國立交通大學

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碩士論文

科技預測模型之檢測：廣泛的羅吉斯模型、費雪模型、

以及甘伯茲模型之比較

The Test of Technological Forecasting Models:

Comparison between Extended Logistic Model,

Fisher-Pry Model, and Gompertz Model

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中華民國九十四年六月

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摘要

這篇論文的目的主要是在檢測一個擁有可變動上限以及彈性轉折點的新模型與其他以廣泛應用的科技預測模型之比較。科技預測對於決策者來說是非常好的補助工具，它除了可以用來找尋科技替代的趨勢以及可能的產品發展的上限之外，也可以被用來檢測產品生命週期的可能趨勢。因此，好的預測可以對未來的趨勢提供一個良好的訊息。費雪模型以及甘伯茲模型是兩種廣泛應用的模型，而且現今也有許多改良的模型被應用在各種科技預測的領域。 因此，這篇論文利用預測圖形以及數理指標來檢測這三種模型的適用性。

本篇論文利用了日本家電產品的滲透率來做模型的比較，此外還利用數位相機的像數的演進來做為實際檢測磨行事用性的依據。結果發現費雪模型在資料點多且可達到百分之百的滲透率或替代率時有較好的預測性，而甘伯茲模型則在當可以適當地找出適合的上限時有較好的表現。至於廣泛的羅吉斯模型不管在預測或者是模型的配適上都有不錯的表現。

關鍵字：科技預測、廣泛的羅吉斯模型、費雪模型、甘伯茲模型

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Abstract

The purpose of this thesis is to find a new technological forecasting model based on time-varied capacity and it has a flexible point of inflection. Technological forecasting is a good auxiliary tool to help managers make a decision whether in technological substitution or technological growth. It can also be used to find out the time that will reach the maximum growth rate and to know the possible trend of product's life cycle. Consequently, a good forecasting can offer useful information to know the possible circumstances in the future. Fisher-Pry and Gompertz model are the most commonly used in this field, and many adapted models also based on their structure. Therefore, this thesis compares these models according to curves and mathematical criteria.

The penetration of durable goods is used to test these three models. The result shows that Fisher-Pry model can fit the data well in some long data sets which reached 100% of capacity and Gompertz model fits data well when the right capacity has been set and data set is asymmetric. The extended logistic model shows a better fit and forecast in most data sets.

Keywords: Technological Forecasting, Fisher-Pry model, Gompertz model, The Extended Logistic Model.

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Chun-Peng Chen

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CHAPTER 1 INTRODUCTION

Technology forecasting has been developed for several decades, and some of methods were derived from other field such as demography. For example, the trend of the adoption of a product will grow slowly at first, and then it will have rapidly growth. Finally, the growth of the adoptions will look like a sigmoid curve. Some quantitative technology forecasting model, such as Fisher-Pry and Gompertz, can offer a precursor when a new product replaces a mature product, and they can offer the probable trend of the market share of a new product. This chapter first presents a brief background to the technology forecasting, and then the problem discussion will be introduced. The research purpose and motivation are also presented with the overall structure of this research in this chapter.

1.1 Background

Technology forecasting has been developing during several decades and is still in the process of improving since many new technological inventions increases the demands of forecasting tools. Many enterprises need these methods in their focus field in order to improve their projection ability and to know the trend as many as possible. Some consultative companies also use technology forecasting method to offer the projection about some products or technologies. Therefore, technology forecasting is comprehensively used now. The technology forecasting can simply divided into two fields according to quantitative or qualitative way. However, these methods truly offer an auxiliary role when managers need to make a decision whether they use quantitative or qualitative method.

Generally speaking, the life cycle of a product or a service will display a bell-shaped curve, and this curve can be divided into five parts- Innovators, Tornado, Main Street, Decline, and Obsolescence. As same as the life cycle, technology adoption life cycle can also fall into five parts- innovators, early majority, late majority, and laggards (Meade and Rabelo, 2004). Therefore, the growth of adopters in new products or services will look like a sigmoid curve, and they will also decline as sigmoid way. Some technology forecasting methods can be a tool to fit and forecast this trend, such as Fisher-Pry and Gompertz. However, there are many adapted or new models proposed in this field (Carrillo and Gonzalez 2002, Bhargava 1995, Rai and Kumar 2003). These methods can fit data sets well in some specific products or services, such as the rate of adoption of mobile phone, and it portrayed the trend of a new product or a service.

1.2 Problem Discussion

Technological forecasting models give the projection and fit in the form of graph according to estimation of points during a period. The forecasters first may observe the market share or sales volumes data, and the time measures can be annual or quarterly. Although many adapted or new technological growth curve models which used to fit the data were proposed in recent years, not every model can fit or forecast well in different kinds of data sets. Technological forecasting models can be classified according their characteristics (e.g. symmetric, asymmetric, and flexible), the method of estimation, and so on. These classification may offer a simply and clearly way to identify which model belongs to which categories. Consequently, some researchers try to find some criteria to identify which model will have better fit or forecast performance in some particular data sets (Young 1993, Meade and Islam 1997). These classifications can reduce the biases which technological growth models made. For example, the symmetric sigmoid curve can be fitted well by symmetrical technological growth model (e.g. Fisher-Pry). However, there are more implications for model selection. Choosing the right model is an important work before researchers prepare to project the time series data points. Some models perform a better forecast but fail to forecast the short term time-series data. Another way to reduce the bias is to use the combined models, and this method will also help the forecasters reducing the error when they did not use an appropriate model to fit the data (Meade and Islam, 1997).

Some growth curve may not reach the saturation level at 100%, because the new technologies may enter the market and adopt the share of the mature one before the mature product reach the saturation at 100%. This phenomenon may make some technological growth models (e.g. Fisher-Pry) can not fit these data well. Therefore, when the limit of capacity is unknown, some models will not fit or forecast well in this situation. Experts' opinion seems a good method to solve this problem if experts suggest a proper limit of capacity. However, if a model can be adjusted its capacity with time-varying, the prediction of limit of capacity can be ignored. Moreover, the number of observations will affect the forecasting accuracy. The more observation points can be obtained, the more accuracy of the curve can be estimated. The stable and robust estimation can be obtained if the data includes the peak of noncumulative adoption curve (Mahajan, et al. 1998).

There are some generalized, adapted, or new technological forecasting models proposed in recent years. The generalized and adapted models are based on the prior common models, such as Fisher-Pry and Gompertz, and adapted them through estimative methods, number of parameters, and so on. Moreover, the saturation level (or called capacity) is limited or not also affect models drawing the fit or predicted performance. There is an interesting idea about the capacity of logistic model, which proposed by Meyer and Ausubel (1999). They argued that the capacity of logistic curve will also vary during time. This research will try to find whether a dynamic capacity will help the model have better performance.

According to the discussion about the technological forecasting model in this section before, some research directions can be found. First, this model will also have a flexible inflection point in order to fit different kinds of data sets. That can improves that one model is suitable for some data sets whether they belong to symmetric or asymmetric form. Second, dynamic saturation level will help researcher save the process to find the possible saturation level. Finally, the number of data will affect the outcome of forecasts.

1.3 Research Motivation

The growth of a product or a service is interesting process, because its shape looks like a sigmoid curve as similar as the growth of population. Therefore, some methods which used in demographic field were comprehensively used in fitting or projecting the technological growth. These methods are generally called the technological forecasting models or technological growth models. However, not all of data will follow the sigmoid curve. They may follow linear-like or other forms. Although there are so many technological growth models can be used in fitting different kind of data sets, some improvements can be found in these models and try to find a better method to fit the data. That is why so many adapted or new models were proposed in decades.

The growth of a product or a new service may not easily be observed when only few data can be gathered, but the similar growth trend may also be happened in mature products. For example, people used canals, railways, road, and airways to be the transportation way, and then the growth of maglev may be seen as similar as these mature technologies (Meyer et al., 1999). Therefore, the fit of a growth model not only can find the trend of data, but also offer a possible way which a new product or service is followed. Moreover, the growth curve can offer some useful information in managerial information. For example, the point of inflection can provide when the products will reach the maximum rate of penetration, and when the curve will reach the saturation level roughly.

1.4 Research Purpose

The growth curve can offer some useful information in managerial information. For example, the point of inflection can provide when the products will reach the maximum rate of penetration, and when the curve will reach the saturation level roughly. There are many quantitative technological forecasting models in this field, and many generalized form were also proposed (Bhargava1995; Rai and Kumar 2003) in order to improve the forecast. Therefore, the way to utilize these models in the right case is more important nowadays in order to get the better fit and forecasting performance, and some scholars have proposed some related paper about the model comparison and selecting criterion (Young 1993, Meade and Islam 1997). Although so many technological forecasting models were proposed, less of them can be used to forecast extensively. The articles proposed about new or modified models explain for a particular time-series data (Young, 1993).

Although there are so many new or adapted models have been proposed in recent decades, some improvements are still can be found in these models. The main purpose of this research will try to find a new technological forecasting model according to the time-varying capacity called the generalized logistic model, and make several comparisons between the extended logistic model and Fisher-Pry model, which is most commonly used in quantitative technological forecasting. Moreover, the comparison will be based on the length of data, and the shape of data.

This thesis will try to find a proper model which capacity growth with time-varying and make a comparison between a new extended logistic model, Fisher-Pry model, and Gompertz model. The goal of this research is to identify the extended logistical model will have a better fit and forecast than Fisher-Pry model and Gompertz model, which are comprehensively used in related area. This research will use the different kind of data sets to test these three models, and data sets will be simply classified in order to identify the performance of these two models in different classification.

1.5 Research Content

The scope of thesis can be divided into several parts, and the basic outline of this thesis was shown (Figure 1). The introduction offers the overview of thesis and the outcome roughly, and talks about the purpose of this research. The literature review gives some definitions such as technology forecasting, technology substitution, the life cycle, and so on. The data collecting will describe why this research collects these data sets and then analyze the outcome of the test in these two models. After analyzing, comparing between these two models will be implemented and set the conclusion.

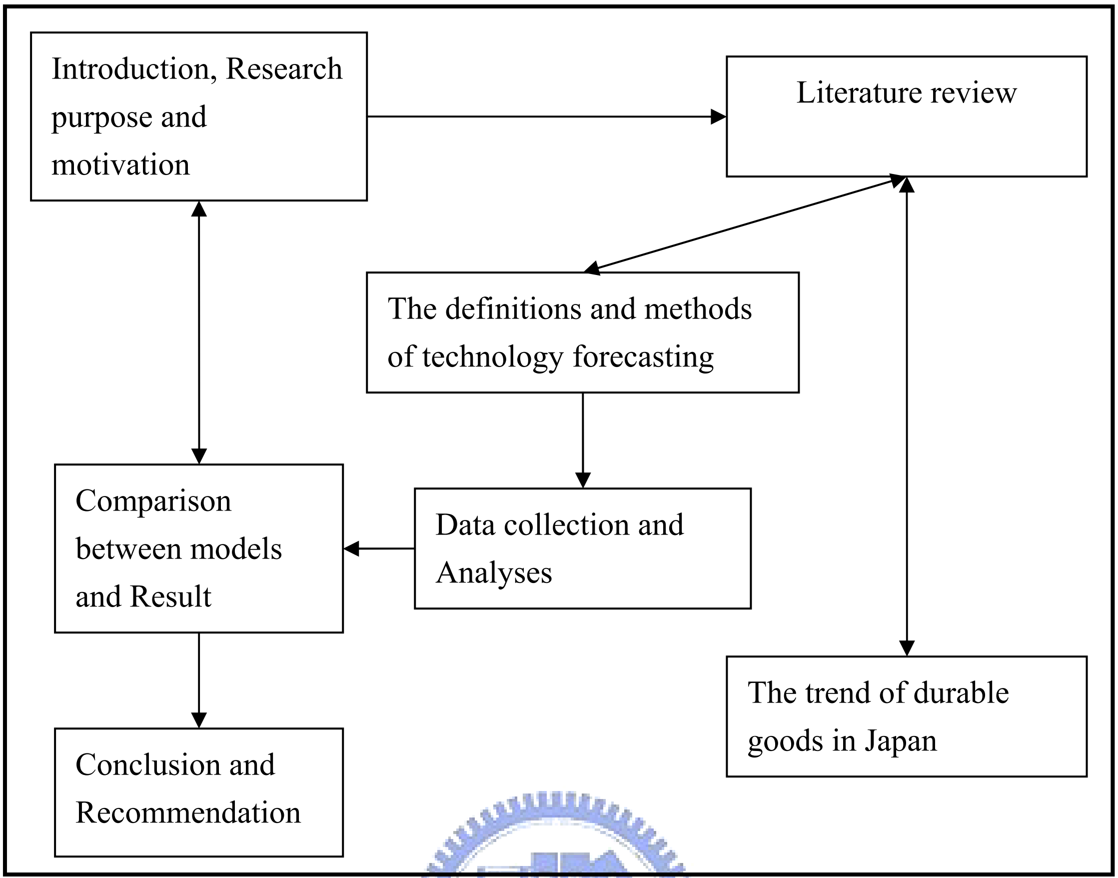

Figure 1. 1 The structure of thesis.

CHAPTER 2 LITERATURE REVIEW

This chapter will introduce some prior researches and some definition related to technological forecasting. Some definitions related to technological forecasting will be introduced in detail, such as technology forecasting, adoption life cycle, and technological substitution. Then some details related to these parts will also be talked about. Finally, this chapter will show some applications of technological forecasting models which some authors proposed before and the selection criteria that prior researches made.

2.1 The Definition of Technology Forecasting

Before talking about the technology forecasting, the words, technology and forecasting, should define first. According to the definition of the American Heritage Dictionary of the English Language, the word "Technology" is the application of science, especially to industrial or commercial objectives, or the scientific method and material used to achieve a commercial or industrial objective. The other word, forecasting, is to estimate or calculate in advance, especially to predict by analysis of meteorological data or to serve as an advance indication of; foreshadow: price increases that forecast inflation.

The definitions of technology and forecasting are described at last section, and some description of technology forecasting will be discussed. Technology grows up sharply in the decade of the end of 21 century, and it is important to project the next new generation products or to see the technology substitution. Technology forecasting can be defined as a prediction of the future characteristics of useful machines, procedures, or technologies. (Martino, 1993) Forecasting is intended to bring the information to the technology management process and to try to predict possible future state of technology or conditions that affect its contribution to corporate goal, so the technology forecasting might be requested to determine what computation power will be on the manager's desk in five or more ten years. However, the attributes of technology most often forecast are:

1. Growth in functional capability
2. Rate of replacement of an old technology by a newer one.
3. Market penetration
4. Diffusion
5. Likelihood and timing of technological breakthroughs.

Forecasters must know the characteristics of growth when they want to forecast the technology or diffusion. They also need to know the attributes of the technology to anticipate how the technology will be used and to choose the legal measures. These measures may be different in forecasting the technology. For example, when using the speed to be the measurement of the products, such as cars, it may not represent the growth in functional capability. Speed may just represent the one of the attributes of the cars. Therefore, choosing right measurement in right technological forecasting methods is important in technology forecasting field. (Porter, Roper, Mason, Rossini, Banks, 1991)

Although the concept and practice of technology forecasting has been proposed for more than three decades, there are two recent developments that have revitalized interest in it. The first of these developments is the enormous increased in the cost of conducting research and development, and the second is the integration of market consideration into technology forecasting process. Indeed, technology forecasts take this reality into consideration, and a major goal is to determine what advances in technology will result in increased sales, enhanced profits, and delighted customers.

(Vanston, 1996)

2.2 The Methods of Technological Forecasting

This section will introduce the classification of technological forecasting, and mainly describe the quantitative technological forecasting, including the type of models, model selection, and application.

2.2.1 The Classifications of Technological Forecasting

When we want to project the future of one industry, we have more than 150 technology forecasting techniques to choose. This section will introduce the classifications of the methods of technological forecasting. However, there are about 18-20 techniques used in various business and others for practical forecasting. Thus, Vanston (1996) thinks that forecasting technologies involve methods. First way is to identify, organize, and extrapolate patterns of past technical development. Second way is to gather and consolidate the opinions of people with special expertise in the areas to be forecast.

The methods of technological forecasting, in a word, can be divided into two simply groups, quantitative and qualitative. However, this classification which just likes dichotomy may not exactly interpret these technological forecasting models. Porter and Rossin (1987) suggested that the hundreds of technology forecasting methods can be categorized in five families, and Porter, Roper, Mason, Rossini, Banks (1991) summarized these methods in their assumptions, strengths, weaknesses, and uses.

1. **Monitoring:** It is the process of scanning the environment for information about the subject of projections, and this method just like a tool for gathering and organizing the information. The sources of information will be identified and

then information

2. **Expert opinion:** The opinion of experts in particular technology field are gathered and analyzed. The assumption of this method is that some experts will know more in particular area than others, therefore the forecasts they projected will more accuracy than others.

3. **Trend extrapolation:** It uses mathematical or statistical technologies to extend time series data into future. This assumes that past conditions and trends will continue in the future.

4. **Modeling:** It represents some structures and dynamics of some parts of real world, and it can be used to forecast the behavior of the system. Models can be flow diagrams, simple equations, and scale models to complicated computer simulations. It assumes that the basic structure and processes of parts of world can be captured by simplified representations.

5. **Scenarios:** It likes the overview of some aspects of the future leading from the present to the future, and it encompasses the possible range of possibilities for some prospects of the future. Scenarios assume that unable forecasts can be constructed from a narrow data base, and future possibilities can be reasonably incorporated in a set of imaginative descriptions.

Table 2. 1 The comparison of technological forecasting methods.

	Strengths	Weaknesses	Uses
Monitoring	Providing large useful information.	Information overload happened without selections.	To provide useful information for structuring a forecast.
Expert Opinion	Tapping high-quality models internalized by experts	Identifying experts is difficult and some extraneous factors will affect experts.	To forecast when experts in this field exists and where data are lacking.
Trend Analysis	A substantial and data-based forecast of quantifiable parameters.	It requires good and enough effective data and it did not explicitly address the causal mechanisms.	To project quantifiable parameters, and analyze adoption and substitutions of technologies.
Modeling	Simplifying the future behavior of complex systems. Building process provides good insight into complex system behavior.	Models that are not heavily data-based may be misleading.	To reduce the complex systems to manageable representations.
Scenarios	It can portrait the possible futures explicitly and incorporate qualitative and quantitative information produced by others.	It may be more fantasy than forecast, unless a firm basis in reality is maintained by the forecasters.	To integrate quantitative and qualitative information and to integrate forecasts form various sources. To provide a forecast when data are too weak to use other methods.

Note: source from Forecasting and Management of Technology, Porter et al, Wiley Interscience.

Porter et al. (1991) categorized technology forecasting methods into three parts:

1. **Direct**: Direct forecasting of parameters that measure an aspect of technology.
2. **Correlative**: Correlative parameters that measure the technology with parameters of other technologies or background forecast parameters.

3. **Structural**: Explicit consideration of cause-and –effect relationships that effect growth.

Many factors will affect the outcome we projected, so there are a lot of models proposed by many people and used in different kind of technology forecasting. Therefore, we can find some ways to view the future.

Vanston (2002) classified the technology methods into five categories, and point out what is quantitative or qualitative clearly (Table 2.2).

Table 2. 2 The map of technology forecasting methods.

Extrapolator	Technology trend analysis, Fisher-Pry analysis, Gompertz analysis, Growth limit analysis, and Learning curve analysis
Pattern Analysis	Analogy analysis, Precursor trend analysis, Morphological matrix, and Feedback models
Goal Analysts	Impact analysis, Content analysis, Stakeholder analysis, Pattern analysis, Roadmaps, and Value chain
Counter Punchers	Scanning, monitors, tracking, Scenarios, Terrain mapping, Decision trees, and Strategic games
Intuitors	Delphi surveys, Nominal group conferencing, Structured and unstructured interview, and Competitor analysis

Note: Source from Technology futures, Inc.

2.2.2 Quantitative Technology Forecasting Methods

As last section has been mentioned, quantitative technology forecasting methods can be classified into extrapolators (Vanston, 2002) and trend extrapolators (Porter el al, 1991). It relies largely on direct time series analysis, and the valid data is necessary. Forecasters believe that the past data contains all the information to project the future (Martino, J.P., 2003). By this reason, forecasters think that the future will represent a logical trend and will be projected by past data. Besides, quantitative technology forecasting methods should be used in conjunction with complementary technology forecasting methods, like expert opinion and monitors. (Porter, Roper, Mason, Rossini, Banks, 1991)

When the variables have been chosen and the necessary data have been obtained, trend analysis can begin. First we have to identify what model we should use, S-shaped growth curves, Learning curve, Exponential growth, or Linear. In the second stage, we have to fit the model to the data through graphical way and to solve for the parameters in the equation. In the third stage, we have to project the future possible situations graphically or mathematically. Finally, we have to compute the confidence intervals and to consider outside factor in order to interpret the projection and perform sensitivity analysis.

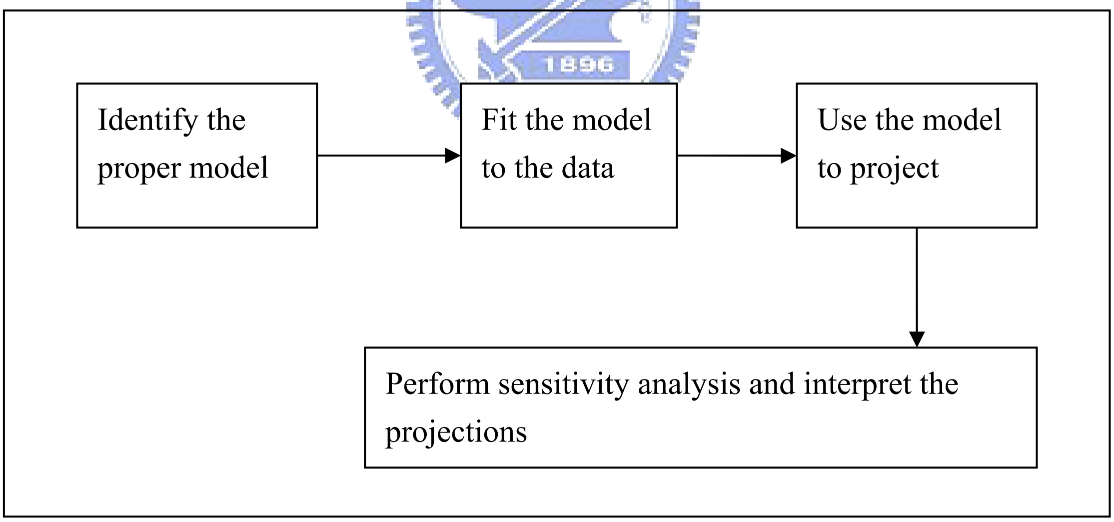

Figure 2. 1 Steps in trend analysis

Martino (2002) also said that forecasting by extrapolation means that the forecaster assumes that the past of a time series contains all the information needed to forecast the future of that time series. That is why some quantitative technological forecasting methods just used time to be the variable, then forecast or fit the data (cumulative market share, penetration, or sales volume).

2.2.3 Type of Quantitative Technological Forecasting Methods

Models which used in technological forecasting can usually be classified according to measurements, shape of curve, and the point of inflection. In prior research, these methods were investigated and found the classification of these methods. The common classification is based on shape, and this research classifies these models based on measurements and shape (Figure 2.2). When forecasters want to project the diffusion trend about a new product, they can use two kinds of measurements, proportion or units, according to what kind of data can be obtained. The Internal-influence model means that the rate of diffusion is viewed as a function of social interaction between prior adopters and potential adopters in the social system. The prior adopters can be seen as initial buyers and potential adopters can be seen as potential buyers. The behavior that initial buyers affect the potential buyers to buy a product can be treated as social interaction. Mix models combined the internal-influence model and external-influence model, which is affected by outside of social system. After deciding which measurement is decided to project, forecasters can choose internal-influence or mix model to be the forecasting tool (Mahajan and Peterson 1985, Frank 2002).

