Frequency Based TEQ Design with Oversampling

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August 9, 2006

Abstract

In our thesis, we design TEQ (time domain equalizer) for DSL (digital subscriber loops) applications using a frequency domain based approach. In DSL applications, usually frequency division multiplexing is used to separate the upstream and downstream signals. In either direction of transmission, some of the frequency bands are not used for transmission. Interference is formulated and minimized in the frequency domain to take advantage of the unused frequency bands. The frequency domain approach allows us to have a more direct control over the frequency response of the resulting TEQ, which is crucial in the final achievable transmission bit rates. Furthermore we will extend the method to the case when there is oversampling at the receiver to gain additional performance improvement. Oversampling is often used for time synchronization at the receiver and it also arises when the number of tones actually used for transmission is less the maximum possible number. The simulation examples demonstrate that the proposed methods (with or without oversampling) shortens the channel effectively and very good transmission bit rates can be achieved.

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Chapter 1

Introduction

The DFT based discrete multitone modulation (DMT) scheme is adopted for high-speed transmission, asymmetric digital subscriber line (ADSL) and very-high-bitrate digital subscriber line (VDSL), [1]-[2]. The modulation and demodulation are done by DFT and IDFT. After IDFT, a cyclic prefix of L samples is added as transmission symbol. When the channel order ν is less than the cyclic prefix length L, there is no interblock interference. The channel coefficients of out-of-window of $L+1$ samples will lead to interference and reduce the transmission rate. The DSL channel is usually longer than the cyclic prefix length. The time domain equalizer (TEQ) will be inserted into the DMT system before demodulation at receiver to shorten the DSL channel within $L+1$ samples. The TEQ plays an important role for DSL transmission. Many TEQ design algorithm is developed in DMT system for DSL applications, but none of them use over-sampling at receiver in DMT system to increase the freedom of TEQ design and consider the location of the zeros of TEQ response to improve transmission rate.

In [3], the authors propose a method for TEQ design by minimizing the energy of the equivalent channel impulse response which is out of the target window to minimize interference called MSSNR (maximum shortening signal-to-noise ratio). In [4], the authors propose a TEQ design consider not only the energy of the equivalent channel out of the target window but depends on it's time index. A TEQ design which considers the intersymbol interference plus interblock interference is given in [5]. The minimum mean-square error (MMSE) based

TEQ designs are shown in [6]-[7]. In [6], it shows the all time-domain equalizer designs which are proposed in time and frequency domain can be expressed in least-square (LS) form and become eigen problems. In [8], the authors design TEQ by exploiting the CP redundancy to force the last samples of CP to be equal to the last samples of system symbols after passing TEQ called MERRY. The MERRY algorithm is a low complexity, blind channel shortening algorithm, and is good trade-off between complexity and performance. A TEQ design criterion that optimizes transmission rate is given in [9]-[10]. The method of BM (bitrate maximizing) is a nonlinear solution, but a fast, near optimal solution [9], minimum-ISI is proposed in [9] and [11]. The TEQ design method of BM by maximizing an approximation to the geometric SNR (MGSNR)[12]-[13] and by optimizing the transmission rate obtained by the adaptive algorithm[6] and [14]. The bit-rate optimized for per-tone TEQ design (PTEQ) is presented in [17] .Many TEQ designs are proposed in time domain. In [15], it notes that the TEQ response will effect the transmission rate. The zeros of TEQ response at transmission bands will cause the poor total transmission rate. The TEQ response has large influence on bit rate. A semi-blind TEQ design method which maximizes the signal-to-interference ratio (SIR) in frequency domain. It exploits the training symbols in DSL initialization and solves it as eigen problem, [16].

