designs.

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