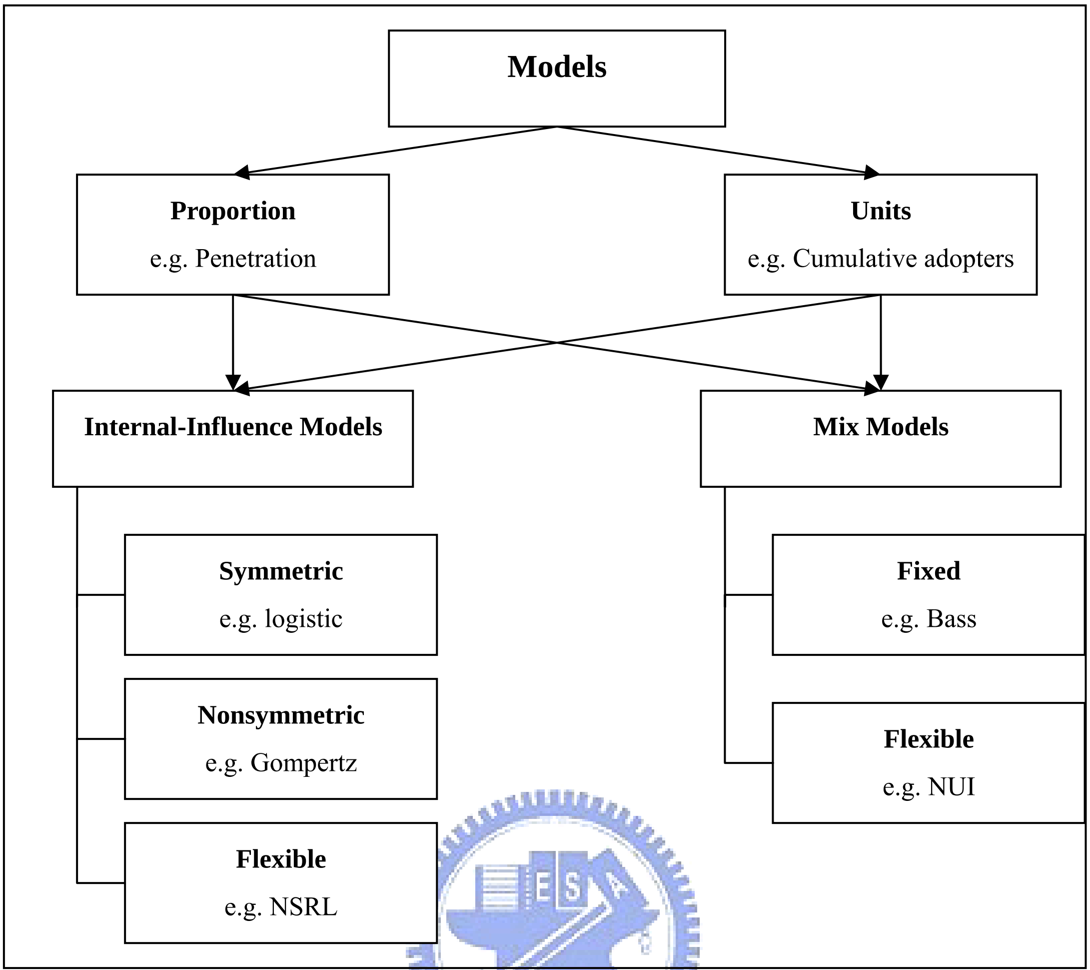

Figure 2. 2 Classification of quantitative technological forecasting models

2.2.4 The selection of technological forecasting models

Many kinds of technological growth curve model have been proposed in decades. This section will describe the selection criteria about the technological growth models. These criteria are based on observing the characteristics of data sets, such as length of data, 50% takeover point, and upper limit. These characteristics can help forecasters know what kind of technological growth model that is appropriate to be the tool of projection. Young (1993) used nine technological growth models to test which one would perform better in different kinds of data sets. He set some criteria about choosing the models in different kinds of data sets. Therefore, he offered selecting criteria before fitting data sets, and this can help the forecasters avoid biases before model fitting and projecting. The process before model fitting will be shown in this section (Figure 2.3). First step is to identify the characteristics of data sets. Then forecasters can choose one or several appropriate models according to the characteristics of data sets.

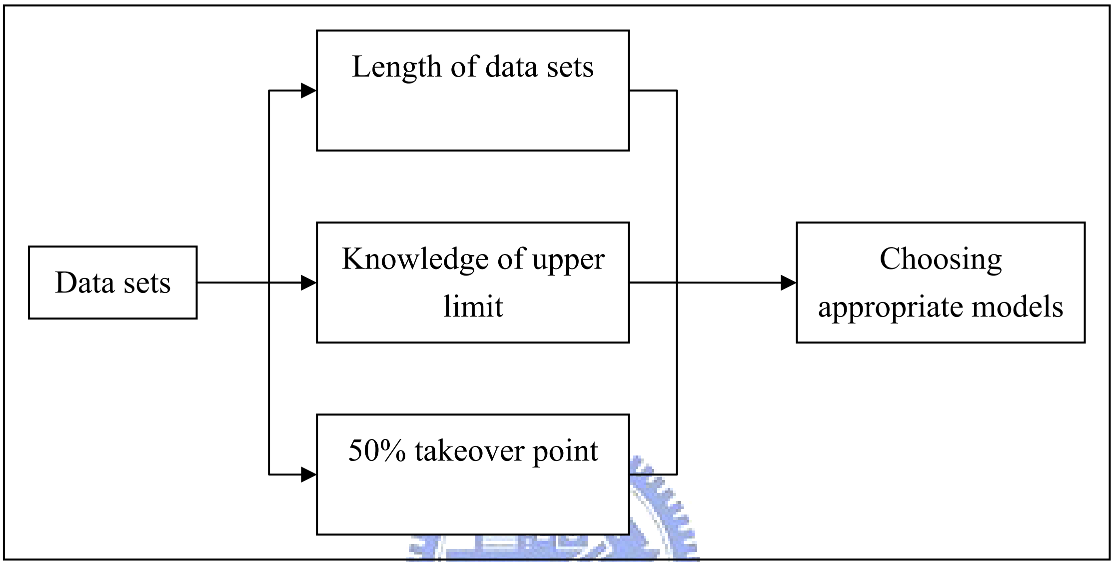

Figure 2. 3 The process before model fitting

Source: From "Technological Growth Curve: A Competition of Forecasting Models", 1993.

2.2.5 Applications of technological forecasting

Technological forecasting methods are commonly used in lots of area, such as the trend of technological growth, and technological substitution. In prior research, wireless communications are popularly investigated through technological forecasting methods. Some of these researches used the penetration to be the measurement (Vanston, 2002), and others used the units (e.g. subscribers) to be the measurement (Frank, 2004). Not only one but also two or more models were used in these researches (Table 2.3). The research subjects of these prior papers do not only focus on a product or an industry, but also investigate into a service. Therefore, quantitative technological forecasting methods are suitable for different kinds of industries, products, and services.

Table 2. 3 The recent researches using quantitative technological forecasting methods

Author	Measurement	Method	Object
Frank (2004)	units	Modified Logistic	Wireless Communications
Hodge (1998)	units/percentage	Fisher-Pry/Gompertz	Analog and Digital Cellular Phone
Rai and Kumar (2004)	percentage	Fisher-Pry/ Rai's Model	Color TV, Railway, and Thermal electricity
Palmer and Williams(1999)	units	Fisher-Pry	Electronic Industry
Victor and Ausubel (2002)	units	Logistic/Fisher-Pry transform	DRAMs
Vanston (2002)	percentage	Fisher-Pry/Gompertz	Residual Broadband

2.3 The Growth Curve

This section will talk about the growth curve, which is commonly used in interpreting the trend of a product, an industry or a service. First part will interpret the definition of growth curve. Then the representative form of growth curve will also be introduced. Finally, the adoption of life cycle which is similar to product life cycle will also be presented in this section.

2.3.1 The Definition of Growth Curve

Every product or service has its own growth curve, and every growth curve may roughly look similar but different in detail. These growth curves may appear in a wide variety, such as parabolic, power, exponential, sigmoid, and so on. Some researchers investigated these phenomena and demonstrated the growth may show not only sigmoid curve but also other forms (Rai and Kumar 2003, Porter et al 1991). However, the most commonly growth curve is sigmoid curve (S-curve). The S-Curve is the well-known concept in the innovation management, and this concept is also applied in interpreting the trend of one technology. The technical progress or performance of products basically follows sigmoid shape of performance versus time, and the measurement can be a product life cycle, a technological life cycle, or some economic performance or technical parameters. Consequently, the S-curve plays an important role in technology forecasting, because it can roughly describes the trend of technology forecasting by a graph. There are many strong evidences that in many cases, the diffusion of technologies demonstrated that it follows the smooth growth pattern just like the S-shaped. For example, Palmer and Williams (1999) employed the Fisher-Pry model to fit the microprocessor clock speed. Hodges (1998) used Fisher-Pry and Gompertz model to fit the analog and digital cellular, and he found that the trend roughly obey the S-shaped curve.

The S-curve shows the revolution of technology and it can be divided into three phases of performance (Figure 2.4). In the first phase of introduction, the new technology may just be developed, and there are still a lot of problems such as financing, process of production, unfamiliar, imperfect and so on. Therefore, the growth of first stage rises slowly and gently. In the second phase of rapid adoption, the new technology has been achieved the economies of scale, and people learn more about the new technology. Besides, the new technology improves and finds more applications. Therefore, the trend of this stage rises sharply in market share. Finally, the third phase of saturation seems to rise more gently than phase 2. The development of technology in this phase may meet the bottleneck, and some new technology may grow stronger and overtake the market share of old technology. In another word, this technology is mature.

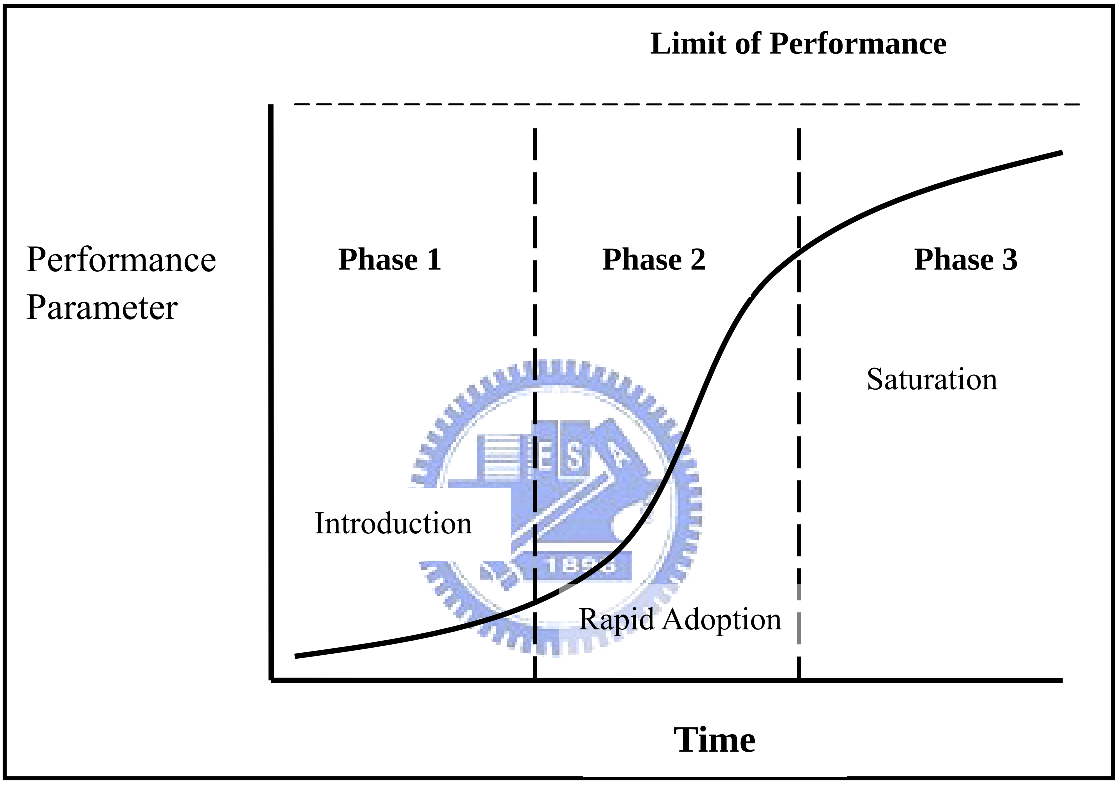

Figure 2. 4 The S-Curve

Source: From "Development of a Methodological Framework for Examining Science and Technology in Flanders," 2000.

The point of inflection is an important characteristic of growth curve (Figure 2.5). It happens when the rate of change reached the maximum value. The point of inflection can be used to judge the growth curve is symmetric or not. In general, the maximum rate of change symmetric curve (e.g. Logistic) is a constant that occurs when 50% of potential adopters have adopted the product. However, the nonsymmetrical curve, such as Gompertz, can also be calculated its point of inflection.

For example, the percentage which the point of inflection reached is about 37%. That means the point of inflection in Gompertz will happen before 50%. Flexible models, which have flexible point of inflection, will reach the maximum rate of change when equal or less than 50%.

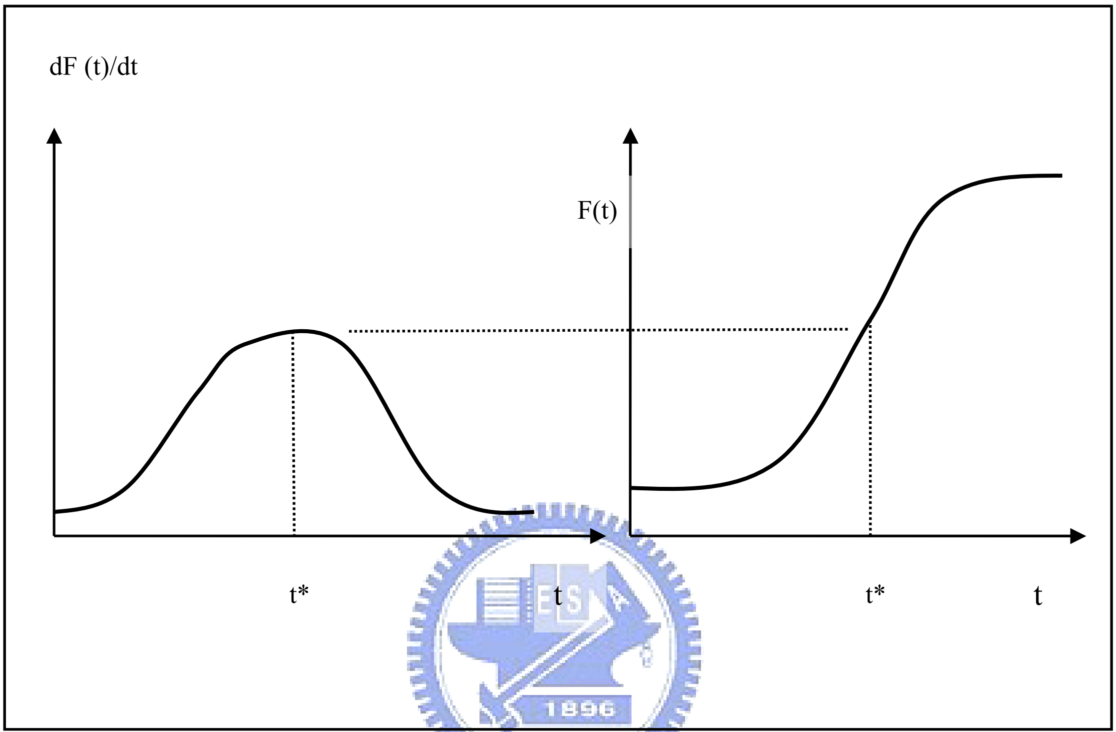

Figure 2. 5 The point of inflection in S-curve.

2.3.2 The Representative of Growth Curve

Some different kinds of growth shape have been mentioned briefly in last section, and some representatives of growth curves will be introduced now. The growth curve is based on calculating the cumulative sales, penetrations, or key performance parameter. For example, calculating the proportion of cumulative adopters who have adopted a product can draw the shape of growth curve. Consequently, here are some questions about the growth curve. What kind of curve may the growth of technology be? Is it an exponential, sigmoid, or other curve? The most commonly curves are exponential and sigmoid, and there are some comparisons of two curves in the underlying section.

The growth of population or technology may shows the exponential growth, but they wouldn't always grow up rapidly (Equation 1). If a negative feedback term is added to this equation, some restrained effect will turn the exponential growth curve into sigmoid curve.

$$p(t) = \beta e^{\alpha t} \tag{1}$$

The most widely used modification of exponential growth is logistic. Adding a negative term will make exponential growth transferring to logistic growth (Equation 2). The feedback term is near to zero when P (t) $\rightarrow$ k and near to 1 when P(t)<<k. Therefore, solving the differential equation can get the solution by integrating, and the solution will be similar to S-shaped curve (Equation 3).

$$\frac{dP(t)}{dt} = \alpha P(t)\left(1 - \frac{P(t)}{k}\right) \tag{2}$$

$$P(t) = \frac{k}{1 + \exp\left(-\alpha(t-b)\right)} \tag{3}$$

When putting a feedback term to the exponential function, some inhibitions will be revealed. The new equation belongs in the technology growth or other field such as population growth. Every technology can't grow infinitely and show a development like the exponential curve. Consequently, it has its limitation in market share, just like the life cycle of technology. The half of life cycle can be divided into three phase; Introduction, Rapid adoption, and Saturation. The logistic function can fit better than exponential function in technology forecasting (Figure 2.6).

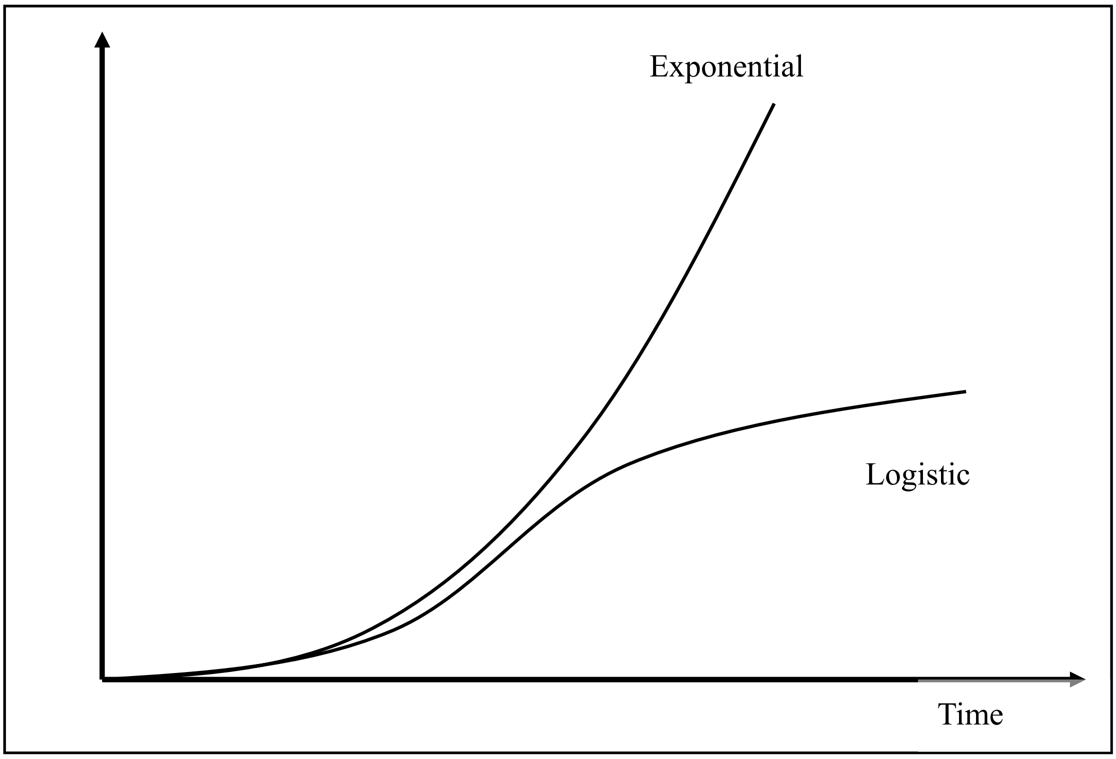

Figure 2. 6 Comparison of exponential and logistic

2.3.3 Technology Adoption Life Cycle

When one new product is thrown into the market, it may success or fails in market. Same as one new technology is proposed. Therefore, the technology adoption life cycle can show what place the technology or product is in. It classifies the market and the reactions to a high-tech product. The technology adoption life cycle is the tool for determining the products, pricing and marketing strategies for high-tech products, and was defined as consisting of six phases, including innovation, chasm, tornado, Main Street, decline and obsolescence.

The phases and the trend of life cycle can be shown (Figure 2.7). The phase of chasm means that it goes down in phase 1. Consequently, some new technology may fall into this phase and can be judged that it may not go to the phase of Main Street. (Meade, P.T., Rabelo, L., 2004)

The S-curve also can be divided roughly into three phase: growth, saturation, and decline. In the first phase, the new technology has to compete with other technologies. In the saturation stage, all technologies have competed, and the growth of new technology faces the limit. In the last phase, no competition will be concerned.

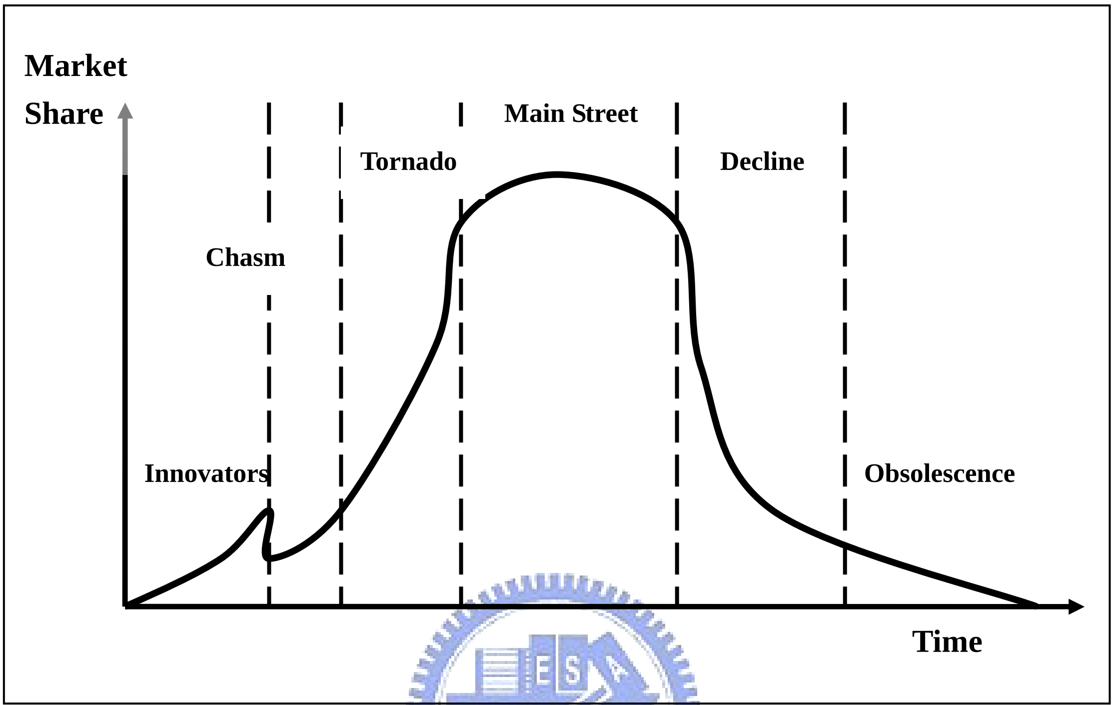

Figure 2. 7 Life cycle phases

Note. From "The technology adoption life cycle attractor: Understanding the dynamics of high-tech markets," by Meade, P.T., Rabelo, L., 2004, Technology Forecasting & Social Change, 71, p.670.

2.3.4 Technological Obsolescence

Obsolescence can be thought simply as the assets loss in value when the market expectation is increasing, the utility of assets is still the same. For example, when the utility of notebook increase, such as wireless function and power saving, the consumer's expectation will also increase. Then the old generation notebook will lose its value gradually in that the consumers' need increases.