On the other hand, in the context of synchronization for DMT systems, oversampling at the receiver is a very useful tool[18]-[20]. In a digital transmission system, synchronization is an essential part of the receiver. In DSL applications, the DMT system is very sensitive to synchronization errors, due to the high number of carriers and large constellations involved. Many methods have been developed [18],[21] to realize a fractional delay in the discrete time domain for DMT system. Usually, the timing correction is performed by means of a finite impulse response interpolation filter. In fact, due to the time-varying filter, FFT outputs would exhibit a time-varying rotation and attenuation. In [18]-[20], oversampling is used to design optimal interpolation filters for timing error correction. Then the correction errors would be reduce to an acceptable level. In addition, oversampling in DMT system can also arise when the number of tones used is less than the maximal possible number of tones. For example, in VDSL system, the number of tones that can be used are 2^{n+8} where $n = 0, 1, 2, 3, 4$, the maximal number of tones is 4096. If less than 4096 tones are used for transmission, the receiving is inherently oversampling.

In this thesis, we propose a frequency domain based TEQ design for DMT system and we will increase the sampling frequency at receiver to advance the freedom of TEQ design. More choice for TEQ design shall get better performance possibly. We minimize the interference which is the set of target tones due to the set of source tones to design our TEQ. And the proposed TEQ design for minimizing interference of just one target tone from other source tones shorten the channel effectively. The advantage of our proposed TEQ design is that the zeros of TEQ response corresponding to the sets of target and source tones. We will directly control the zeros of TEQ response by choosing the target and source tones. Then the transmission bands will free from the zeros by choosing the sets of target and source tones at unused band and we will get better transmission rate than other TEQ desigm methods. We will also modify our objective function by considering to minimize TEQ response at null tones. The zeros of TEQ response are also related with the tones which is selected to minimize TEQ response. The much better transmission rate will be obtained for considering frequency criterion. The objective function of our proposed TEQ design can be expressed in terms of TEQ coefficients and it can be obtained by solving eigen problem.

1.1 Outline

In Chapter 2, the block diagram and filterbank representation of DMT system model and introduction of VDSL will be shown. The equivalent discrete block diagram and filterbank representation of DMT system model with oversampling will be derived in Chapter 3. A survey of TEQ designs will be introduced in Chapter 4. The TEQ design with oversampling method is proposed in Chapter 5. Chapter 6 shows some computer simulations and comparisons. Finally, Conclusions and discussions will be presented.

1.2 Notation

1. Boldfaced lower case letters represent vectors and boldfaced upper letters are reserved for matrices. The notation $\mathbf{A}^{\dagger}$ denotes transpose-conjugate of $\mathbf{A}$.

2. The notation $\mathbf{W}$ represents the DFT matrix and W_M represent the value given by
$$W_M = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{M}}.$$

3. The notation $\mathbf{I}_M$ represent the $M \times M$ identity matrix.

4. The notation $(A(z))_{\downarrow N}$ denotes N-fold decimation of $X(z)$. In the time domain $B(z) = (A(z))_{\downarrow N}$ means $b(n) = a(Nn)$.

Chapter 2

System Model no oversampling

2.1 DMT System Model

The block diagram of the DFT based DMT system is as shown in Fig. 2.1. The channel can be modeled as a FIR (finite impulse response) filter of order ν with additive noise $q(n)$. The input symbol block $\mathbf{s}(n)$ is a $M \times 1$ vector with QAM (Quadrature amplitude modulation) symbols where M is the number of tones or subchannels. The input symbol block passes through the M-point IDFT and 'P/S' (parallel to serial) operation converts the parallel sample into serial sample train. Then the cyclic prefix (CP) of L samples is added as each block, the last L samples of the block are copied to put at the beginning of the block. After adding CP, the transmitted block passes through the channel plus noise $q(n)$. At the receiver, the received block discards the cyclic prefix and the 'S/P' (serial to parallel) operation converts the serial samples into parallel form. The value of d represents the synchronization delay before the removal CP. Followed by the 'S/P' operation , the block passes through the DFT and M parallel one-tap frequency domain equalizers (FEQ). The FEQ with one-tap coefficient ,$\frac{1}{\lambda_k}$, where λ_k is the M-point DFT of channel $C_1(z)$, $\lambda_k = \sum_{n=0}^{\nu} c_1(n) W_M^{kn}$. And then the final output symbols are obtained.

When channel length is less than the CP length, there is no interblock interference. If not, the channel coefficients of out-of-window of $L+1$ samples will lead to interference and reduce the transmission rate. At this situation, the time domain equalizer (TEQ) will be inserted into the system before demodulation at receiver to shorten the within $L+1$ samples. In order to generator the real transmitted signals, the input symbol block $\mathbf{s}(n)$ has to be conjugate symmetric, $s_k(n) = s^*_{M-k}(n),\ k = 1, \cdots, M-1$. As a result, only $\frac{M}{2}$ QAM symbols can be transmitted at the same time.

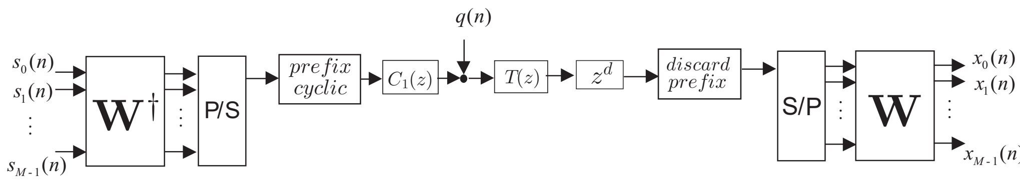


Figure 2.1: Block diagram of the DMT system

2.2 Filterbank Representation

The block diagram of the DFT based DMT system is as shown in Fig. 2.1. The modulation and demodulation are done by IDFT and DFT. The $M \times M$ DFT matrix denoted by $\mathbf{W}$, with $[\mathbf{W}]_{m,n} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{M}mn}$. The length of cyclic prefix is L and the parameter d represents the synchronization delay. $C_1(z)$ is the equivalent discrete time channel with respect to a sampling period T_s and $q(n)$ is additive noise. $T(z)$ is time domain equalizer with length T which is in order to shorten the original channel to avoid interference. The insertion of cyclic prefix can be viewed as an $N \times M$ matrix $\mathbf{F}_0$ where $N = M + L$,

$$\mathbf{F}_0 = \begin{pmatrix} \mathbf{0} \quad \mathbf{I}_L \\ \mathbf{I}_M \end{pmatrix} \tag{2.1}$$

And the discard prefix also can be viewed as a matrix,

$$\mathbf{F}_1 = \begin{pmatrix} \mathbf{0} & \mathbf{I}_M \end{pmatrix} \tag{2.2}$$

Then we can exchange the operation 'P/S' and prefix insertion. On the other hand, we also exchange the operation 'S/P' and the 'discard prefix' at the receiver. From the above, the DMT system in Fig. 2.1 can be redrawn as Fig. 2.2(a). Let us define,

$$\mathbf{G} = \mathbf{F}_0\mathbf{W}^{\dagger}, \quad \mathbf{S} = \mathbf{W}\mathbf{F}_1 \tag{2.3}$$

Notice that $\mathbf{G}$ is a constant matrix; we can exchange $\mathbf{G}$ and expanders. Similarly, we can also exchange $\mathbf{S}$ and decimators at receiver. The system in Fig. 2.2(a) becomes the one given in Fig. 2.2(b). We combine the $\mathbf{G}$ and delay train and the transmitting filter is

$$\mathbf{f}(z) = \begin{pmatrix} F_0(z) & F_1(z) & \cdots & F_{M-1}(z) \end{pmatrix} \tag{2.4}$$

then

$$\mathbf{f}(z) = \begin{pmatrix} 1 & z^{-1} & \cdots & z^{-(N-1)} \end{pmatrix} \mathbf{G} \tag{2.5}$$