The technology S-curve was introduced in the last section. Consequently, technological obsolescence will happen following with the technology change, and this phenomenon can be observed by using technological substitution analysis, such as Fisher-Pry model, which based on the adoption of potential adopters. The old technology has economics of scale, and it is well known and mature in the first stage. Basically, the appearance of obsolescence is not very clear in old technology at early stage when a new technology entries in the market. However, when new technology grows up and achieves its economics of scale and has more improvements, the old technology can't enhance its market share and start to follow a downward tendency. In the final stage, the old technology will decline to zero and be displaced by new technology (Figure 2.8).

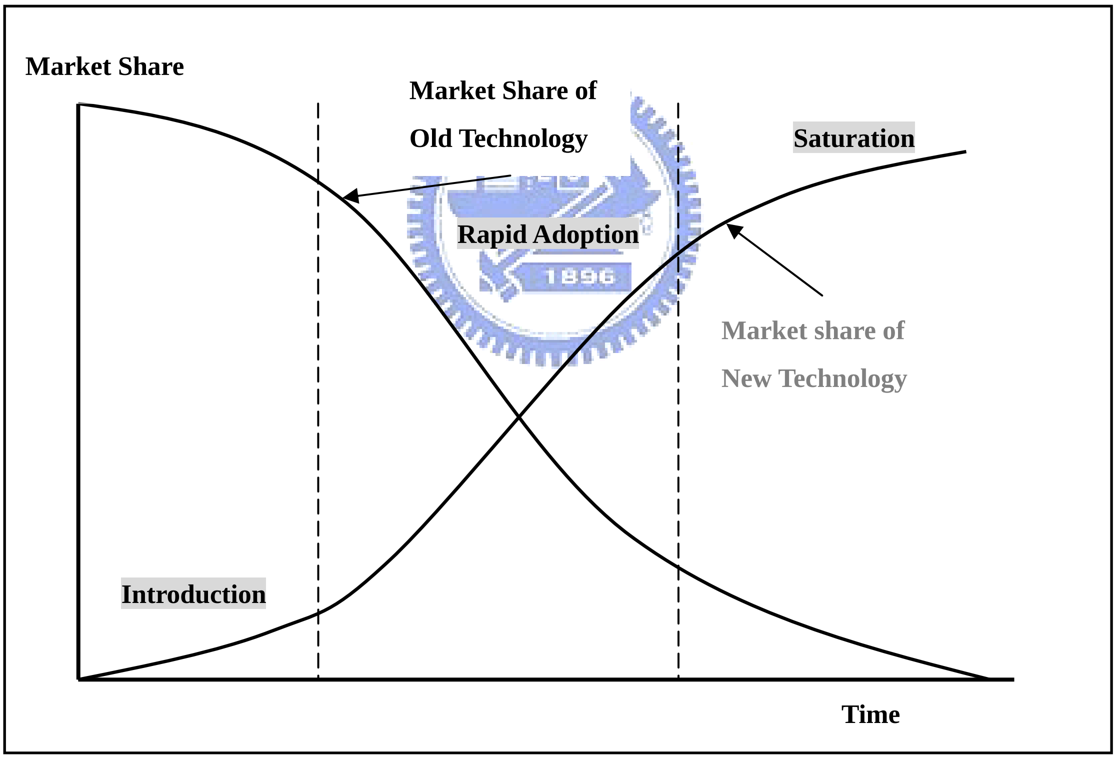

Figure 2. 8 The typical obsolescence and S-curve chart

2.4 Technological Substitution

Technological change is a dynamic and multidimensional phenomenon involving substitution and diffusion of technologies (Rai, A.P., Kumar, N., 2003). Technology substitution can be defined as a process by which an innovation is replaced partially or completely by another in terms of its market share or sales volumes over a period of time. This process follows the adoption life cycle and can be listed in several emphases.

- New technology enters the market and grows as a logistic curve (e.g. Fisher-Pry type for two competitors).
- Only one technology will reach the saturation level at any given time.
- A technology in saturation level will follow a non-logistic path and then will begin to decline which follows a reverse logistic form.
- A declining technology still follows a reverse logistic form (e.g. Reverse Fisher-Pry for two competitors) and becomes no competition gradually. (Meyer, Yung, and Ausubel, 1999)

For example, .Hodges (1998) investigated that the transfer of the subscribers between the analog and the digital systems in the U.S. was obeyed the technological substitution process. This process was fitted and projected by some technological substitution models (e.g. Fisher-Pry model). In general, we can use the mathematical methods to predict the trend of the technology substitution, and substitution models proposed by different researchers are based on the assumption that the replacement factor is a function of the market share captured by the technology (Rai and Kumar, 2003). Basically, fitting and projecting the substitution of technology is important, and it can let the enterprisers know how they should adjust their strategies and know the timing of launching a new product. However, Morris and Pratt (2001) said that forecasting technological substitution requires a model that generates intuitive understanding of the factors affecting substitution but that also has good predictive ability.

2.5 Hypotheses

The development of technological forecasting is popularized in decades, and not only qualitative but also quantitative methods have been investigated comprehensively. This research focuses on one branch of quantitative technology forecasting model that based on estimating the penetration or the market share of a product. This research proposes a new model based on a dynamic capacity, and use this new model to compare with Fisher-Pry model and Gompertz model.

The derived literatures proved that put a right technological forecasting model into an appropriate data set will increase the accuracy of fit performance. There are two commonly technological forecasting models, Fisher-Pry and Gompertz, and these two models can fit well in symmetric (e.g. Fisher-Pry) and asymmetric (e.g. Gompertz) data sets. However, not all the products or technologies will grows as symmetric sigmoid curve. This research will test the Fisher-Pry model and Gompertz model in some data sets which grows not only as symmetric but also asymmetric and other forms, and the extended logistic will also be examined in same condition. In order to examine whether Fisher-Pry, Gompertz or extended logistic model can offer a better fit performance, the hypothesis will be set as below:

H1: The extended logistic model will have a better fitting performance than Fisher Pry model and Gompertz model in both long and short data sets.

The characteristics of growth curve of each data set may have influence on fitting performance when a technology forecasting model is used to fit the data set. According to prior researches about quantitative technological forecasting, Fisher-Pry model offer a good fit in data sets which grows as symmetric, and Gompertz model can offer a better fit in asymmetric data sets. On the other hand, the new model, extended logistic, is supposed that it can offer a good fitting performance whether the data set is symmetric or not. Therefore, the hypotheses will be listed as below:

H2: The extended logistic model has better fit performance in symmetric, asymmetric, and flexible data set than Fisher-Pry model and Gompertz model.

According the prior research, many research papers draw some data sets before checking the fitting performance to examine the predicted performance. The predicted performance of a data set is more important than fitting performance of a data set. The good fit of growth curve only can observe the trend of products or technologies in specified time period, but a good forecast can offer an omen of the adoption of new products. This research will test whether a good fit can product a good forecast and this will be put into the hypotheses. Moreover, the data which are drawn form different kind of data will used to examine the predicted performance in Fisher-Pry model and extended logistic model.

H3: The extended logistic model has better predicted performance in both long and short data set than Fisher-Pry model and Gompertz model.

If Fisher-Pry model can offer a good fit in symmetric data, it may also offer a good predicted performance in symmetric data, and so does Gompertz model. The extended logistical model will also be tested whether it can offer a good predicted in symmetric data. The other characteristic of data set, such as asymmetric data, will also be discussed in this research. Therefore, the comparison between these models will be hypothesized in this research.

H4: The extended logistic model has better predicted performance in symmetric, asymmetric, and flexible data set than Fisher-Pry model and Gompertz model.

H5: A good fit performance will lead a good predicted performance.

CHAPTER 3 METHODOLOGY

This chapter will introduce the methodologies used in this thesis in detail. There are many quantitative models proposed using in technology forecasting area. Some of them are statistical models, and others are time-related models. Besides, an adapted way to improve the fit in technological forecasting models will be introduced in this chapter.

There are many forecasting methods in technology forecasting field, and some criteria can make sure of which one is better. First, matching the model to the fundamental phenomena can make researchers knowing what extent can be represented in the actual area. Second, it is important to tailor and select models to the horizon. Finally, some time-series related models should be found forecast validity in order to improve forecast more accuracy. (Armstrong, J.C., 2001)

This thesis will use some quantitative models and compare with these models in the penetration of durable goods in Japan, and test the technological substitution of the pixels of Digital Camera of Taiwan. Furthermore, the research structure and the introduction of data collecting will be introduced in this chapter.

3.1 Research Structure

The figure 4.1 shows the process of research structure in this thesis. In the first step, some proper methods will be chosen and one combined model which built by combining some proper models will also be proposed. According to the combining models proposed by Meade and Islam (1997), they classified the technological forecasting model by two characteristics, the point of inflection and shape. Some existent technological forecasting models and a combining model will be adopted in this research. Then some data will be collected and put these data into the models to fit them. The process will be followed to draw the trend and project it. In this part, three different kinds of software can offer the forecasting tools to fit and draw the data.

However, comparing with models can extrapolate which one is better. Some criteria can be used to make the judgments. Furthermore, we can observe the level of fit by viewing the visible figure of trend in these growth curves.

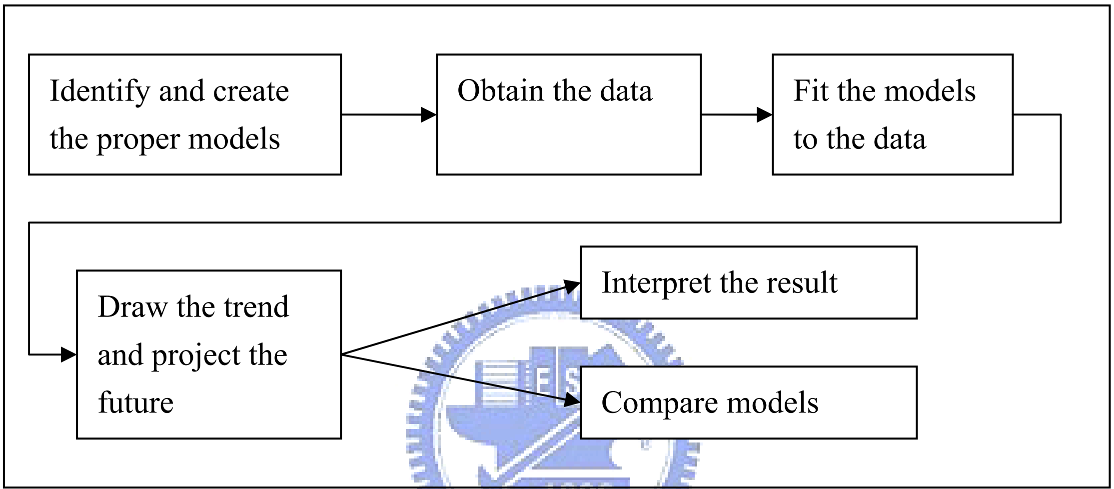

Figure 3. 1 The process of research

3.2 Data Collection

The quantitative technology forecasting models need data like sales volumes, market share, and some other secondary data (e.g. Annual survey by statistical bureau). Finding the correct data is important to project the data more accuracy. Therefore, data can be collected by some authorities and government organizations, such as IT IS, MIC (Market Intelligence Center), DGBAS (Directorate-General of Budget, Accounting, and Statistics), and some other statistical bureau in other countries. This research will use the penetration of durable goods in Japan which were gathered from the Statistics Bureau of Japan, and test the technological substitution of digital camera which collected from MIC annual report.

In order to examine the fits of models, some penetrations of durable goods in Japan is used in this research, such as Microwave ovens, space heater, room air conditioner, color TV sets, videotape recorder, radio cassette, and CD player. These data sets will be divided into two parts. One is used to test the fit performance of models, and the other is used to test the accuracy of forecasts. Therefore, every data set will be retained last 5 data in order to test the accuracy.

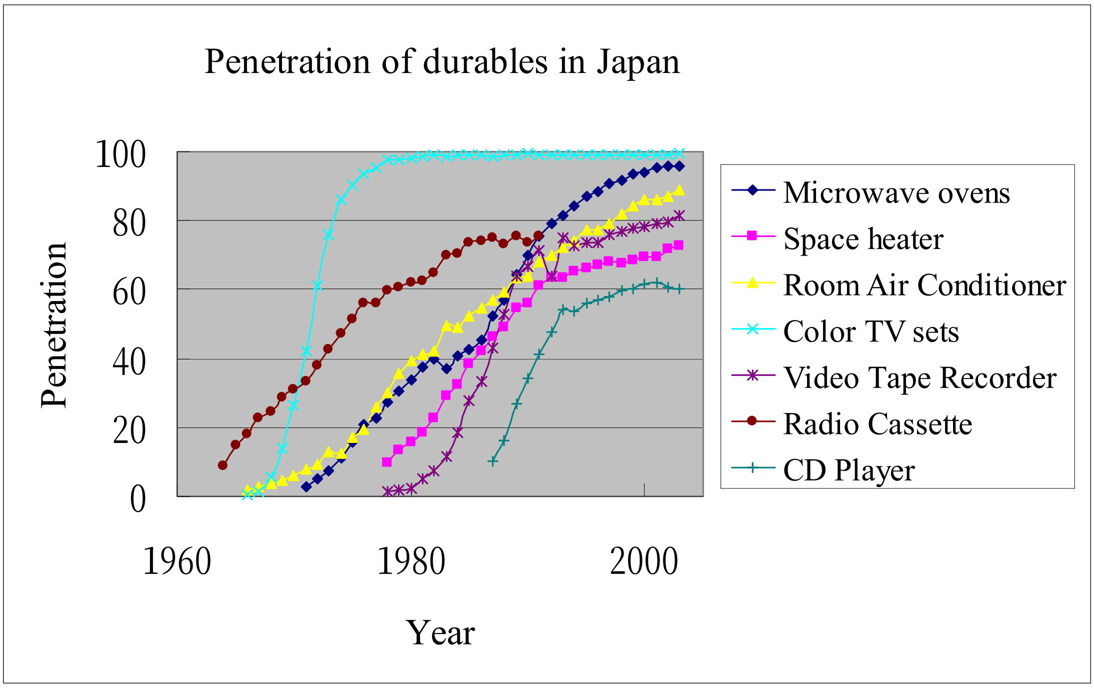

Figure 3. 2 The penetration of durable goods in Japan

(Source: These data sets were gathered from the Statistics Bureau of Japan. http://www.stat.go.jp/)

These seven data sets can be classified according to the length of data, the point of inflection, and the knowledge of upper limit (Table 3.1). First, this research considers that a data set can be thought as long data if the number of observations is more than 25 in a data. Second, the type of graph can be roughly considered symmetric, asymmetric, or other forms. Finally, this thesis considers whether the graph reach the 100% level or not.

Table 3. 1 The classification of data sets

		Long data sets	**Short data sets**
Length of data sets		Microwaver Oven, Room Air Conditioner, and Color TV	Space Heater, Video Tape Recorder, Radio Cassette, and CD Player
	Asymmetric	**Symmetric**	**Other**
The point of inflection	Color TV, and Video Tape Recorder	Microwaver Oven, and Room Air Conditioner	Space Heater, Radio Cassette, and CD Player
		100%	**Less than 100%**
Upper Limit		Microwaver Oven, Room Air Conditioner, and Color TV	Space Heater, Radio Cassette, Video Tape Recorder, and CD Player

Digital Camera grows rapidly in recent years, especially in technological growth in pixels. Data was collected from MIC annual electronic industrial report. This research simply divided the data into two groups, less than two million pixels, and more than two million pixels. The purpose of this classification is to test the trend of technological substitution between the old and new generation.

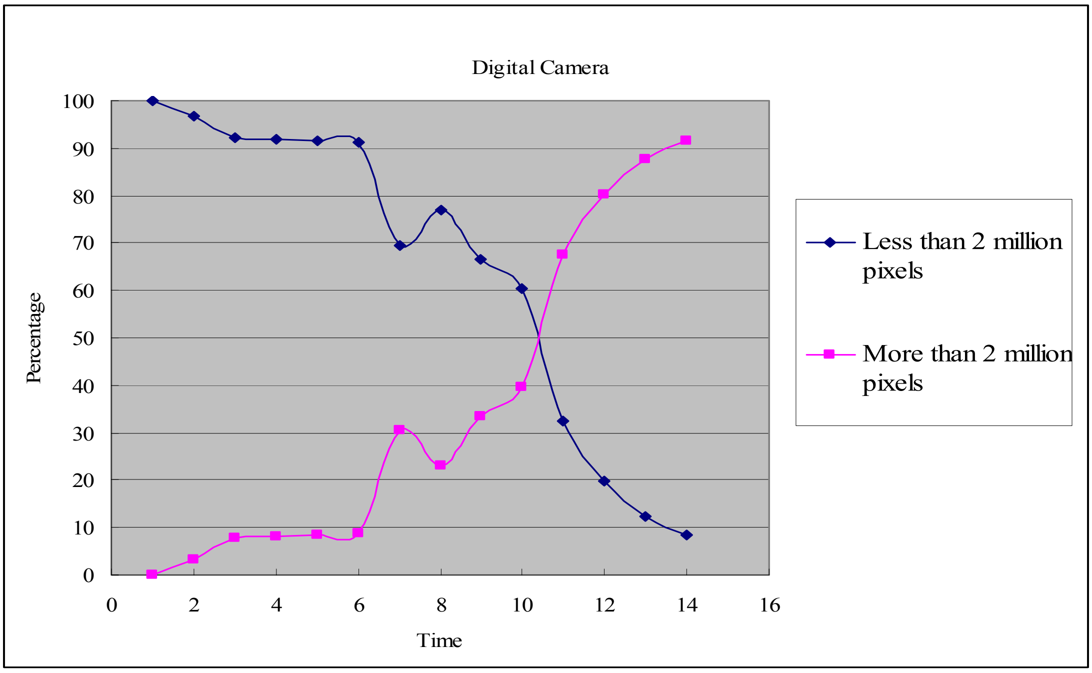

Figure 3. 3. The percentage of pixel-change of Digital Camera

3.3 Fisher-Pry Model

In 1971 Fisher and Pry published a paper that described a simple model of technological change. This model is equivalent to biological growth model, because the technological growth is similar to biological growth. Therefore, this model is a common model in projecting the technological substitution, such as the rate at which one technology will replace another. This model is based on the observation that most growth trends follow an “S-shape” curve or “logistic”, in time with an exponential growth rate in the early years.

Williams, Palmer, and Hughes (1999) mentioned that using this model can be justified on three assumptions:

- Many technological advances can be considered as a competitive substitution, a superior method take the place of an inferior method.
- If the market share of an older technology has progressed as far as a few percent, it will proceed to completion and new technology will reach the saturation level.
- The fraction rate of substitution of new for old is proportional to the remaining amount of old to be substituted.

It is convenient to take the fraction of potential market penetration, f, as the dependent variable where $f = Y/L$ and L is the upper bound for the growth of the variable Y. The f value is between 0 and 1, and variable y can be the sales volumes or the market share. Using the sales volumes as variables is a good assumption to project the future, because it was influenced by many factors, such as service, distribution, and so on. Consequently, the rate of sales volumes can reflect the trend appropriately when the quantitative technology forecasting model is used.

3.3.1 The Mathematical Inference of Fisher-Pry Model

The mathematical inference will be discussed in this section, and the model will be explained by mathematical expression. Before discussing about the Fisher Pry model, the mathematical inference can start at the general idea of models for innovation diffusion. The process of mathematical inference about the Fisher Pry model will be shown in appendix 1. Therefore, this section will directly discuss the general inference of Fisher Pry model.

According to the Fisher Pry original model, the equation can formulate as:

$$\frac{df}{dt} = b\left(f \times (1 - f) \right) \tag{4}$$

The basic fundamental of this model is that the rate of change of f is proportional to both f and (1-f). The parameters, b, is effective to detect the interactive effort when the potential market share, (1-f), converted into accepting the new technology through the fraction, f, of the market share that accepted the new technology. Thus, we can use the integration to solve the f and get the common equation of Fisher Pry model. The function can be written by:

$$f = \frac{1}{1 + \exp\left[-b(t - t_0)\right]} \tag{5}$$

Raymond Peal proposed a forecasting model which is similar to Fisher Pry model, and it is called the Pearl curve. He utilized this model to forecast in demographic forecasting. Moreover, it can be used in examining the absolute growth of a technology rather than the fraction of market share that a technology was taken over by another technology. (Equation 3)

$$f = \frac{1}{1 + c\exp(-bt)} \tag{6}$$

Fisher-Pry model also can be changed from the nonlinear form into the linear one. (Equation 4)Where f is the fraction of market share of the new technology, and

(1-f) is the fraction of the market share of the old technology; α is half the annual fractional growth in the early years, and t_0 is the time at which f =1/2. Fisher Pry is a symmetric model. That means its point of inflection will happen at f=0.5 (Figure 3.3).

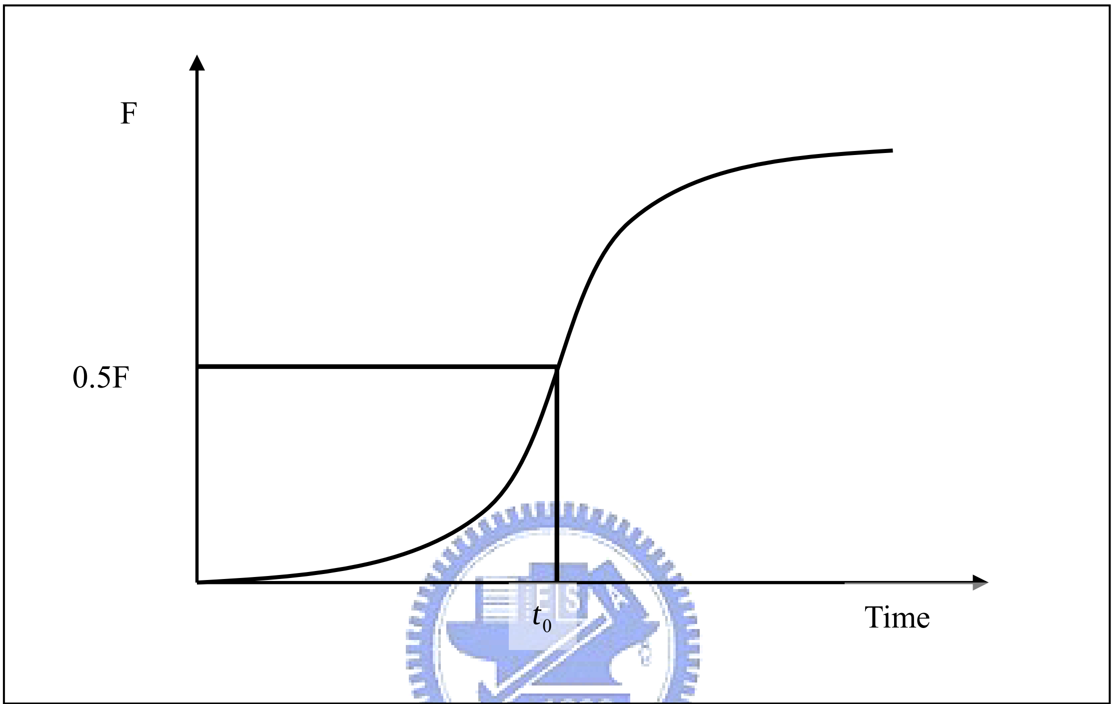

Figure 3. 4 The point of inflection of Fisher Pry

Furthermore, the parameters in equation 3 can be calculated by the transformed Fisher-Pry linear model. This linear model is directed transformed by equation 3, and the parameters can be easily estimated. The OLS method used in linear regression can find the proper estimators in transformed Fisher-Pry model. (Equation 8)

$$\ln\left[\frac{f}{(1-f)}\right] = 2\alpha\left(t - t_0\right) \tag{7}$$

$$Z = \ln\left[{(1-f)}/{f}\right] = \ln(c) - bt \tag{8}$$

3.3.2 The variants of Fisher-Pry model

The Fisher-Pry model can be modified to fit the trend; therefore, Bhargava (1995) proposed a generalized form of the Fisher-Pry model. He believe that the substitution rate, b, may not be always a constant; furthermore, he considered that the parameters b is a time dependent variable. B (t) is a time related function, and this function can has various forms. (Equation 9)

$$\ln\left[\frac{f}{1-f}\right] = \int B(t)dt \tag{9}$$

Some possible forms can be express as equation 6, and we can solve the integration to some possible forms of generalized Fisher-Pry description.

$$\ln\left[\frac{f}{1-f}\right] = a + b(t)^c$$
$$\ln\left[\frac{f}{1-f}\right] = a + bt + ct^2$$
$$\ln\left[\frac{f}{1-f}\right] = a + be^{ct} \tag{10}$$

3.4 Gompertz Model

The Gompertz model is occasionally referred to as a natality model and it is a little different from Fisher-Pry model, which is characterized as a growth model. When researchers want to use the Gompertz model in technology forecasting, some conditions that the equipment replacement is driven by equipment degeneration rather than technology innovation. The curve of Gompertz model represents an asymmetric form. Therefore, the Gompertz model can exhibit a good projection in technical performance, such as the speed of CPU or the storage of hard disk.

3.4.1 The mathematical Inference of Gompertz Model

This section will introduce and verify the Gompertz model. The mathematical inference can be started at the original form of Gompertz model. The original form can be expressed as:

$$\frac{df}{dt} = b \times f \times (\ln(L) - \ln(f)) \qquad (11)$$

An explicit mathematical inference of Gompertz model is shown in appendix 2, and this inference is proven by internal-influence model. After integrating the formulation (Equation 11), the equation of Gompertz model can be represented by:

$$f = \exp[-b\exp(-kt)] \qquad (12)$$

where f= Y/L, and L is the upper bound for the number of units, y. The parameters b and k can be estimated from fitting the curve of the data. This equation is the variation of S-shaped curve, and so does Fisher-Pry. However, the Gompertz curve is not symmetrical about the central inflation point, and its inflation point occurs at t= (ln b)/k. Gompertz model will reach the points of inflection before 0.5F, and this can be the way to make sure that Gompertz model is a asymmetric model. As same as Fisher-Pry model, we can obtain the coefficient of parameters b and k by linear regression. The regression model can be showed by underlying equation:

$$Z = \ln\left[\ln\left(L/f\right)\right] = \ln(b) - kt \qquad (13)$$

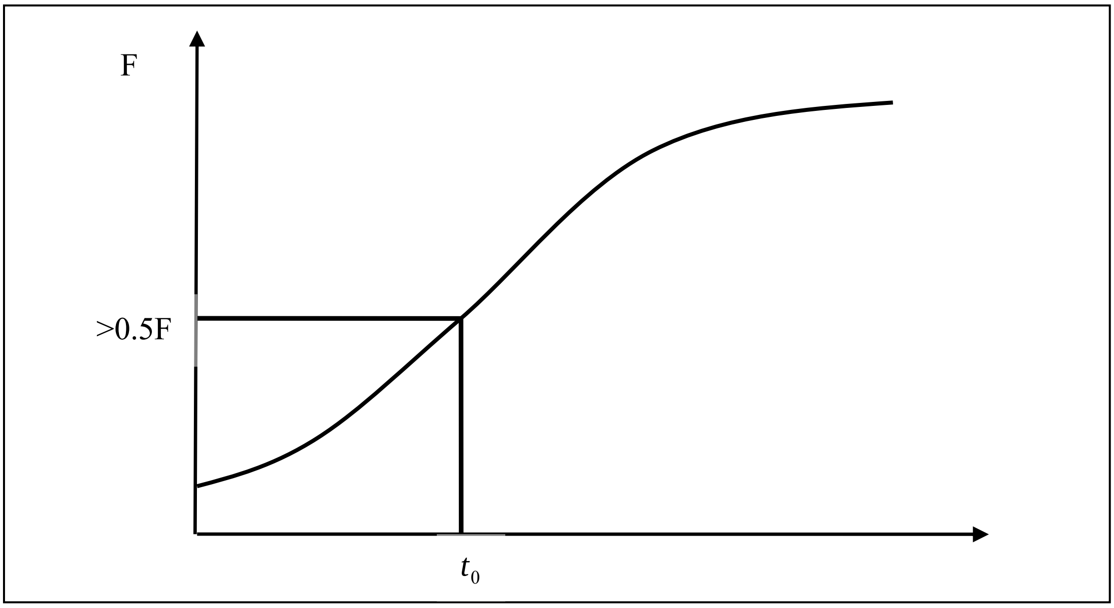

Figure 3. 5 The point of inflection of Gompertz

3.5 The Extended Logistic Model in This Research

The new growth model will be introduced in this research, and the mathematical inference will also be presented. In prior research, the technological forecasting models always based on this equation or its integration;

$$\frac{df}{dt} = b\left(f \times (1-f)\right) \tag{14}$$

$$\ln\left(\frac{f}{1-f}\right) = a + bt \tag{15}$$

where a is the constant and b in equations is still the same.

However, many variants based on growth model were proposed in last two decades (Bhargava 1995, Rai and Kumar 2003). Bhargava argued that the parameters may not always be a constant, and Rai and Kumar proposed several new models which changed the form of parameter, t, to be a logarithmic form or exponential form. Moreover, Meyer and Ausubel (1999) proposed a new thinking about the capacity of logistic curve. They considered that the capacity may also fluctuate according to time and grow as same as logistic curve (Equation 14).

$$\frac{df}{dt} = bf\,(1 - f/k(t)) \tag{16}$$

where $k(t)$ is the function which is similar to logistic curve.

Now the idea based on fluctuant capacity about the logistic curve will be introduced in this research. As the equation mentioned before (Equation 14), this research modified the equation as:

$$\ln\left(\frac{f}{k(t) - f}\right) = (a + bt) \tag{17}$$

This research assumes that $k(t) = 1 - D \times \exp(-A \times t)$, where the value of D can be any number and the value of A must bigger than zero. This research assumes that the capacity will fluctuate. A product may reach the saturation level at 80 or 60%, because the market share or substitution rate of this product may be affected by new products. It may not achieve the 100% of market share and may collapse earlier than expected.

3.5.1 The mathematical Inference of Extended Logistic Model in This Research

This section will be described the mathematical inference of the new model in order to make more clear about the new model. In last section, the adapted form of equation has been defined (Equation 14). The first step is using some mathematical calculation in order to cancel out the logarithm. Therefore, the exponential term is added in both sides of equation. Then the equation will become as:

$$\frac{k(t) - f}{f} = \exp(-a) \times \exp(-bt) \tag{18}$$

In order to simplify the equation, some algebraic expressions will be used. Let the letter, C, equals to $\exp(-a)$, and put this transformation into the equation. After that, the equation can be expressed as:

$$\frac{k(t) - f}{f} = C \times \exp(-bt) \tag{19}$$

Now the simplified equation has been demonstrated, and the next step is to find the final equation. Therefore, this research will calculate the equation step by step and list the equation of every mathematical procedure. After doing a transformation to move the letter f in the denominator in the left side of equation, it becomes easier to find the final form of new model.

$$k(t) - f = f \times C \times \exp(-bt) \tag{20}$$

Finally, the final form of the model can be found through several calculations, and the equation can be solved. The equation, $k(t)$, can also be putted in the model. Therefore, two forms of new model can be expressed as:

$$f = \frac{k(t)}{1 + C \times \exp(-bt)} \quad \text{or}$$

$$f = \frac{1 - D \times \exp(-At)}{1 + C \times \exp(-Bt)} \tag{21}$$

3.5.2 The Variants of Extended Logistic Model

There are many models related to the technological forecasting, and some models can be treated as the variants of other models. For example, Bertalanffy model can be expressed as:

$$f = \frac{1}{(1 + c \times \exp(-bt))^{1/\alpha}} \tag{22}$$

This model can be transformed into some common technological forecasting models. It gives the Fisher-Pry model when $\alpha = 1$, and it becomes the Gompertz model as $\alpha \to 0$. Therefore, the new model also can be taken as some variants of other models. The new model is similar to Fisher-Pry model when $D \to 0$, and it becomes the extended logistic model (Mahajan and Peterson, 1978) as D=1 and A=B. Consequently, the new model can also be taken as the variants of other models.

3.6 Compare Method

This section will introduce how this thesis estimates the parameters of each model. Estimating correct coefficients can make the forecast or fit better. Another part will describe the measurements which test the fit and forecast performance. These measurements can be seen as tools to help research test the hypotheses.

3.6.1 The Estimation of Parameters

This research uses the nonlinear least square method to estimate the parameters. The basic idea of least square can be expressed as:

$$Q = \sum_{i=1}^{n} \left[Y_i - f(X_i, \kappa) \right]^2 \tag{23}$$

Where Y_i is the actual data, X_i is the number of variables, and κ is the matrix of parameters.

There are two kinds of methods which based on least square method can find the value of parameters, normal equation and Gauss-Newton method. This research uses the Gauss-Newton method to find the appropriate solution. This method uses the Taylor series to find a linear regression model which approximate to nonlinear regression model, then use the ordinary least square method to estimate the parameters. However, this method needs to set initial values($g_0^0, g_1^0, ..., g_{p-1}^0$) of parameters, and they can be roughly found by some ways such as the related research before, the theoretical expectation, and the value which can make the least square, Q, smaller.

$$f(X_i, \gamma) \cong f(X_i, g^0) + \sum_{k=0}^{p-1} \left[\frac{\partial f(X_i, \gamma)}{\partial \gamma_k} \right]_{\gamma = g^{(0)}} \left(\gamma_k - g_k^{(0)} \right) \tag{24}$$

where $g_{p\times 1}^{(0)} = \begin{bmatrix} g_0^{(0)} \\ g_1^{(0)} \\ . \\ . \\ g_{p-1}^{(0)} \end{bmatrix}$

This equation is approximate to the linear regression form without the intercept; therefore, the ordinary least square method can be used to calculate the SSE (sum of square error). Gauss-Newton method use the adjust value of $g^{(1)}$ to do the same procedure above and get the new SSE. Therefore, repeating these procedures and examining the every SSE value. The criterion of stop these procedures is to examine whether the value of $SSE^{t+1} - SSE^{t}$ can be omitted. This research use 0.0001 to be the criterion to stop the process. The final value of parameters is the matrix of g^{t} when $SSE^{t+1} - SSE^{t}$ is less than 0.0001.

3.6.2 The Measurements of Fitting and Forecast Performance

There are three common measurements to inspect whether the model performed a good fit or a good forecast. Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) are commonly used in this field. Therefore, the mathematical representation will be shown as below:

$$MAD = \frac{\sum_{i=1}^{n} \left| f_i - \hat{f}_i \right|}{n} \tag{25}$$

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{f_i - \hat{f}_i}{f_i} \right|}{n} \tag{26}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} \left(f_i - \hat{f}_i \right)^2}{n}} \tag{27}$$

where f_i is the actual value at time t, $\hat{f}_i$ is the estimate at time t, and n is the number of observation.

These three measurements are based on the residuals, which represent the distance between real data and predicted data. Consequently, if the value of these three measurements is small, the fitting and predicted performance is better. These three measures will be taken as tools to examine the hypotheses.

CHAPTER 4 RESULTS

This chapter can be divided into two parts, fitting performance and forecasts, and Fisher-Pry model, Gompertz model, and new model will be tested in these two parts. The residuals can be an index to measure fitting performance, and three criteria, MAD, RMSE, and MAPE will be tested in this research in order to compare between Fisher-Pry, Gompertz and extended logistic model. Moreover, the trend curve between these three models will also be drawn. These figures will offer an easily way to find visibly which model is better.

4. 1 Fitting Performance between Models

The first step of using technological forecasting model is to find the coefficients in each model, and then these models will be examined in each data set. The projected points will be drawn and compare the curve with original data sets. Checking residuals is another way to inspect the fitting performance between models, and some measurements based on calculating the residuals are used to measure the performance.

4.1.1 The Test of Fisher Pry Model and Gompertz Model

This section will introduce the fit result of Fisher Pry, Gompertz Model. These two models are commonly used in technological forecasting field, such as technological growth and technological substitution. This research will first test 7 data sets in order to examine the fitting performance in these three models, and they can be classified according to the length of data sets and a point of inflection. After testing these models in these 7 data sets, the coefficients can be estimated by nonlinear least square (NLLS) method (Table 4.1). The NLLS method needs an initial value of each parameter, and these two models can be easily found the initial value through using the first data set (t=0).

Table 4. 1 Coefficients of Fisher-Pry and Gompertz Model

Microwave Oven	C	B	R^2	**Room Air Conditioner**	C	B	R^2
Fisher-Pry	11.65	0.172	0.981	Fisher-Pry	14	0.138	0.980
Gompertz	3.294	0.115	0.970	Gompertz	3.817	0.092	0.995
Space Heater	C	B	R^2	**Color TV**	C	B	R^2
Fisher-Pry	4.992	0.142	0.944	Fisher-Pry	50.734	0.179	0.998
Gompertz	2.148	0.149	0.994	Gompertz	9.364	0.493	0.999
Video Tape Recorder	C	B	R^2	**Radio Cassette**	C	B	R^2
Fisher-Pry	14.128	0.242	0.926	Fisher-Pry	50.734	0.719	0.970
Gompertz	8.423	0.298	0.991	Gompertz	2.057	0.135	0.993
CD Player	C	B	R^2				
Fisher-Pry	3.771	0.188	0.857				
Gompertz	1.701	0.268	0.969				

The fitting performance can be checked by the value of R^2. This measurement mainly indicates the degree of explain ability of independent variable. The data set of color TV shows the best R^2 value (0.998) in all data when Fisher-Pry model is used, and it is the one which achieves the saturation level. On the contrary, the lowest R^2 value is the data set of CD player, which seems to reach the saturation level at 60%. Because of the limitation of capacity, Fisher-Pry model may not have better fitting performance in some data sets which are similar to the data of CD player. On the other hand, Gompertz model allows the change of saturation level; therefore, it gets better performance than Fisher-Pry in some data sets. In some data sets which have got the 100% saturation level (e.g. Microwave oven, Room air conditioner, and Color TV), Fisher-Pry model has similar performance to Gompertz model. On the contrary, Gompertz will have better in data sets which reach the saturation level before 100%.

4.1.2 The Test of Extended Logistic Model

This section will examine the extended logistic model in these seven durable goods in Japan. The main purpose of checking these data sets is to look over the fitting performance in these phenomena. As this thesis mentioned before, these seven data sets not all have the same shape. Therefore, this research uses these seven data sets to examine the model in order to compare with Fisher-Pry and Extended logistic model.

Table 4. 2 The coefficients and R-square of extended logistic model

	Microwave Oven	**Room Air Conditioner**	**Space Heater**	**Color TV**	**Video Tape Recorder**	**Radio Cassette**	**CD Player**
Coefficients							
D	3	0.949	0.419	0.166	0.496	0.537	0.504
A	0.9	0.051	0.017	0.146	0.04	0.034	0.02
C	12.895	5.403	5.089	61.521	71.996	3	3.966
B	0.178	0.142	0.291	0.794	0.568	0.223	0.677
R^2	0.966	0.996	0.998	0.999	0.994	0.994	0.996

The coefficients of extended logistic model are calculated by nonlinear least square method, and R-square is also counted (Table 4.2). All R-square of these seven data sets is near 1 when the extended logistic model is used to fit the data sets. Consequently, the extended logistic model can offer a good fit in these seven data sets.

4.1.3 Comparison between models

This section will make some comparison between these two models by using the testing criteria, RMSE, MAD, and MAPE. The fit curve can offer a growth trend of one technology; therefore, the fitting performance is also an important part to do the technology forecasting. These data sets can be classified according to length, the saturation level, and the growth rate during 10% to 90% penetration level. This section will use the estimated value of these three models to draw the curve in order to see the fit performance through figures. The fitting performance between models also can be performed in other way. The testing criteria will be listed by a tabular form. These criteria which used to test the fitting performance will be described first, and they will help the researcher easily to find which model is better than others through the number form.

Now the long data sets (e.g. Microwave oven, Room air conditioner, and Color TV sets) will be discussed first. The extended logistic model can compare with other two existent models by drawing each projected curve (Figure 4.1 to 4.3). Although these three models all look close to the original data points in these three data, some difference between models can also be found through projected curves. The penetration of microwave oven shows a bi-logistic form for the growth lagging at 1980-1985; therefore, these models all overestimate the penetration in that stage (Figure 4.1). This phenomenon makes Gompertz underestimate the introduction and saturation level, but Fisher-Pry and extended logistic model are more close to data points than Gompertz.

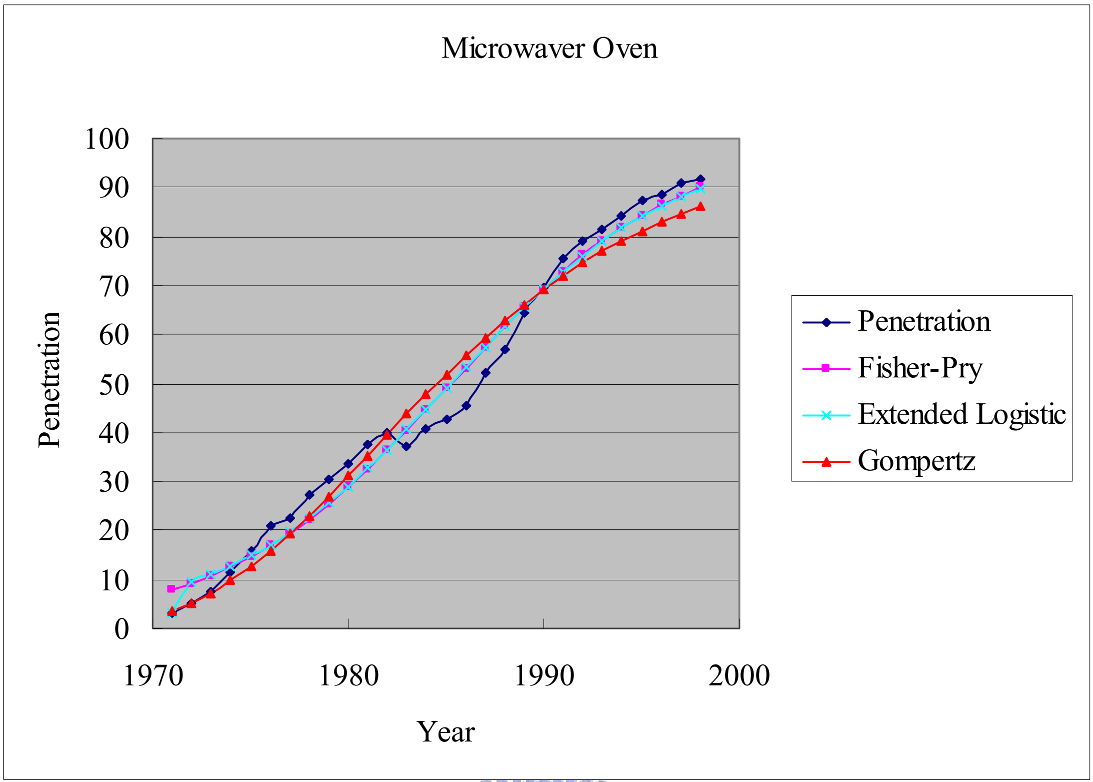

Figure 4. 1 The penetration and estimate value in Microwave Oven data.

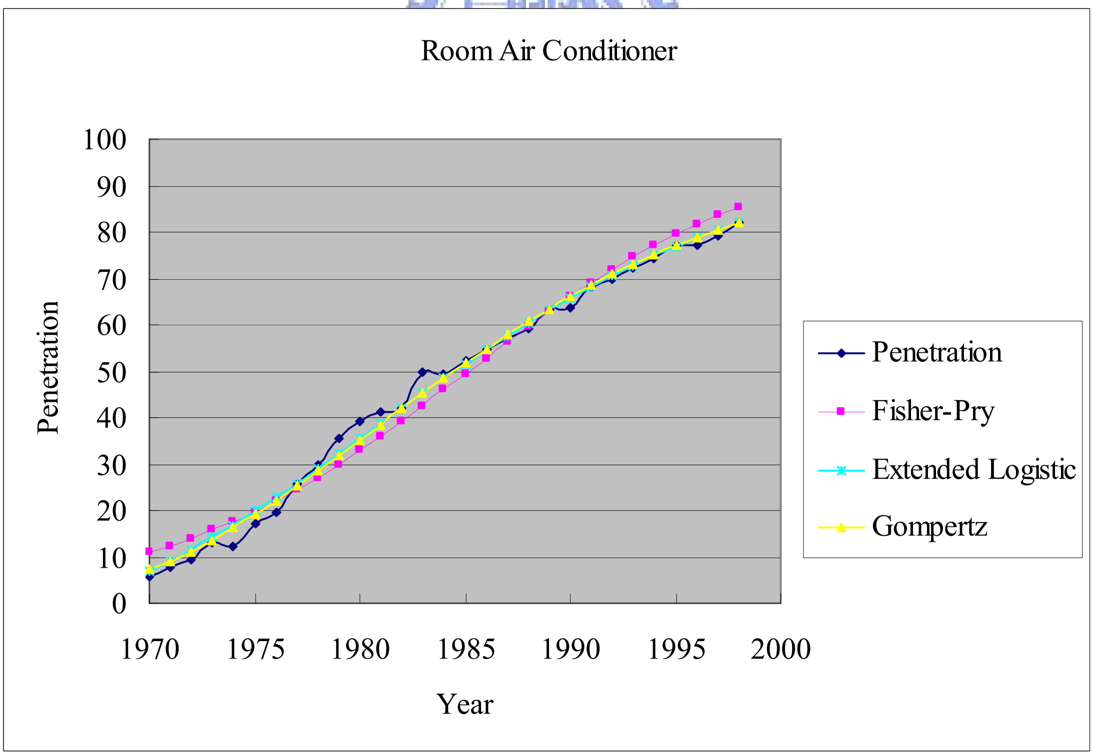

Figure 4. 2 The penetration and estimate value in Room air conditioner data.

Saturation level plays an important role in order to make forecasts precisely. However, Fisher-Pry supposes a saturation level at 100%. In the case of room air conditioner, extended logistic and Gompertz perform better than Fisher-Pry model according to the projected curve (Figure 4.2). Fisher-Pry obviously overestimates the trend in introduction and saturation level. This case shows that Fisher-Pry is only appropriate to some data sets whose saturation level reached 100%. The data set of color TV sets shows a rapid growth, and it only takes 6 years from 10% to 90%. These three forecasting models all looks better in fitting performance, because most of data points fall in between 90% to 100% (Figure 4.3). Consequently, the only way to check the fitting performance is to look the criterion.

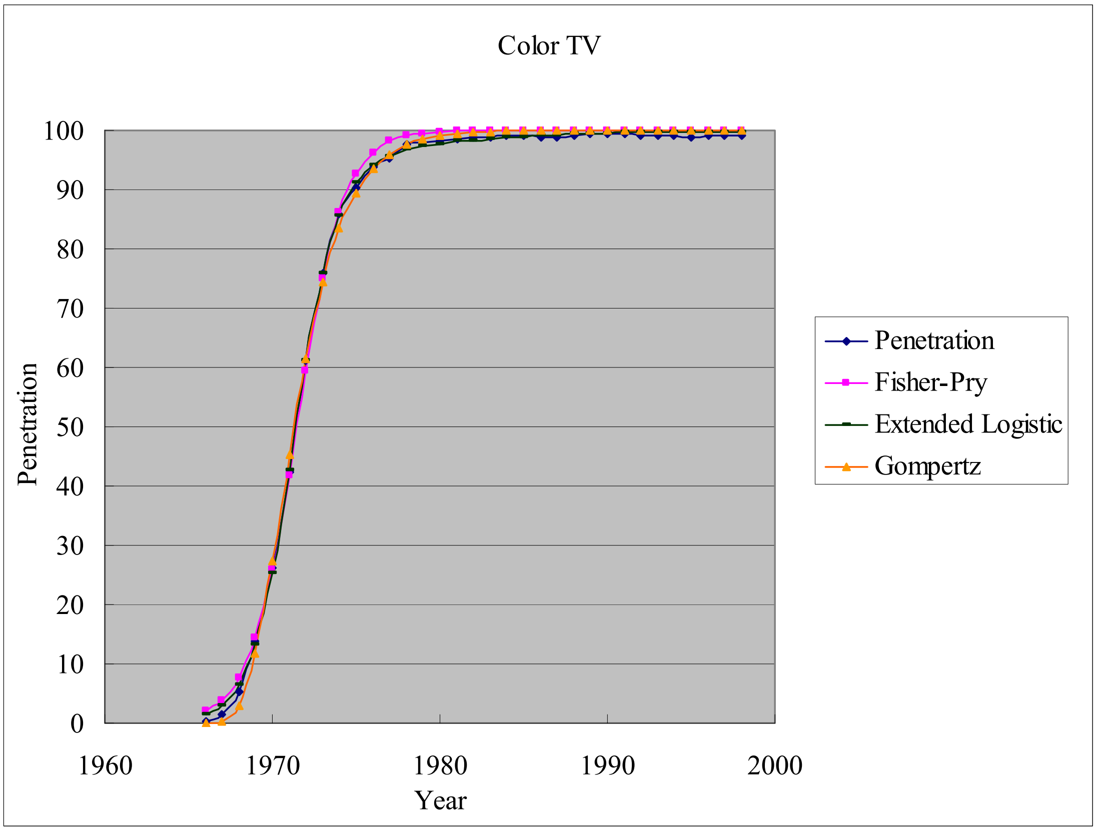

Figure 4. 3 The penetration and estimate value in Color TV data.