The impulse response of k-th transmitting filter is $F_k(z) = \frac{1}{\sqrt{M}} \sum_{n=0}^{N-1} W_M^{-k(M-L+n)} \cdot z^{-i}$. Let us combine $\mathbf{S}$ and advance train to be $\mathbf{r}$(z). The receiving filter is

$$\mathbf{r}(z) = \begin{pmatrix} R_0(z) & R_1(z) & \cdots & R_{M-1}(z) \end{pmatrix} \tag{2.6}$$

then $r(z)$ can be written as

$$r(z) = \mathbf{S} \begin{pmatrix} 1 \\ z \\ \vdots \\ z^{N-1} \end{pmatrix} \tag{2.7}$$

The impulse response of i-th receiving filter is $R_i(z) = \frac{1}{\sqrt{M}} \sum_{l=L}^{N-1} W_M^{i(l-L)} \cdot z^l$. The filterbank representation of DMT system is shown in Fig. 2.2(c). The transmitting and receiving filters are both DFT filters. At transmitter, the prototype filter is $F_0(z)$ and all the other transmitting filters are the frequency-shifted version of $F_0(z)$,

$$F_k(z) = W^{Lk} F_0(zW^k), \ k = 1, 2, \cdots, M-1. \tag{2.8}$$

On the receiver side, the prototype filter is $R_0(z)$ and all the other receiving filters are the frequency-shifted version of $R_0(z)$,

$$R_k(z) = W^{-Lk} R_0(zW^k), \ k = 1, 2, \cdots, M-1. \tag{2.9}$$

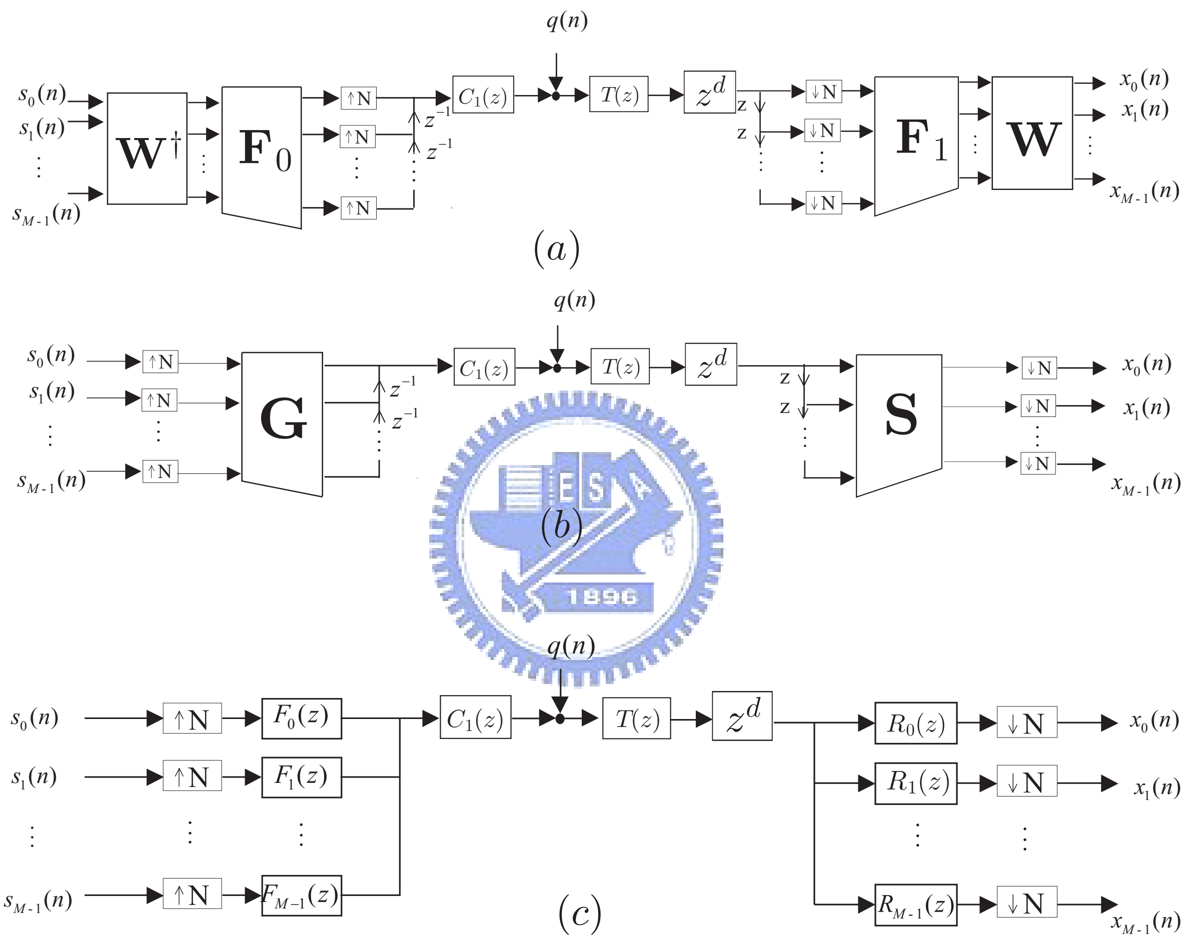


Figure 2.2: Derivation of the filterbank representation of the DMT system

From the input symbol $s_k(n)$ to the output symbol $x_i(n)$ is LTI, suppose the transfer function is $P_{ik}(z)$. The system from the input $\mathbf{s}(n)$ to output $\mathbf{x}(n)$ is a $M \times M$ LTI system $\mathbf{P}(z)$ with $[\mathbf{P}(z)]_{ik} = P_{ik}(z)$. Using polyphase identity between the input symbol $s_k(n)$ and the output symbol $x_i(n)$, the transfer function from k-th input symbol to i-th output symbol can be written as

$$P_{ik}(z) = (F_k(z)R_i(z)H(z))_{\downarrow N} \quad . \tag{2.10}$$

When the channel length is short enough, there is no interblock interference occurred. The subchannel gain from $s_k(n)$ to $x_k(n)$ is λ_k, that is $P_{ik}(z) = \lambda_k \delta(i-k)$. The outputs $x_i(n)$ multiple a one-tap frequency domain equalizer equal to the $\frac{1}{\lambda_k}$. In this case, the receiver outputs are the same as the transmitter inputs in the absence of noise. As the result, the DFT based DMT system can be viewed as M parallel subchannels as show in Fig. 2.3.

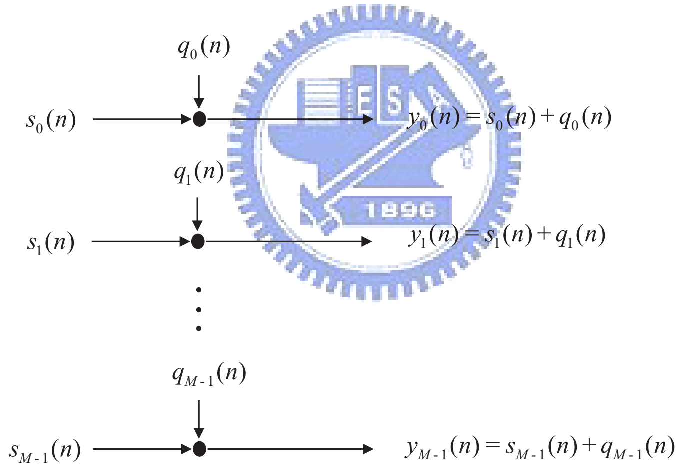


Figure 2.3: Equivalent M parallel subchannels

2.3 VDSL System

The VDSL system use Frequency Division Duplexing (FDD) to separate upstream and downstream transmission. The frequency plan shall consist of two upstream bands denoted as 1U, 2U and two downstream bands denoted as 1D, 2D. The bands shall be allocated as shown in Fig. 2.4. The values of the splitting frequency

Separating Frequencies	f_0	f_1	f_2	f_3	f_4	f_5
(MHz)	0.025	0.138	3.75	5.2	8.5	12

Table 2.1: VDSL band separating frequencies

f_i shall be as given in Table 2.1. The use of the band between 25 kHz and 138 kHz shall be negotiated during the initialization to indicate if the capability