After checking these long data sets, the extended logistic model shows the trend that it fits better than other two models (Table 4.3). The extended logistic model performs better than Gompertz and Fisher-Pry model in all data sets.

Table 4. 3 The comparison between long data sets.

	Microwave oven	**Room air conditioner**	**color TV**
Gompertz and Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic
Fisher-Pry and Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic

Now the short data sets will be discussed in order to compare the performance between models. The data set of space heater grows slowly after reaching 60% of penetration. Fisher-Pry seems to lose its fit ability in this case, but Gompertz and extended logistic model still have good fit ability (Figure 4.4). The reason is that Gompertz and extended logistic model can change it capacity. If the right capacity is chosen, the fitting performance will be better.

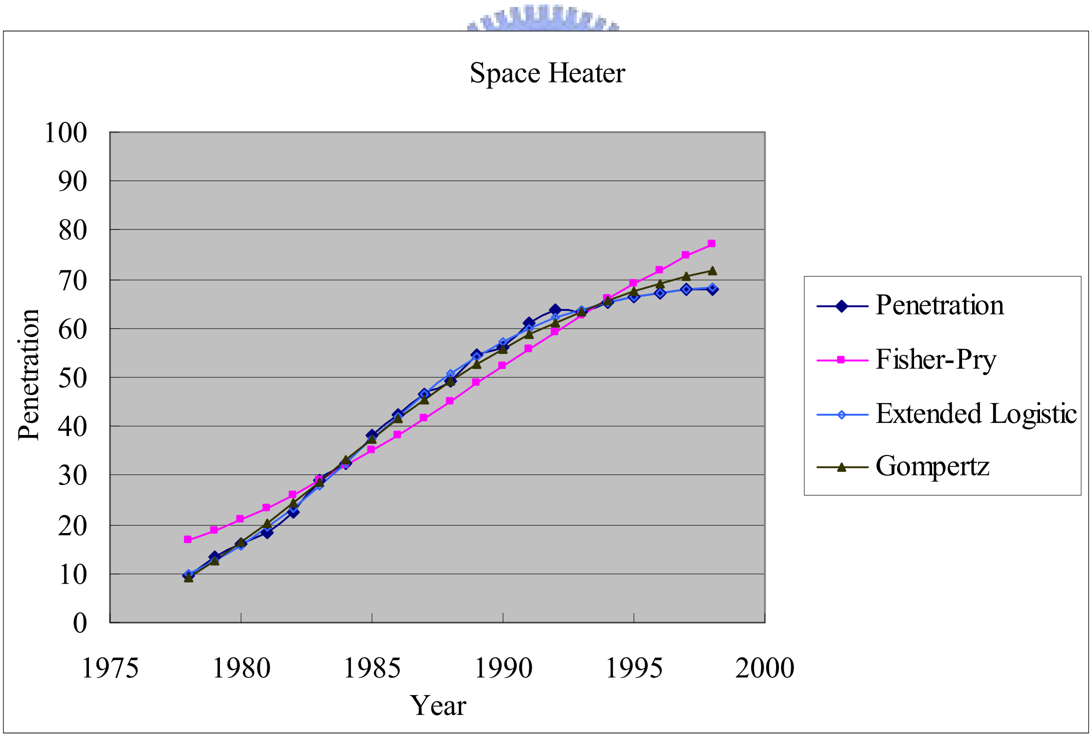

Figure 4. 4 The penetration and estimate value in Space heater data.

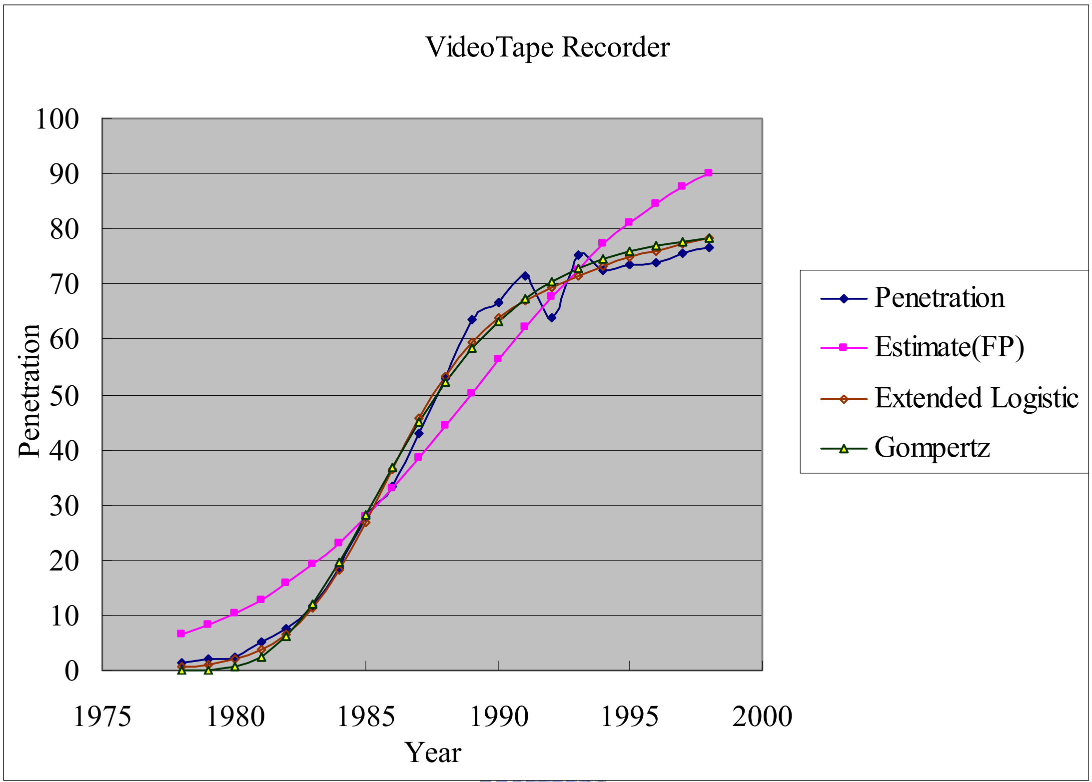

Figure 4. 5 The penetration and estimate value in Space heater data.

Same situation happened in two cases below, videotape recorder and radio cassette (Figure 4.5 to 4.6). The case of videotape recorder shows a standard sigmoid form, and grows slower after 1993. The extended logistic and Gompertz model both have good fitting performance in this case, and Fisher-Pry still meets the same problem. Although the case of radio cassette does not look like a sigmoid curve, the extended logistic model also can fit this case well, and so does Gompertz model. The extended logistic decide the capacity from the observed data points, but Gompertz model needs to decide the capacity manually. Although both extended logistic and Gompertz model have good fitting performance, the extended logistic can avoid deciding capacity in order to decrease the personal bias.

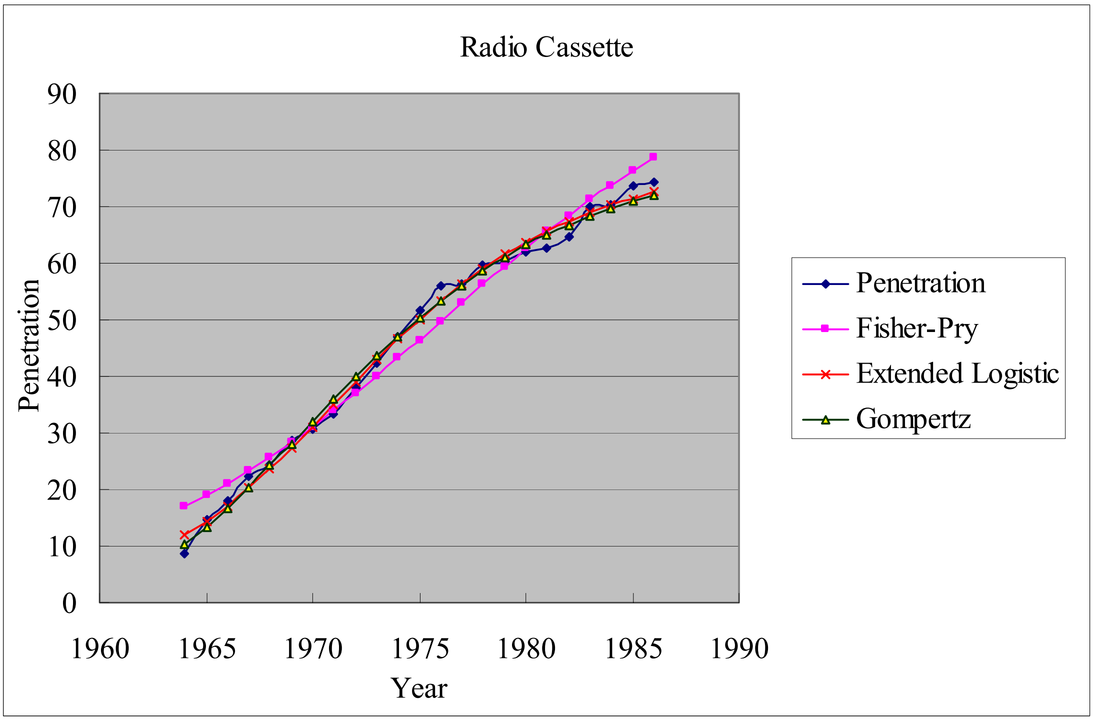

Figure 4. 6 The penetration and estimate value in Radio Cassette data.

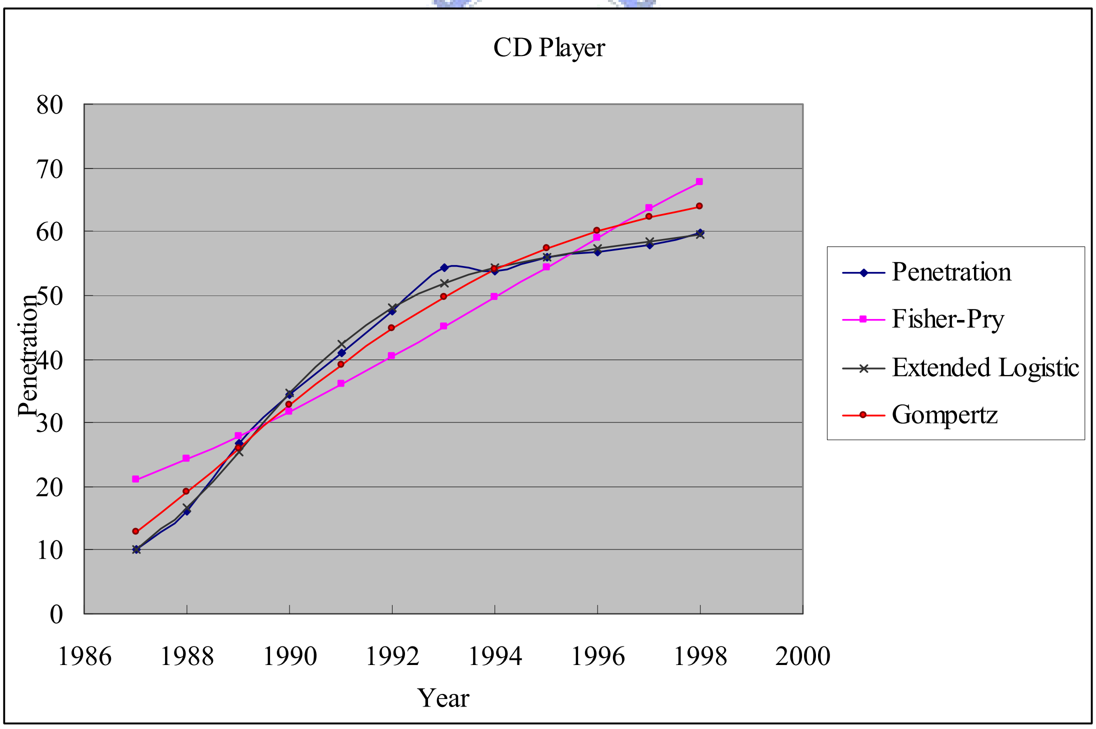

Figure 4. 7 The penetration and estimate value in CD player data.

The case of CD player has the least number of data points in these 7 data sets (Figure 4.7). The extended logistic model shows the better fitting performance than other two forecasting models. The reason that extend logistic performs better is that it can adjust the capacity according to data points, not adjust it artificially. Fisher-Pry has the worst performance than other models, and Gompertz model can adjust its fitting performance by changing the capacity. However, changing the capacity may affect projections when Gompertz model is used in this case.

This research tests the short data sets (Space Heater, Video Tape Recorder, Radio Cassette, and CD player), and this step is to inspect whether the extended logistic model also fit well in short data sets. Because most of the short data sets do not reach 100% saturation level, this research will set the possible upper limit. For example, Space heater will reach the saturation level about 80%; therefore, 0.8 is the possible upper limit. Therefore, the extended logistic model performs better than other two models in these four short data sets (Table 4.4).

Table 4. 4 The comparison between short data sets.

	Space Heater	**Video Tape Recorder**	**Radio Cassette**	**CD player**
Gompertz and Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic
Fisher-Pry and Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic

The classification of data sets in this thesis has been introduced at Chapter 3 (Table 3.1), and the shape of curve can simply divided into three forms, symmetric (Microwave Oven, and Room Air Conditioner), asymmetric (Color TV, and Video Tape Recorder), and flexible (Space Heater, Radio Cassette, and CD Player). The extended logistic model fits well in different kinds of points of inflection (Table 4.5), and judgmental criteria are according to the measurements (Table 4.6). The most factor that influence the Fisher-Pry model is upper limit. Because Fisher-Pry model assumed that old products or technologies will complete substitute by new technologies, it will make large biases in fitting some data sets whose saturation level are less than 100%.

Table 4. 5 The rank of models in symmetric, asymmetric, and other data sets

	Symmetric		**Asymmetric**		**Flexible**		
	Microwave Oven	Room Air Conditioner	Color TV	Video Tape Recorder	Space Heater	Radio Cassette	CD Player
Extended Logistic	1	1	1	1	1	1	1
Fisher-Pry	2	3	2	3	3	3	3
Gompertz	3	2	3	2	2	2	2

Table 4. 6 The criterion between models in durable goods.

	Model	**MAD**	**RMSE**	**MAPE**
Microwave Oven (28)	Fisher-Pry	3.491	3.8629	0.1821
	Gompertz	4.1594	4.9	0.1082
	Extended	3.3284	3.7454	0.1253
Room Air Conditioner(33)	Fisher-Pry	3.4078	3.8123	0.3195
	Gompertz	1.34	1.8	0.068
	Extended	1.267	1.5957	0.076
Color TV(33)	Fisher-Pry	1.2104	1.3689	0.2336
	Gompertz	1.0088	1.1472	0.0818
	Extended	0.5167	0.6059	0.1426
Space Heater(21)	Fisher-Pry	4.1426	4.7339	0.1412
	Gompertz	1.2302	1.5921	0.0309
	Extended	0.213	0.7862	0.0189
Video Tape Recorder(21)	Fisher-Pry	7.0779	7.9655	0.7375
	Gompertz	2.3444	2.8108	0.1893
	Extended	1.9434	2.4643	0.1043
Radio Cassette(21)	Fisher-Pry	2.794	3.4465	0.1014
	Gompertz	1.3965	1.6142	0.041
	Extended	1.3313	1.6252	0.047
CD Player (12)	Fisher-Pry	5.4321	6.2623	0.2104
	Gompertz	2.5618	2.8868	0.0799
	Extended	0.7339	0.9895	0.019

4.2 Predicted Performance between Models

The predicted performance will be introduced in this section, and it is mainly including of the test of Fisher-Pry and Gompertz model, the test of extended logistic model, and the comparison between models. These classifications can easily found the performance of each model and find the difference between models.

4.2.1 The Test of Fisher-Pry Model and Gompertz Model

This section will mainly talk about the predicted ability of Fisher-Pry and Gompertz model in these data sets. The first three data sets which have been mentioned before are divided from the long data sets; therefore, the Fisher-Pry model can be checked whether it has a good prediction or not. Fisher-Pry model will overestimate the data which does not achieve the 100% saturation level, but it can offer a good prediction in Microwave oven and color TV which reach or close to the 100% of saturation level (Table 4.7). However, when the short data sets were predicted by Fisher-Pry model, such as space heater and CD player, the different situation will make the prediction of Fisher-Pry model bias. In these four predictions which belong to the short data set, the Fisher-Pry model overestimated these four data sets. The first reason is that Fisher-Pry model assumes that the penetration will finally reach 100%, but there are some data sets are not confirmed in this assumption, such as Video Tape recorder. The other reason is that some data sets may follow a bi-logistic form, and this phenomenon will make the prediction of Fisher-Pry overestimate. However, Fisher-Pry still offers a good prediction in long data sets which close to 100% of penetration.

Gompertz model looks like underestimating the data when the long data sets were tested. It not only underestimate the long data sets but also short ones. Although

Gompertz model allow the researchers to set the possible saturation level, it still underestimate the data sets in this research. However, the most important rule in Gompertz model is to find a proper capacity of data sets. Right capacity can help the Gompertz model improve the accuracy of fits and forecasts. The capacity is decided by subjective judgments in this research. Although this subjective capacity also helps Gompertz have a good result, there is no evidence to show that subjective judgments can be used to decide the capacity. Therefore, the best strategy of deciding capacity is to ask experts in related area.

Table 4. 7 The penetration and prediction in last five points (Fisher-Pry and Gompertz).

		1999	**2000**	**2001**	**2002**	**2003**
Microwave Oven	Penetration	93.3	94	95.3	95.7	95.8
	Fisher-Pry	91.378	92.640	93.730	94.668	95.473
	Gompertz	87.668	88.931	90.071	91.100	92.027
Room Air Conditioner	Penetration	84.4	86.2	86.2	87.2	88.8
	Fisher-Pry	87.157	88.624	89.943	91.125	92.179
	Gompertz	83.250	84.603	85.855	87.014	88.084
Color TV	Penetration	98.9	99	99.2	99.3	99.4
	Fisher-Pry	100	100	100	100	100
	Gompertz	100	100	100	100	100
Space Heater	Penetration	68.3	69.4	69.5	71.8	72.5
	Fisher-Pry	79.805	81.997	83.999	85.817	87.459
	Gompertz	72.823	73.776	74.608	75.332	75.962
Video Tape Recorder	Penetration	77.8	78.4	79.3	79.6	81.4
	Fisher-Pry	91.937	93.558	94.872	95.929	96.776
	Gompertz	78.72	79.048	79.292	79.474	79.609
Radio Cassette	Penetration	74.9	73	75.4	73.5	75.5
	Fisher-Pry	80.657	82.619	84.421	86.067	87.565
	Gompertz	72.954	73.808	74.563	75.228	75.815
CD Player	Penetration	60.1	61.8	62.1	60.5	60.3
	Fisher-Pry	72.005	75.634	78.930	81.887	84.510
	Gompertz	65.38	66.440	67.260	67.895	68.384

4.2.2 The Test of Extended Logistic Model

This section will introduce the predicted performance of extended logistic model through different kinds of data sets. In the prior section, the fit performance of extended logistic model has been mentioned. This section will also follow the format of last section which described the predicted performance of Fisher-Pry model. Therefore, long data sets will be discussed first and then the short data sets will also be mentioned. Only one of the long data sets is underestimated by extended logistic model (Room air conditioner), and the other data sets are estimated well (Table 4.8). The extended logistic model offers a good fit in the data set whose growth curve just likes the sigmoid shape whether it is symmetric or not, but it may underestimate the trend of growth curve when the data does not look like sigmoid. In the short data sets, the extended logistic model also offers a good prediction in these data, but it still overestimates two data sets (Radio cassette and CD player). Nevertheless, these over-estimations are not serious for less than five percentages. The extended logistic model predicts well in the data sets of space heater and video tape recorder, and the growth shape of these data sets looks like the sigmoid curve. These short data does not reach the maximum saturation level (100%) and are just saturated less than 90%, but they are both predicted well. Therefore, the extended logistic model can offer a better forecast in some data sets just looked like sigmoid curve It also may overestimate or underestimate in some data sets, but the biases in these data are smaller than the biases that Fisher-Pry made.

Table 4. 8 The penetration and prediction in last five points (Extended Logistic).

		1999	2000	2001	2002	2003
Microwave Oven	Penetration	93.3	94	95.3	95.7	95.8
	Prediction	91.888	93.119	94.176	95.079	95.848
Room Air Conditioner	Penetration	84.4	86.2	86.2	87.2	88.8
	Prediction	80.869	81.965	82.987	83.940	84.831
Color TV	Penetration	98.9	99	99.2	99.3	99.4
	Prediction	99.866	99.884	99.900	99.913	99.925
Space Heater	Penetration	68.3	69.4	69.5	71.8	72.5
	Prediction	69.891	70.578	71.210	71.799	72.352
Video Tape Recorder	Penetration	77.8	78.4	79.3	79.6	81.4
	Prediction	78.550	79.405	80.221	81.001	81.749
Radio Cassette	Penetration	74.9	73	75.4	73.5	75.5
	Prediction	74.116	75.185	76.181	77.113	77.989
CD Player	Penetration	60.1	61.8	62.1	60.5	60.3
	Prediction	60.283	61.103	61.890	62.653	63.397

4.2.3 Comparison between Models

The predictions of each data set by these three models are clearly described at last two sections. The comparison between models will be discussed in this section through the mathematical criteria. In the long data sets, the value of criteria in room air conditioner and color TV in Fisher-Pry and extended logistic models are closing, but the extended logistic model outperform Fisher-Pry model in the data set of Microwave oven. On the other hand, Gompertz has better forecast in the data of room air conditioner than Fisher-Pry and Extended Logistic. The fitting performance in these long data sets between models reveals that the extended logistic model outperformed Fisher-Pry model in these three long data sets. However, Extended Logistic model also outperform than Gompertz in Microwave oven and color TV. Although there are no difference between these models in the case of color TV by observing the fitting curve, mathematical criteria can offer a judgmental way to inspect which model is better in whether fitting performance or predicting performance. (Table 4.9)

Table 4. 9 The comparison between long data sets (Forecasts)

	Microwave oven	Room air conditioner	color TV
Gompertz and Extended Logistic	Extended Logistic	Gompertz	Extended Logistic
Fisher-Pry and Extended Logistic	Extended Logistic	Fisher-Pry	Extended Logistic

The extended logistic model outperform Fisher-Pry model in all short data sets, and the criteria values of Fisher-Pry model are about ten times bigger than extended logistic model (Table 4.11). All of these short data sets do not like the long data set which has higher saturation level (near 100%); therefore, Fisher-Pry may not have good predicted performance in these data according to its assumptions. However, the extended logistic model offer a more flexible saturation level and this term make this model can offer a better predicted performance than Fisher-Pry model.

Although Gompertz model does not have better forecasts than Extended Logistic, it still has one better forecast than Extended Logistic model (Table 4.10). Because Gompertz model can set the saturation level artificially, the accuracy of model is based on the appropriate upper limit. This way is not the only one step to improve the accuracy, and it just offer a way to increase the model forecasts.

Table 4. 10 The comparison between short data sets.

	Space Heater	Video Tape Recorder	Radio Cassette	CD player
Gompertz and Extended Logistic	Extended Logistic	Extended Logistic	Gompertz	Extended Logistic
Fisher-Pry and Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic	Extended Logistic

Table 4. 11 The criteria between models in prediction part.

	Model	MAD	RMSE	MAPE
Microwave Oven	Fisher-Pry	15.31	15.33	0.1615
	Gompertz	4.86	4.903	0.0513
	Extended	0.89	0.95	0.0093
Room Air Conditioner	Fisher-Pry	3.25	3.3	0.0374
	Gompertz	0.7987	0.9528	0.0093
	Extended	3.64	3.66	0.042
Color TV	Fisher-Pry	0.84	0.86	0.0085
	Gompertz	0.84	0.86	0.0085
	Extended	0.74	0.75	0.0074
Space Heater	Fisher-Pry	13.52	13.58	0.192
	Gompertz	4.2003	4.2465	0.0599
	Extended	0.93	3.71	0.0133
Video Tape Recorder	Fisher-Pry	15.31	15.33	0.1931
	Gompertz	4.2003	4.2465	0.0087
	Extended	0.89	0.95	0.0122
Radio Cassette	Fisher-Pry	9.81	10.1	0.1318
	Gompertz	1.127	1.2829	0.01516
	Extended	1.97	2.25	0.0265
CD Player	Fisher-Pry	17.63	18.22	0.2896
	Gompertz	6.11	6.26	0.1004
	Extended	1.27	1.72	0.0209

Moreover, this research also tries to test these data sets before point of inflection in order to examine the predicted performance between models. First, the point of inflection will be set manually. Second, the parameters in these models will be found and they are used to find the predicted value in each data set. Finally, the criteria will be calculated based on residuals and be compared between models. In the long data sets, the predicted value of extended logistic model is between Fisher-Pry model and Gompertz model, and it has better forecast than the other two models in the case of room air conditioner. That shows the extended logistic model can reduce the overestimation in Fisher-Pry model. However, Gompertz model performs well than other models in other conditions, and the extended logistic model performs second in these three models, especially in short data sets. The result in predicted performance before point of inflection shows the model with manual capacity will perform better than the model with time-varied capacity. The more data will offer more information when technological forecasting model is used in some researches. Therefore, the outcome of predicted value can't be fixed by first test. It should be changed when more information is gathered.

Table 4. 12 The criteria between models in prediction part before point of inflection.

	Model	MAD	RMSE	MAPE
Microwave Oven	Fisher-Pry	6.51	6.722	0.077
	Gompertz	12.182	12.378	0.142
	Extended *	11.231	11.414	0.132
Room Air Conditioner	Fisher-Pry	11.016	11.182	0.157
	Gompertz	3.785	3.943	0.054
	Extended*	2.228	2.529	0.031
Color TV	Fisher-Pry	1.24	1.43	0.013
	Gompertz*	0.806	0.83	0.008
	Extended	1.074	1.151	0.011
Space Heater	Fisher-Pry	16.48	18.09	0.244
	Gompertz*	2.681	3.123	0.04
	Extended	5.685	6.907	0.083
Video Tape Recorder	Fisher-Pry	17.133	18.65	0.23
	Gompertz*	6.566	6.93	0.09
	Extended	8.224	9.058	0.111
Radio Cassette	Fisher-Pry	11.668	13.333	0.168
	Gompertz*	2.459	3.099	0.037
	Extended	8.012	9.543	0.115
CD Player	Fisher-Pry	23.268	24.234	0.386
	Gompertz	9.434	9.63	0.157
	Extended*	6.323	6.98	0.104

4.3 A case of Technological Substitution in Digital Camera Industry

Digital camera industry grows rapidly in Taiwan, and DC itself also advance in it functions such as pixels. The data from MIC has been introduced in chapter 3, and these three models will be tested in this case. This research divided the substitution of technology in pixels of digital camera into two parts, more than 2 million pixels and less than 2 million pixels. In the case of growth technology in digital camera (more than 2 million pixels), Fisher-Pry and extended logistic model shows better predicted performance than Gompertz model.

Table 4. 13 The criteria between models in pixels (more than two million pixels)

	MAD	RMSE	MAPE
Fisher-Pry	20.27	20.367	0.2502
Gompertz	27.725	27.933	0.339
Extended Logistic	20.27	20.367	0.2502

On the other case, the decay of technology in digital camera (less than 2 million pixels), Gompertz model has better predicted performance than the other two models. Some interesting things can be observed in these two cases. First, the parameter, A, in extended logistic model is always no significant in these two cases. Therefore, the extended logistic model changes to Fisher-Pry model. Second, one model performs well in growth of technology can not promise that it also can has better performance in the opposite case.

Table 4. 14 The criteria between models in pixels (less than two million pixels)

	MAD	RMSE	MAPE
Fisher-Pry	20.19	20.285	1.4282
Gompertz	14.34	14.756	0.914
Extended Logistic	20.19	20.285	1.4282

4.4 Results of the Tested Hypotheses

Table 4.12 shows the result of tested hypotheses which have been mentioned in chapter three. In the parts of fitting performance, the extended logistic model outperform Fisher-Pry model and Gompertz model in both long and short data sets and so does in symmetric and asymmetric data. That means the extended logistic model offers a better fit performance in these data sets than Fisher Pry and Gompertz, and the hypothesis 1 and 2 can be accepted. In the parts of predicted performance, the extended logistic model commonly offer better forecasts than Fisher-Pry and Gompertz, especially in some short data sets or some data which the capacity did not reach 100%. However, the extended model proposed by this report does not predict the data well in room air conditioner. It underestimates the data set when this data set is fitted by the extended logistic model. Although it still has lower mathematical criteria (e.g. MAPE) than Fisher-Pry and Gompertz, it still underestimates the forecasts to the data of room air conditioner. Therefore, this research rejected the hypothesis because the extended logistic model may underestimate the data set when it is used to forecast the long data sets. Nevertheless, the extended logistic still offers a good fit and prediction in other data sets.

Moreover, when all of the data sets have been tested in these two models, this report will checked whether a good fit performance will lead a good predicted performance. Table 6 shows the rank of fit and predicted performance in Fisher-Pry, Gompertz and extended logistic model. It shows a regular pattern between fit performance and predicted performance in these 7 data sets. Consequently, this research accepts the hypothesis 5, which assumes a better fit performance will help the forecast better.

Table 4. 15 The rank of Fisher-Pry and extended logistic model in performance

	Fisher-Pry		**Gompertz**		**Extended Logistic**	
	Rank of fit	**Rank of predict**	**Rank of fit**	**Rank of predict**	**Rank of fit**	**Rank of predict**
Microwave Oven	3	2	3	3	1	1
Room Air Conditioner	3	2	2	1	1	3
Color TV	3	3	2	2	1	1
Space Heater	3	3	2	2	1	1
Video Tape Recorder	3	3	2	2	1	1
Radio Cassette	3	3	2	1	1	2
CD Player	3	3	2	2	1	1

Table 4. 16 The test of hypotheses

Hypothesis	Description of hypotheses	Result
Hypothesis 1	The extended logistic model will have a better fitting performance than Fisher Pry model and Gompertz model in both long and short data sets.	Accepted
Hypothesis 2	The extended logistic model has better fit performance in symmetric, asymmetric, and flexible data set than Fisher-Pry model.	Accepted
Hypothesis 3	The extended logistic model has better predicted performance in both long data and short data set than Fisher-Pry model.	Rejected
Hypothesis 4	The extended logistic model has better predicted performance in symmetric, asymmetric, and flexible data set than Fisher-Pry model.	Rejected
Hypothesis 5	A good fit performance will lead a good predicted performance.	Accepted

CHAPTER 5 CONCLUSIONS

This chapter will make a final conclusion of this research and will talk about several parts, including conclusions, contributions, research limitations, and further research. The test of extended logistic model, Fisher-Pry model, and Gompertz model will be concluded with the result of these tests in chapter four, and the contributions of this research will be explicitly introduced. Moreover, the research limitations which include of data collection, model choosing, and so on will also be described. Finally, the further research will introduce several directions to do the further research.

5.1 Conclusions

This research has examined the fitting and predicted performance in several data sets between extended logistic model and Fisher-Pry model. As this report assumed, the extended logistic model has better fitting performance in all data sets than Fisher-Pry model and Gompertz model. Although prior research investigated that Fisher-Pry has better performance in some data sets whose growth curves were looked like sigmoid (Rai and Kumar, 2004), the extended logistic model still can offer a better fit than Fisher-Pry. The result of this research also supports this point. Hypothesis 1 to hypothesis 2 all accept the hypothesis, and that means the extended logistic model indeed has better performance in fitting the data. Moreover, whether a data belongs to what kind of data sets, the extended logistic model still can fit well in these data sets.

In the part of predicted performance, hypothesis 3 to hypothesis 4, this research hold the last five points of each data sets in order to examine the predicted performance of extended logistic and Fisher-Pry model. According to the mathematical criteria, MAPE, RMSE, and MAD, the outcome shows that the extended logistic model can offer better forecasts than Fisher-Pry except the data of room air conditioner. In the original data set of room air conditioner, its growth curve is similar to sigmoid curve. However, the extended logistic did not outperform Fisher-Pry model in forecasting, even though it fit the data of room air conditioner better than Fisher-Pry model. A point has to be noticed in this data set. Fisher-Pry just outperform the extended logistic a little bit, and it overestimated the penetration of room air conditioner. The extended logistic underestimates the penetration of room air conditioner; therefore, the extended logistic model just looks like a pessimist. This phenomenon also happened when the extended logistic model was used to predict in other data sets. On the other side, Fisher-Pry overestimates the forecast in these data sets except the data set of Microwaver oven. Even though the data set of Microwaver oven did not be overestimated, this research still considers that Fisher-Pry model looks like an optimist in these data sets.

The same condition has happened in comparison between extended logistic model and Gompertz model. The extended logistic model has better forecasts than in two cases of long data sets, but Gompertz model still has one better forecast (Room air conditioner). In short data sets, Gompertz model only outperform the extended logistic model in the case of radio cassette. Deciding a right capacity before running Gompertz model is hard to forecaster but it is very important. This research set the capacity of Gompertz model arbitrarily; therefore, this may be one of the reasons why Gompertz model forecast better than extended logistic model.

Moreover, this study also discusses the predicted performance before point of inflection. Gompertz shows better predicted performance in five data sets, and extended logistic model shows better performance in two data sets. This result reveals that extended logistic model may not performs better before point of inflection, so does Fisher-Pry model. However, the predicted performance of extended logistic model will increase when the more data has been observed. The result of predicted performance after point of inflection also supports this observation. Although extended logistic model may perform better than the other two models before point of inflection, it has robust performance than other models.

This research also considered whether a better fit can brings a better forecast, and the hypothesis 5 is used to test this condition. In these data sets no matter what kind of data set it is, the ranks of fit and predicted performance show a regular pattern. This result shows that a good fit may offer a good forecast when a model is used to test the data. Therefore, a good fit performance may useful in long data sets, and this fit curve offer a general trend of a product or a service in specific time period. A good fit model may also offer an explicit forecast in a data set.

5.2 Contributions

This research proposed a new technological forecasting model based on the time-varied capacity. The simply idea of this model is that the capacity may not always be constant. Therefore, this research proposed that the capacity will fluctuate in early stage and be gradually stable. In the test of this research, the extended logistic indeed outperforms Fisher-Pry and Gompertz model in most data sets no matter what kinds of data. Not only in projecting the trend but also the fit performance, the extended logistic model always performed better than Fisher-Pry and Gompertz model. The most contribution of this research is that the extended logistic model can adjust the capacity of data sets automatically. This process can make the model fit data better than some models which have fixed capacity (e.g. Fisher-Pry) or some models which need setting a capacity (e.g. Gompertz model). Every product or technology has its capacity, and this capacity is not always 100%. The capacity may be 50%, 60%, or 40%. On a one hand, a model which has fixed capacity will have a big problem when a data set does not match the capacity which models assumed. On the other hand, forecasters will worried about setting a capacity and rely on some experts to set it in some models like Gompertz model. Wrong setting will damage the predicting performance. Consequently, this model offers an objective way to find a proper capacity.

5.3 Research Limitation

This research uses the penetration data to test the extended logistic, Fisher-Pry, and Gompertz model. The penetration data is defined as the percentage of each household adopted a product, and it is always a survey data. These data sets may have biases when collecting the original data. These biases may not affect the model estimation, because these biases are inherent. Therefore, this research may have some inherent biases which made by the data itself because the penetration data were used. However, when a real market share or penetration is not easily gathered, the penetration data is a good data to test the model. Another pitfall is that penetration data does not consider the repeat purchase. Consequently, the penetration data may underestimate the real purchase.

Another consideration is about the estimation. This research uses the Gauss-Newton methods to estimate the parameters, but it needs to set the initial value. A proper initial value will lead a good performance, and this is an important part in this research. Moreover, the Gauss-Newton may find a minimum sum of square value in specific range, and this minimum value is not the absolute minimum value in all regions.

5.4 Further Research

This report proposed an extended logistic model base on the time-varied capacity. This research uses the extended logistic model to fit different kind of data in order to examine the performance of extended logistic model. However, this research just uses seven data sets to examine the fit and predicted performance. The prior research argued that a new or adapted model always fit well in particular data sets. Therefore, testing this model in more data sets will make sure the accuracy when the model is tested in different kinds of data. This research just compares with Fisher-Pry mode, but there are many other models in technological growth field, consequently, the further research can compare the extended logistic model with other models.

There are many data can be used in the extended logistic model, such as the cumulative adopters who buy a new products. Therefore, if the percentage of cumulative adopters who bought products or services can be collected, the extended logistic model can be used to fit the data. On the other aspect, there are other factors will affect the market share or penetration. Frank (2004) proposed a modified logistic model used in wireless telecommunication. He added some macroeconomic factors (e.g. GDP) and some related data such as penetration of telephone per household in his proposed model. Therefore, the extended logistic model can also be test by adding some other factors.

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Appendices

Appendix 1: The Mathematical Inference of Fisher-Pry Model

According to the internal-influence model, the mathematical expression can be shown as:

$$\frac{dN(t)}{dt} = bN(t)\left[\overline{N} - N(t)\right]$$

In the internal influence model, the rate of diffusion is treated as a function of interpersonal communication or social system between prior adopters and potential adopters in the social system. Interpersonal communication or social interaction can be shown as $N(t)\left[\overline{N} - N(t)\right]$; therefore, the equation can just be called the pure imitation diffusion model. The $N(t)$ can be solved by the equation through integration as:

$$\int_{t_0}^{t} \frac{1}{N(t)} \times \frac{\overline{N}}{\overline{N} - N(t)} dN(t) = \int_{t_0}^{t} b\overline{N} dt$$

Then the transformation of left side equation can let the integration more easily as:

$$\int_{t_0}^{t} \frac{1}{N(t)} + \frac{1}{\overline{N} - N(t)} dN(t) = \int_{t_0}^{t} b\overline{N} dt$$

In order to remove the nature log, both equations between the equal sign have to multiple the exponents.

$$\ln \frac{\overline{N} - N(t)}{N(t)} = -b\overline{N}(t - t_0) + \ln \frac{\overline{N} - N_0}{N_0}$$

After arranging the equation, the final solution can be obtained by the equation below:

$$\frac{\overline{N}-N(t)}{N(t)}=\frac{\overline{N}-N_0}{N_0}\times\exp\left[-b\overline{N}(t-t_0)\right]$$

$$N(t)=\frac{\overline{N}}{1+\frac{\overline{N}-N_0}{N_0}\exp[-bN(t-t_0)]}$$

The Fisher-Pry model can be proven through the internal-influence diffusion model. In order to find the appropriate way to shown the Fisher Pry model, $F={N(t)}\big/{\overline{N}}$ can be used to change the internal-influence model into Fisher Pry model.

$$F=\frac{1}{1+\frac{\overline{N}-N_0}{N_0}\exp\left[-b\overline{N}(t-t_0)\right]}$$

where $N(t=t_0)=N_0$

In other aspect, the point of inflection can be solved by the original equation of internal influence diffusion model which changed the equation form $N(t)$ into $F={N(t)}\big/{\overline{N}}$.

$$\frac{dF}{dt}=bF(1-F)$$

Doing the differential process to the equation, the F value at the point of inflection can be obtained as:

$\frac{d}{dF}=[bF(1-F)]=0$ then corresponding value, F, is 0.5.

Appendix 2: The Mathematical Inference of Gompertz Model

The internal-influence diffusion model can be used to express the Gompertz model as:

$$\frac{dN(t)}{dt}=bN(t)\left[\ln\overline{N}-\ln N(t)\right]$$

First, the equation can be expressed as below one:

$$\int_{t_0}^{t} \frac{1}{N(t) \times \ln \frac{N(t)}{\overline{N}}} dN(t) = -bdt$$

then integrate the equation 2 can obtain $N(t)$ and let $N(t_0) = N_0$;

$$\frac{1}{\overline{N}} \ln\left(\ln\left(\frac{N(t) - N_0}{\overline{N}} \right) \right) = -b(t - t_0)$$

The $N(t)$ can be expressed as simpler form as:

$$N(t) = \overline{N}\left[-\ln\left(\frac{\overline{N}}{N_0} \right) \exp[-b(t - t_0)] \right]$$

Therefore, when the parameter in the cumulative adopter distribution of the Gompertz form is changed form $N(t)$ to F (let $F = N(t) / \overline{N}$), the generalized Gompertz form can be obtained. In addition, the constant $\ln \overline{N} / N_0$ can be simplified as $a = \ln \overline{N} / N_0$.

$$F = \exp\left[-\left(\ln \frac{\overline{N}}{N_0} \right) \exp[-b(t - t_0)] \right]$$

$$F = \exp[-a \exp[-b(t - t_0)]]$$

Appendix 3: The Penetration of Durable Goods in Japan

	Microwave ovens	Space heater	Room Air Conditioner	Color TV sets	Video Tape Recorder	Radio Cassette	CD Player
1964						8.7	
1965						14.6	
1966			2.0	0.3		17.9	
1967			2.8	1.6		22.5	
1968			3.9	5.4		24.5	
1969			4.7	13.9		28.6	
1970			5.9	26.3		30.8	
1971	3.0		7.7	42.3		33.4	
1972	5.0		9.3	61.1		38.1	
1973	7.5		12.9	75.8		42.4	
1974	11.3		12.4	85.9		47.0	
1975	15.8		17.2	90.3		51.6	
1976	20.8		19.5	93.7		55.9	
1977	22.6		25.7	95.4		56.2	
1978	27.3	9.6	29.9	97.7	1.3	59.6	
1979	30.6	13.4	35.5	97.8	2.0	60.5	
1980	33.6	15.9	39.2	98.2	2.4	61.9	
1981	37.4	18.4	41.2	98.5	5.1	62.7	
1982	39.9	22.7	42.2	98.9	7.5	64.7	
1983	37.2	29.0	49.6	98.8	11.8	70.1	
1984	40.8	32.3	49.3	99.2	18.7	70.2	
1985	42.8	38.3	52.3	99.1	27.8	73.6	
1986	45.3	42.3	54.6	98.9	33.5	74.2	
1987	52.2	46.4	57.0	98.7	43.0	74.9	10.0
1988	57.0	49.1	59.3	99.0	53.0	73.0	16.1
1989	64.3	54.6	63.3	99.3	63.7	75.4	26.8
1990	69.7	56.1	63.7	99.4	66.8	73.5	34.3
1991	75.6	61.2	68.1	99.3	71.5	75.5	41.0
1992	79.2	63.6	69.8	99.0	63.8		47.5
1993	81.3	63.4	72.3	99.1	75.1		54.3
1994	84.3	65.2	74.2	99.0	72.5		53.8

1995	87.2	66.3	77.2	98.9	73.7		55.9
1996	88.4	67.0	77.2	99.1	73.8		56.8
1997	90.8	67.9	79.3	99.2	75.7		57.9
1998	91.7	67.8	81.9	99.2	76.8		59.9
1999	93.3	68.3	84.4	98.9	77.8		60.1
2000	94.0	69.4	86.2	99.0	78.4		61.8
2001	95.3	69.5	86.2	99.2	79.3		62.1
2002	95.7	71.8	87.2	99.3	79.6		60.5
2003	95.8	72.5	88.8	99.4	81.4		60.3

Appendix 4: The Procedures of estimating parameters

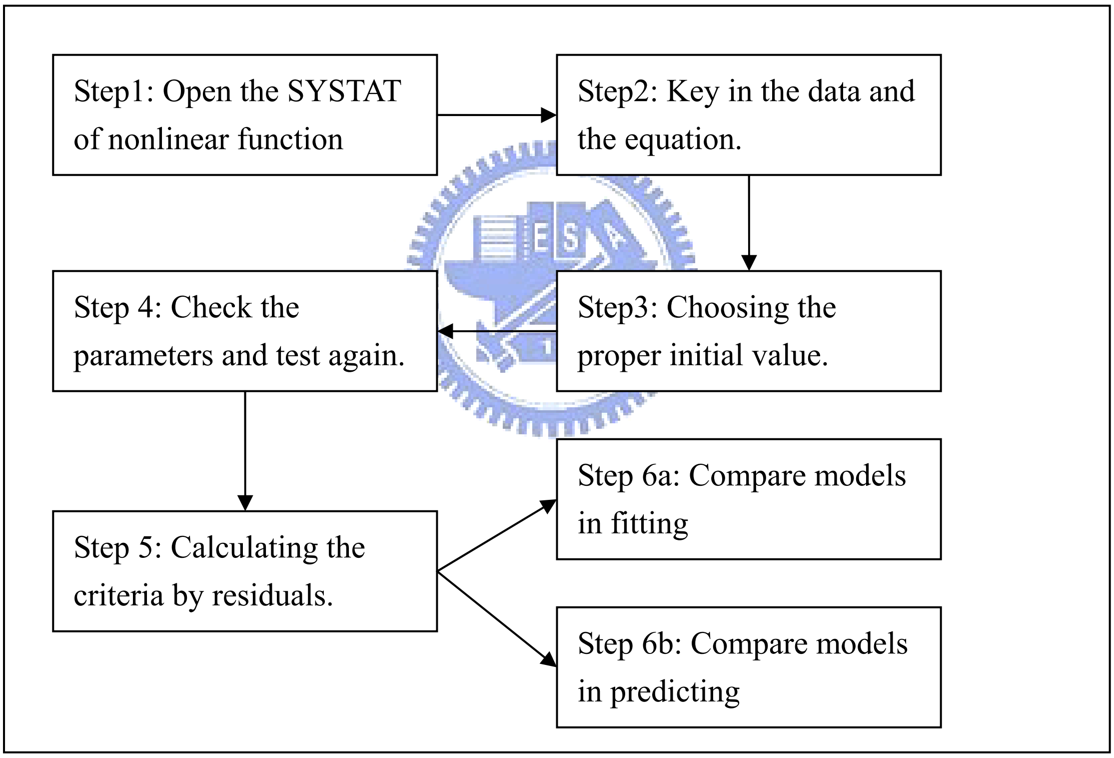


Figure 1.The process of testing the Fisher-Pry model.

The step one it to open the SYSTAT software of nonlinear regression and key in the data sets. When all of the data has been keyed, depend variable should be selected and key the equation in the column of model expression. After that the initial value of parameters should be set. The proper initial value can be easily found by using first two data points. The equations below can help us to find the initial parameters.

$$f_0 = \frac{1}{\left(1 + C \times \exp\left(-B \times t_0\right)\right)}$$

$$-B \times t = \ln\left(\frac{1 - f_1}{C}\right)$$

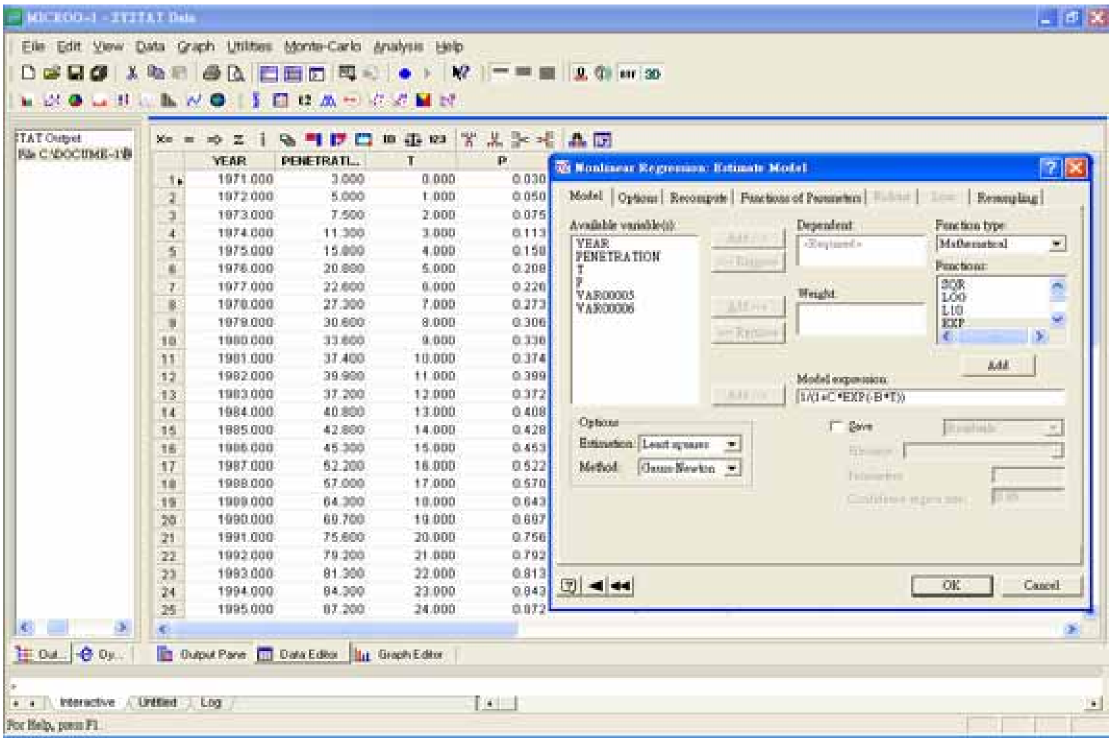


Figure 2.The window of nonlinear regression form in testing Fisher-Pry model.

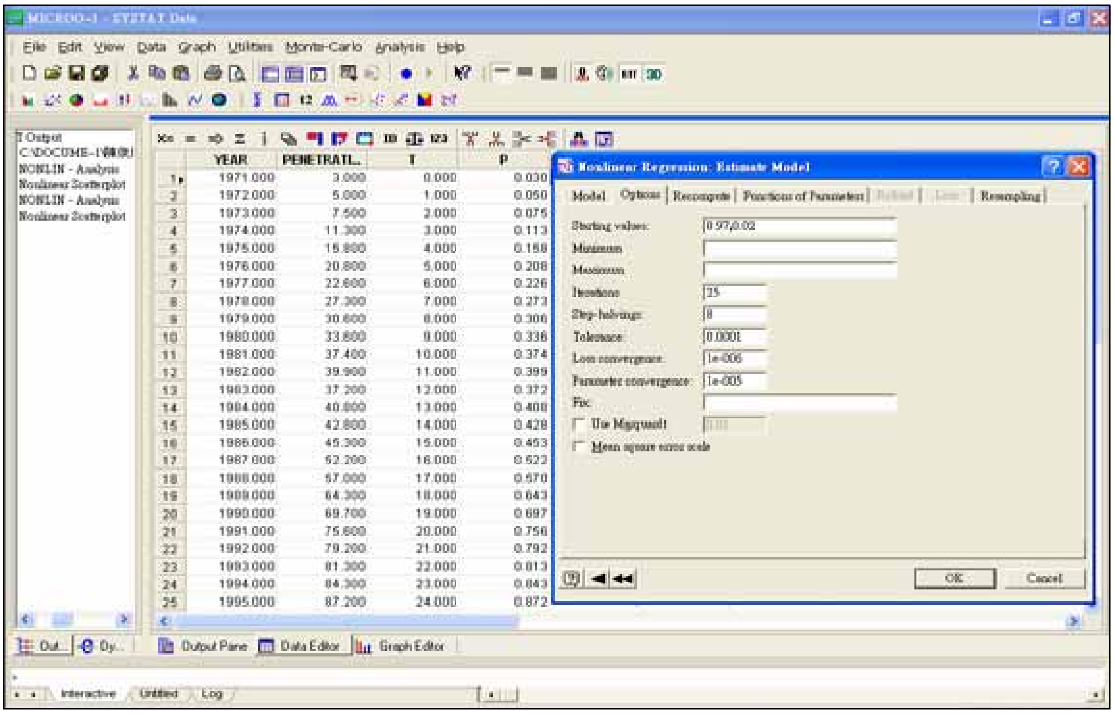


Figure 3. Set the initial value of Fisher-Pry model in nonlinear process.

The outcome of Fisher-Pry model is shown as below. First, the Raw R^2 (Regression sum of square / total sum of square) is the proportion of the variation in y that is explained by the sum of squares due to regression. Some researchers object to this measure because the means are not removed. The Mean corrected R^2 tries to adjust for this. Many researchers prefer the last measure of R^2 (R (observed vs. predicted) squared). It is the correlation squared between the observed values and the predicted values. Second, the equations below show how the sum of squares be calculated.

Regression: $\sum wy^2 - \sum (y-f)^2$

Residual: $\sum w(y-f)^2$

Total: $\sum wy^2$

Mean Corrected: $\sum (y-\bar{y})^2$

The last process in estimating the models is to check the parameters. The proper initial value will help the Gauss-Newton process find the convergence in fewer iterations and the minimum value in all region. If the initial value can be set appropriately, the Gauss-Newton method will diverge or converge at one partially minimum value. After find the right parameters, the residuals can be shown at another file. Then the criteria that used to judge which model is better can be calculated. The SYSTAT has a good function that can help the forecasters calculate the MAD, MAPE, and RMSE quickly.

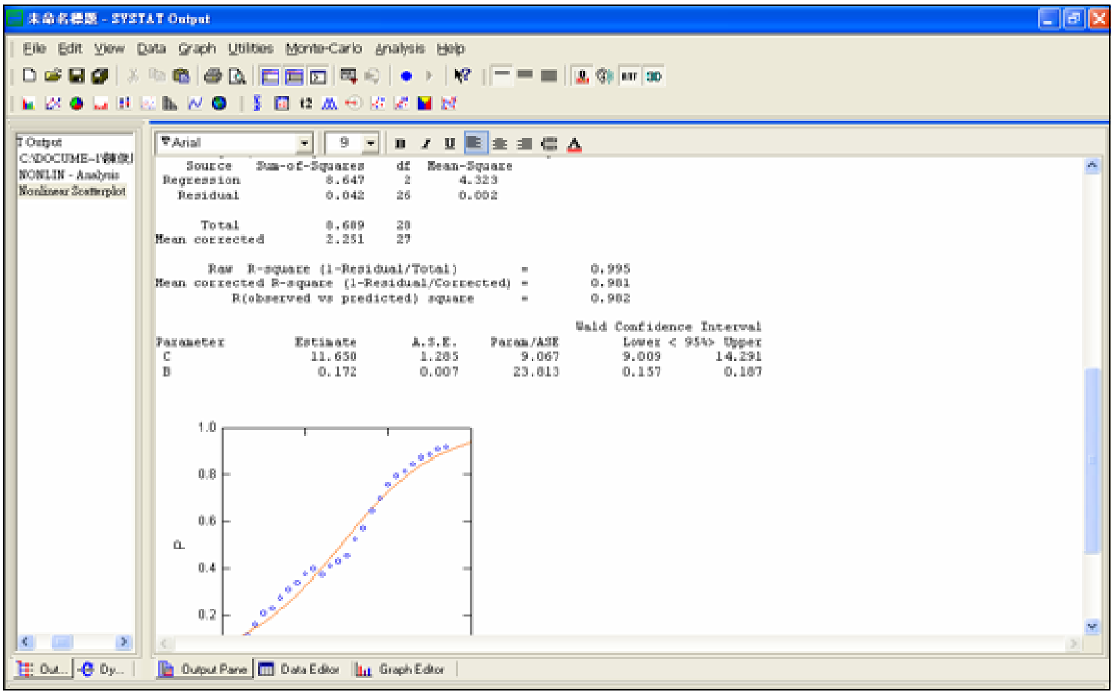


Figure 4 Result of estimating the parameters of Fisher-Pry model in nonlinear process

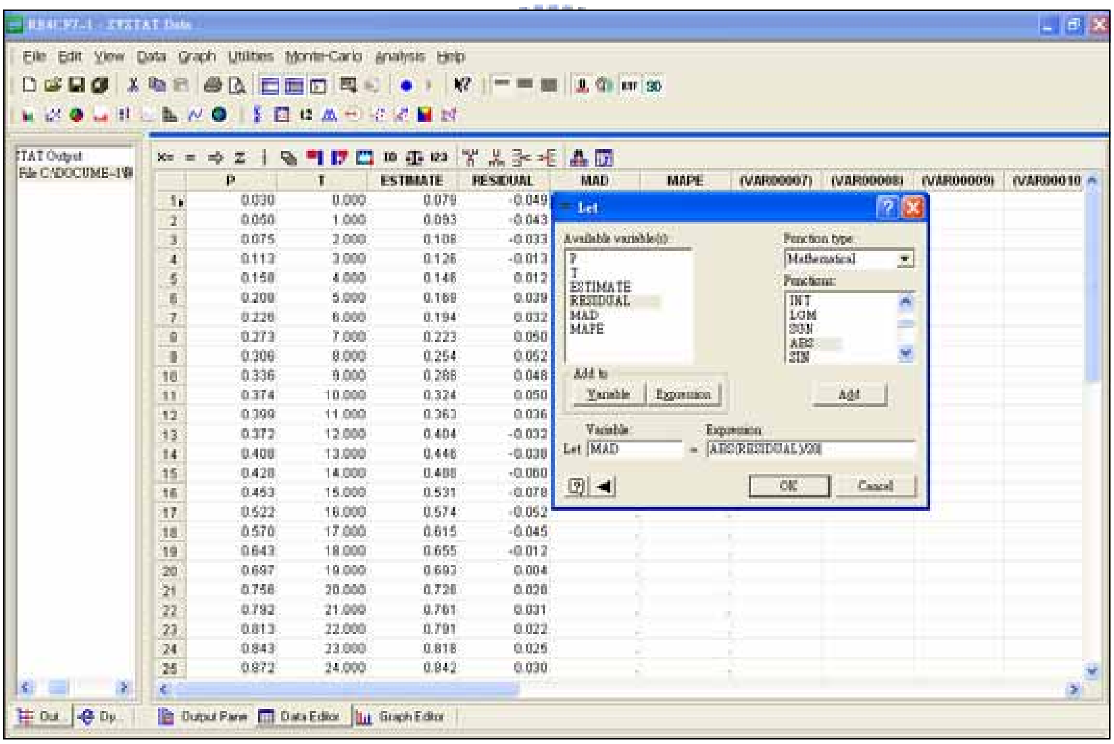


Figure 5 The way to calculate the criteria in SYSTAT (Fisher-Pry)

Appendix 5 The Process of Running the Gompertz Model

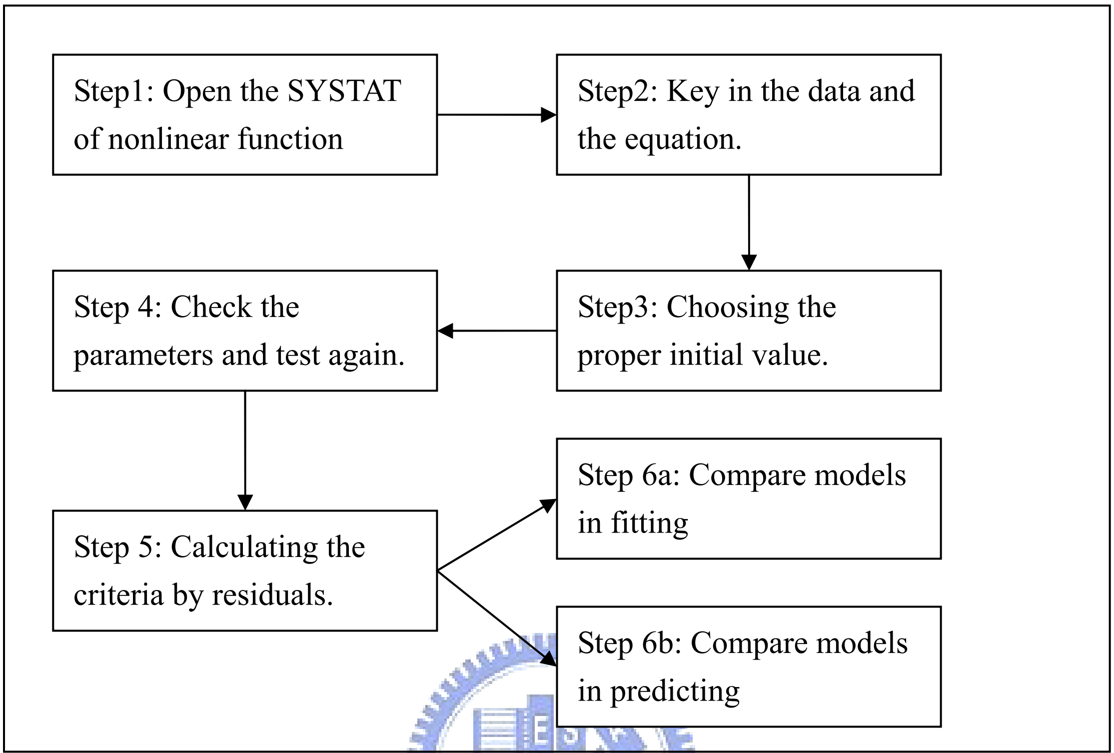


Figure 6 The process of testing the Gompertz model.

The step one it to open the SYSTAT software of nonlinear regression and key in the data sets. When all of the data has been keyed, depend variable should be selected and key the equation in the column of model expression. After that the initial value of parameters should be set. The proper initial value can be easily found by using first two data points. The equations below can help us to find the initial parameters.

$$\frac{\ln(f_0)}{\exp(-B * t_0)} = -C$$

$$\ln\left(\frac{\ln(f_1)}{-C}\right) = -B * t_1 \quad \text{, where } t_0 = 0, t_1 = 1 .$$

One thing should be noticed here. The original Gompertz form includes the capacity, L, in the equation, but the capacity is supposed that it equals to one. Therefore, the capacity, L, is omitted in this equation. If the capacity may reach at 80% or less, it can be set by 0.8.

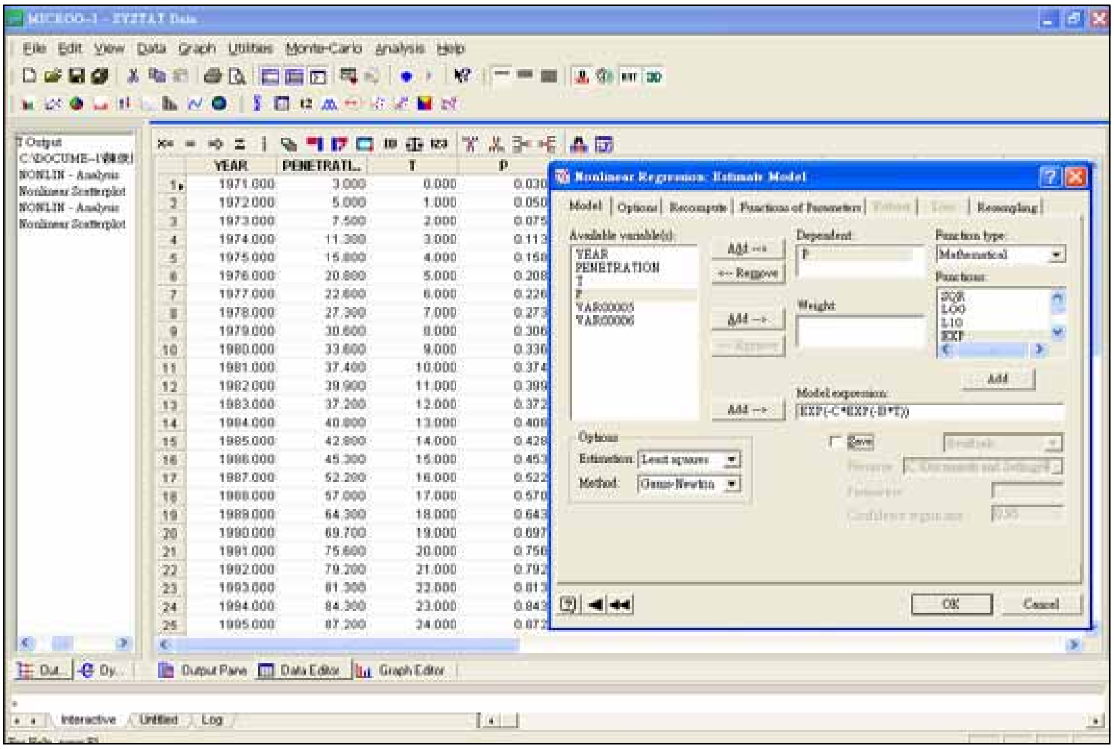


Figure 7 The window of nonlinear regression form in testing Gompertz model.

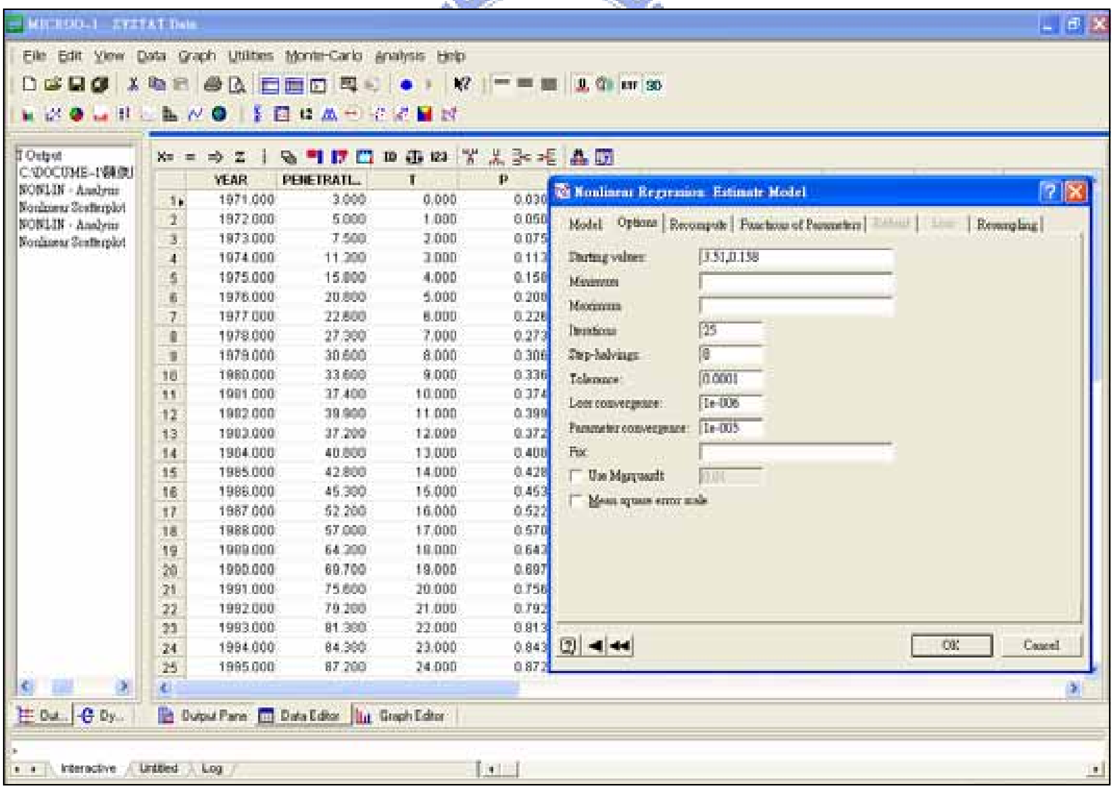


Figure 8 Set the initial value of Gompertz model in nonlinear process.

The outcome of Gompertz model is shown as below. The meaning of R^2, sum of square, parameters, etc has been mentioned at last appendix. Consequently, the points for attention are same as Fisher-Pry model.

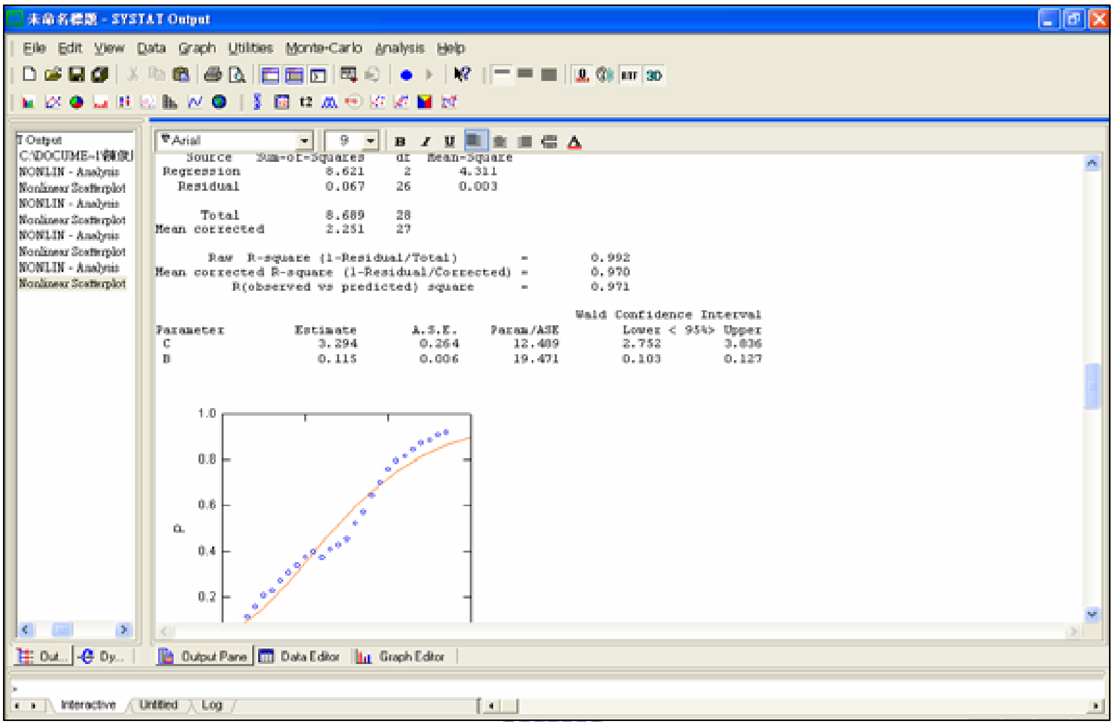


Figure 9 Result of estimating the parameters of Gompertz model in nonlinear process

The criteria can also be calculated by SYSTAT, and the process is same as Fisher-Pry model. Knowing the mathematical equations of criteria will help forecasters get the data of criteria fast.

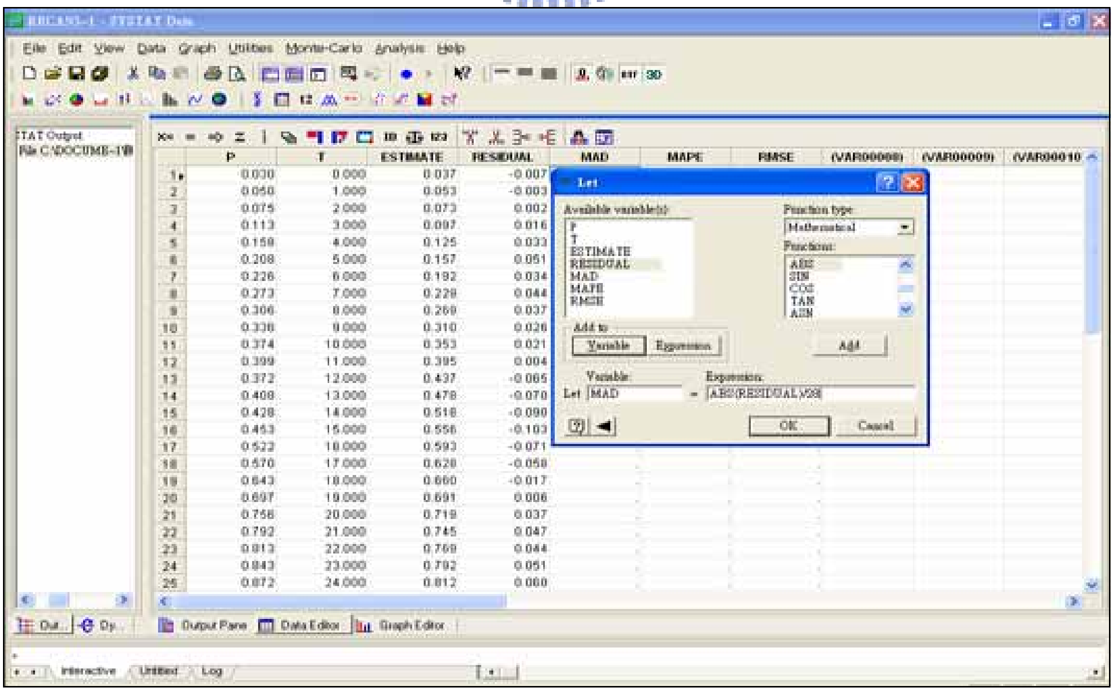


Figure 10 The way to calculate the criteria in SYSTAT (Gompertz)

Appendix 6 The Process of Running the Extended Logistic Model

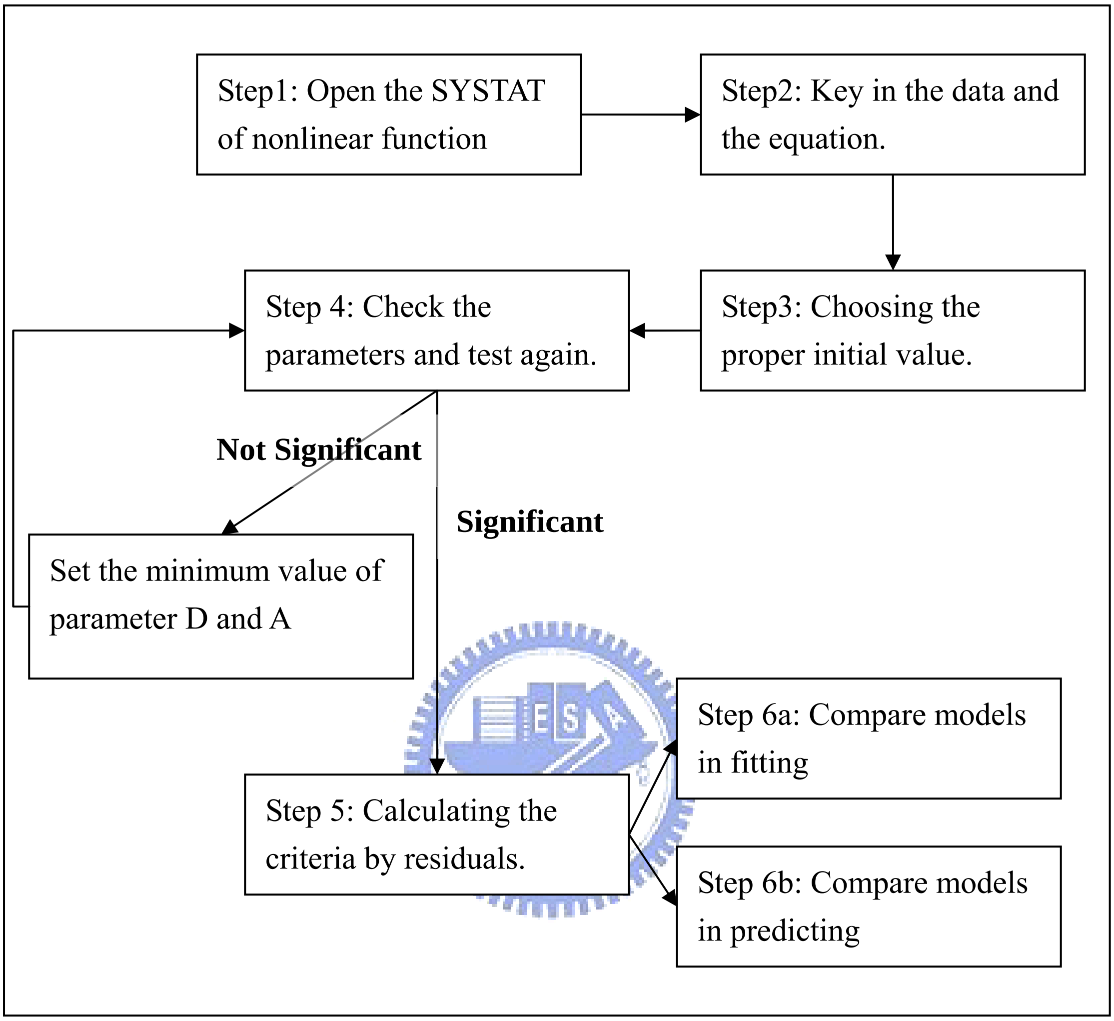


Figure 11 The process of testing the extended logistic model.

The first step of estimating the parameters of extended logistic is to key in the data sets and equation in nonlinear regression process in SYSTAT. The most important thing in nonlinear regression process is to find the proper initial value. However, extended logistic model does not like the other models that can easily find the initial parameters. The initial value will be set in some small values.

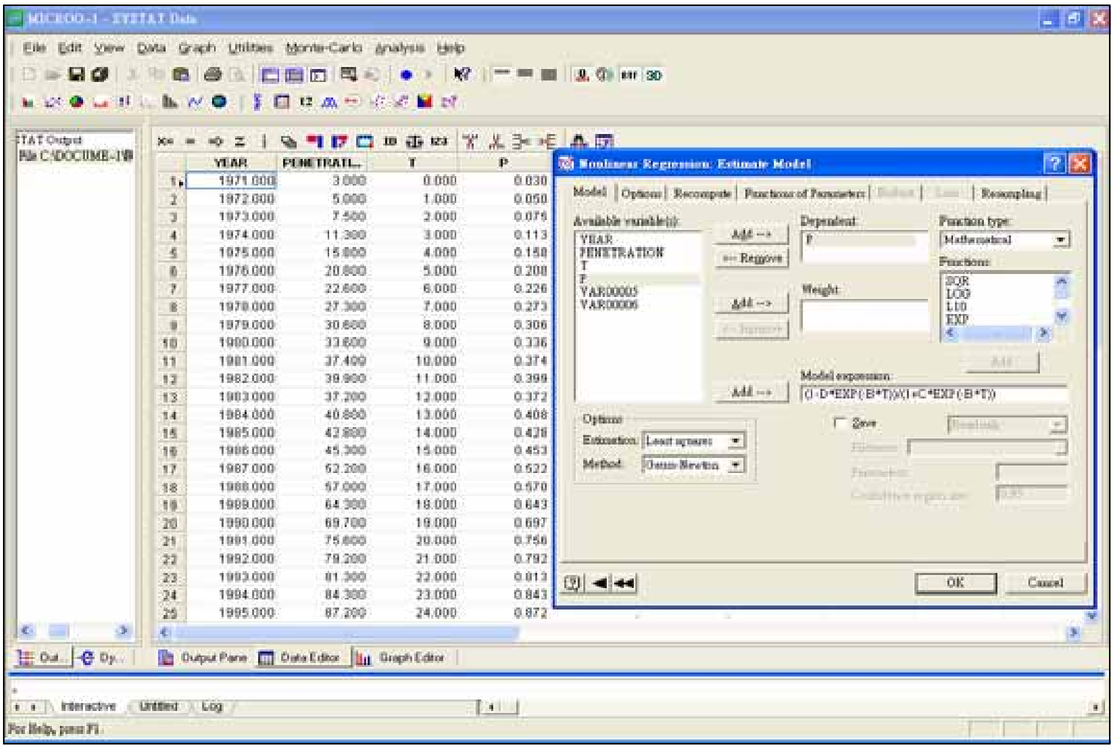


Figure 12 The window of nonlinear regression form in testing extended logistic model.

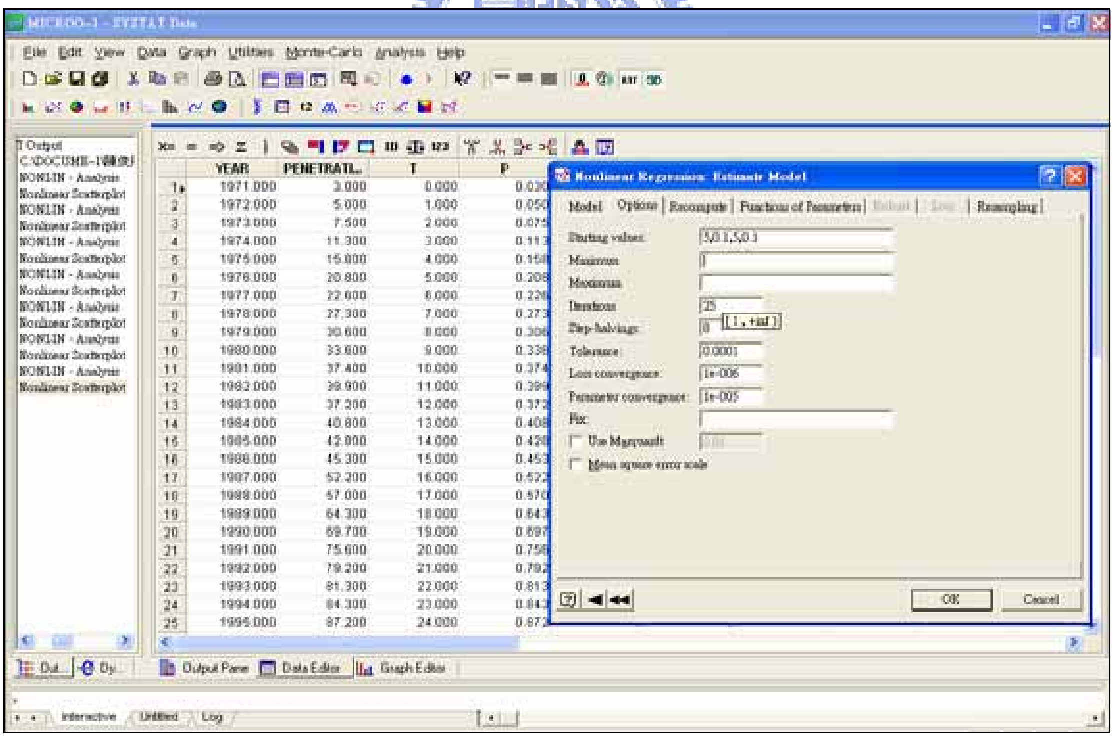


Figure 13 Set the initial value of extended logistic model in nonlinear process.

The first test of the extended logistic model shows that the parameter D and A is not significant (Figure 14). The value of parameter/ASE, the estimate of each parameter divided by its asymptotic standard error, is roughly as t test. Consequently, giving the limitation of minimum initial value will help the capacity rise up and increase the pram/A.S.E. value. After setting the minimum value of parameter, D, at 0.8, the pram/A.S.E. value increases less to be significant (Figure 14). Therefore, set the minimum value of the parameter D or A once again.

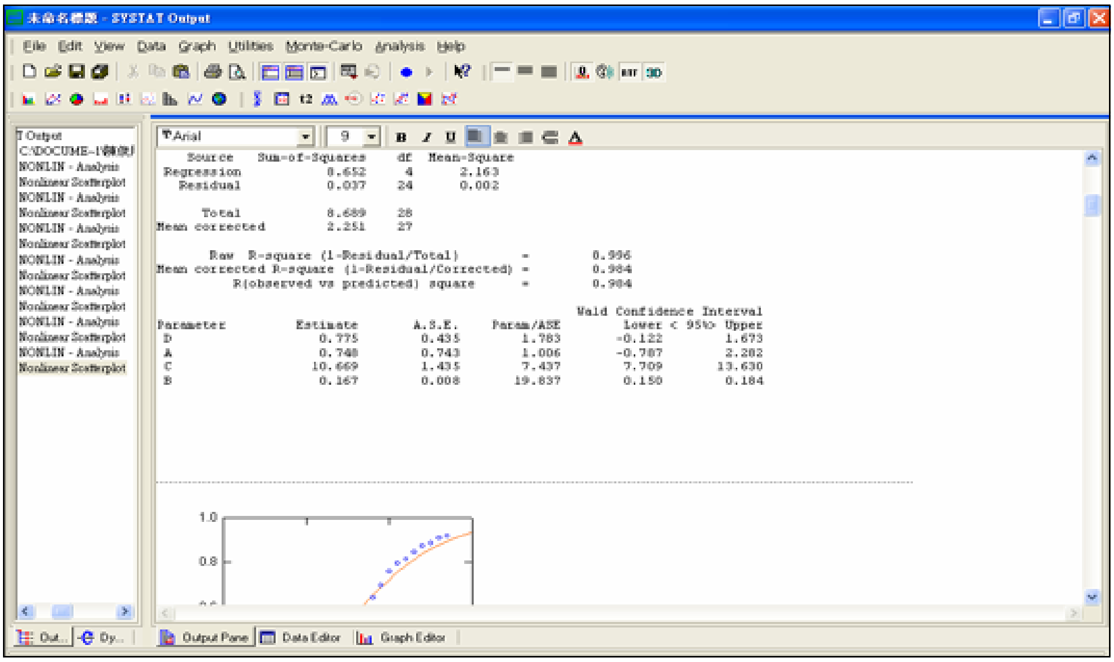


Figure 14 The first outcome of estimation in extended logistic model

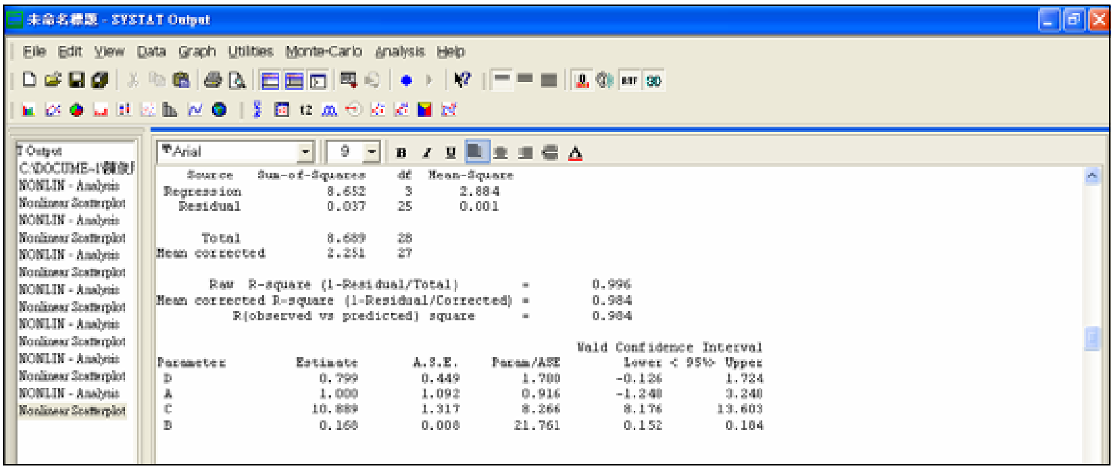
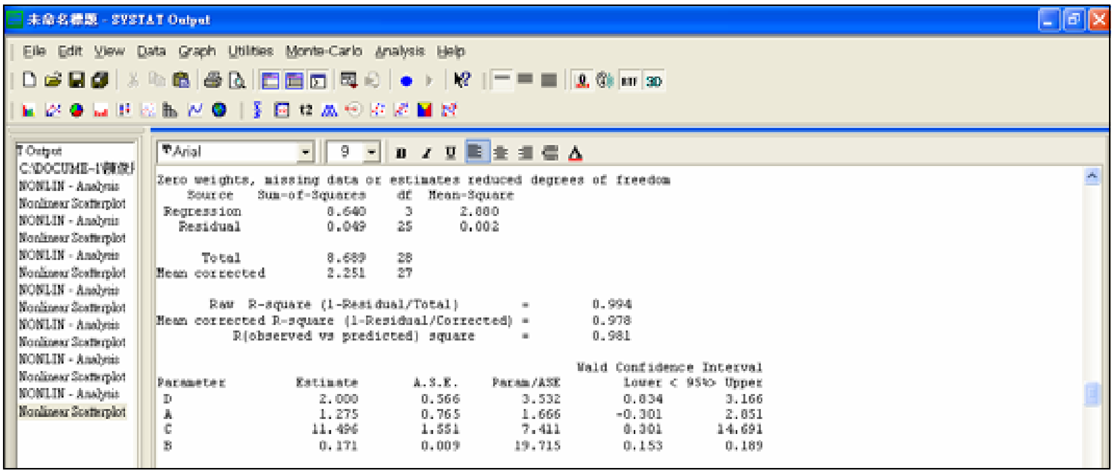


Figure 15 The second outcome of extended logistic model.

Then the figure 16 shows the outcome of changing both parameters D and A. The parameter/A.S.E value is bigger than the outcome before, but these two measurements both does not significant. After more than three repeated test, all the parameters shows significant (Table 17-19).



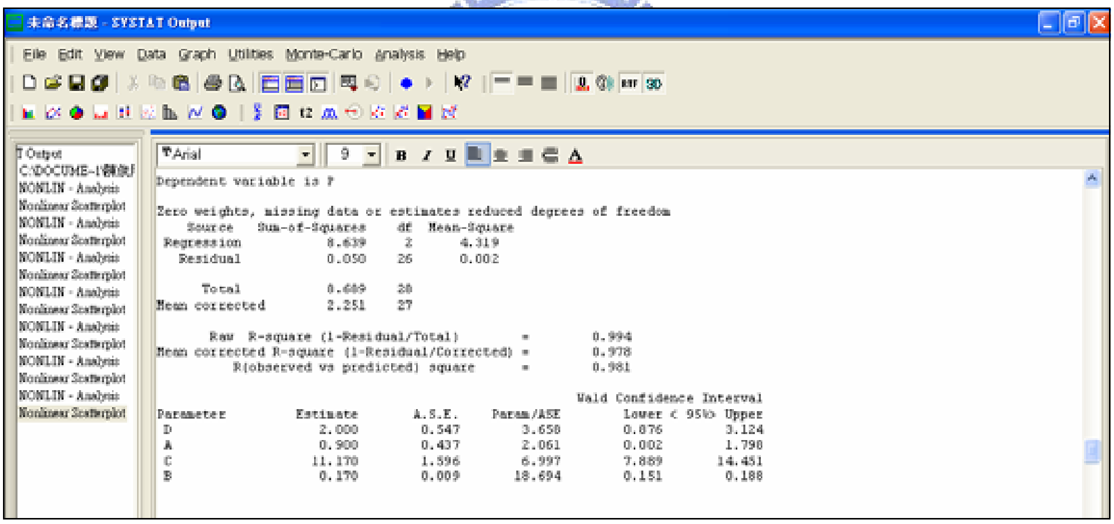

```
Zero weights, missing data or estimates reduced degrees of freedom
    Source    Sum-of-Squares    df   Mean-Square
 Regression           0.640      3       2.880
   Residual           0.049     25       0.002

      Total           8.689     28
Mean corrected        2.251     27

       Raw  R-square (1-Residual/Total)           =        0.994
Mean corrected R-square (1-Residual/Corrected) =        0.978
          R(observed vs predicted) square         =        0.981

                                                   Wald Confidence Interval
Parameter          Estimate      A.S.E.     Param/ASE     Lower < 95%> Upper
 D                    2.000       0.566         3.532      0.834       3.166
 A                    1.275       0.765         1.666     -0.301       2.851
 C                   11.496       1.551         7.411      0.301      14.691
 B                    0.171       0.009        19.715      0.153       0.189
```

Figure 16 The fourth outcome of extended logistic model.




```
Dependent variable is P

Zero weights, missing data or estimates reduced degrees of freedom
    Source    Sum-of-Squares    df   Mean-Square
 Regression           8.639      2       4.319
   Residual           0.050     26       0.002

      Total           8.689     28
Mean corrected        2.251     27

       Raw  R-square (1-Residual/Total)           =        0.994
Mean corrected R-square (1-Residual/Corrected) =        0.978
          R(observed vs predicted) square         =        0.981

                                                   Wald Confidence Interval
Parameter          Estimate      A.S.E.     Param/ASE     Lower < 95%> Upper
 D                    2.000       0.547         3.658      0.876       3.124
 A                    0.900       0.437         2.061      0.002       1.798
 C                   11.170       1.596         6.997      7.889      14.451
 B                    0.170       0.009        18.694      0.151       0.188
```

Figure 17 The fifth outcome of extended logistic model.

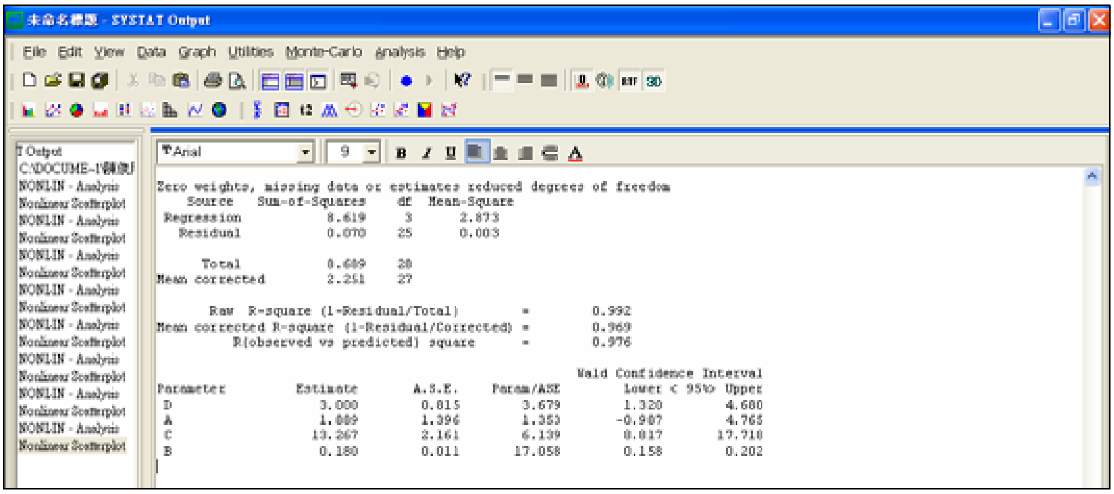


Figure 18 The sixth outcome of extended logistic model.

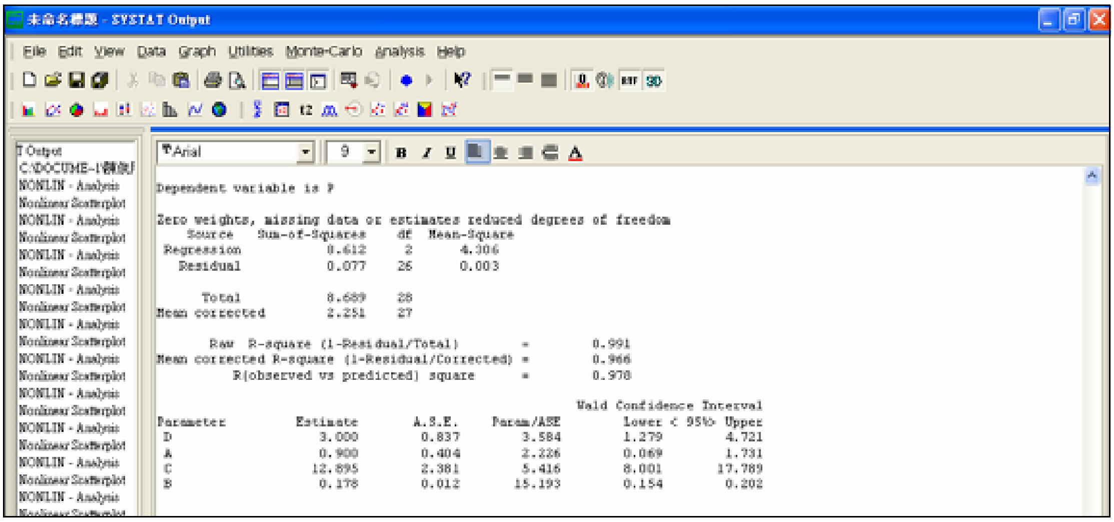


Figure 19 The seventh outcome of extended logistic model.

Some issues should be noticed when using this new extended logistic model. First, the estimated method of Gauss-Newton will not promise that one test will be done in this model. It may need more than three times to find the proper parameters. Second, limiting the parameters values is in order to let the parameters significant. Although the value of parameter/A.S.E just can be tested as t-test roughly, this can help us to identify the capacity.