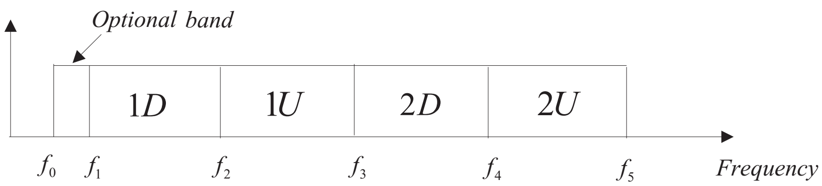


Figure 2.4: VDSL band allocation

exists and select one of the following option:

– Use of the band for upstream transmission;

– Use of the band for downstream transmission;

– The band is not used.

The modulation shall use a maximum number of sub-carrier equal to $\frac{M}{2} = 2^{n+8}$, where n can take the values 0, 1, 2, 3, 4. Disjoint subsets of the $\frac{M}{2}$ sub-carriers shall be defined for use in the downstream and upstream directions. The frequency spacing, Δf, between the sub-carriers shall be 4.3125 kHz. The sub-carriers shall be centered at frequencies $f = k \cdot \Delta f$. The tone index k can take the values $k = 0, 1, 2, \ldots, \frac{M}{2} - 1$. The downstream band 1D between 138 kHz and 3.75 MHz is tone $\{32 - 869\}$ and 2D between 5.2 MHz and 8.5 MHz is tone $\{1206 - 1971\}$. The upstream band 1U between 3.75 MHz and 5.2 MHz is tone $\{870 - 1205\}$ and 2D between 8.5 MHz and 12 MHz is tone $\{1972 - 2783\}$. The downstream and upstream band are both transmission band but one is transmission band and the other is unused band.

An algorithmic constellation encoder shall be used to construct sub-channel QAM constellations with a minimum number of bits equal to 1. The maximum number of bits that shall be supported is negotiated during initialization. And it shall be constrained between 8 and 15 bits. For a given sub-channel, the encoder shall select an odd-integer point (X, Y) from the square-grid constellation based on the b bits. For example, for $b = 2$, the four constellation points shall be labeled 0, 1, 2 and 3 corresponding to (0,0), (0,1), (1,0) and (1,1), respectively shown in Fig. 2.5.

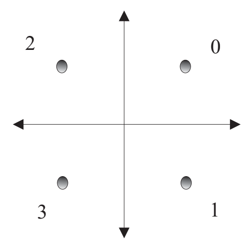


Figure 2.5: Constellation for 4QAM

Chapter 3

System Model for DMT with Oversampling

In this chapter, we will introduce the system model for DMT with oversampling that is the system for our proposed TEQ design. In section 3.1, we will derive the discrete equivalent system model with oversampling from continuous system model. Section 3.2, we will introduce how to generate two kinds of noise, one is white noise and the other is crosstalk noise, in VDSL system.

3.1 Equivalent Discrete DMT System with Oversampling

In this section, the system model with oversampling is discussed next. First we derive the equivalent discrete time channel model when there is oversampling. Fig 3.1(a) shows the part of the system from the D/C (discrete to continuous) converter at the transmitter to the C/D (continuous to discrete) converter at the receiver. The transmitter D/C converter corresponds to a sampling period T_s while the receiver C/D converter uses a sampling period $\frac{T_s}{Q}$; the oversampling factor is Q. The continuous time channel $h_c(t)$ is sandwitched in between. An ideal D/C converter with sampling period T_s can conceptually be considered as the inter-connection of the two blocks shown in Fig. 3.1(b), conversion of discrete-time sequence to impulse and a reconstruction filter $H_r(j\Omega)$. The filter $H_r(j\Omega)$ is an ideal lowpass filter with gain T_s and cut-off frequency $\frac{\pi}{T_s}$. Furthermore,

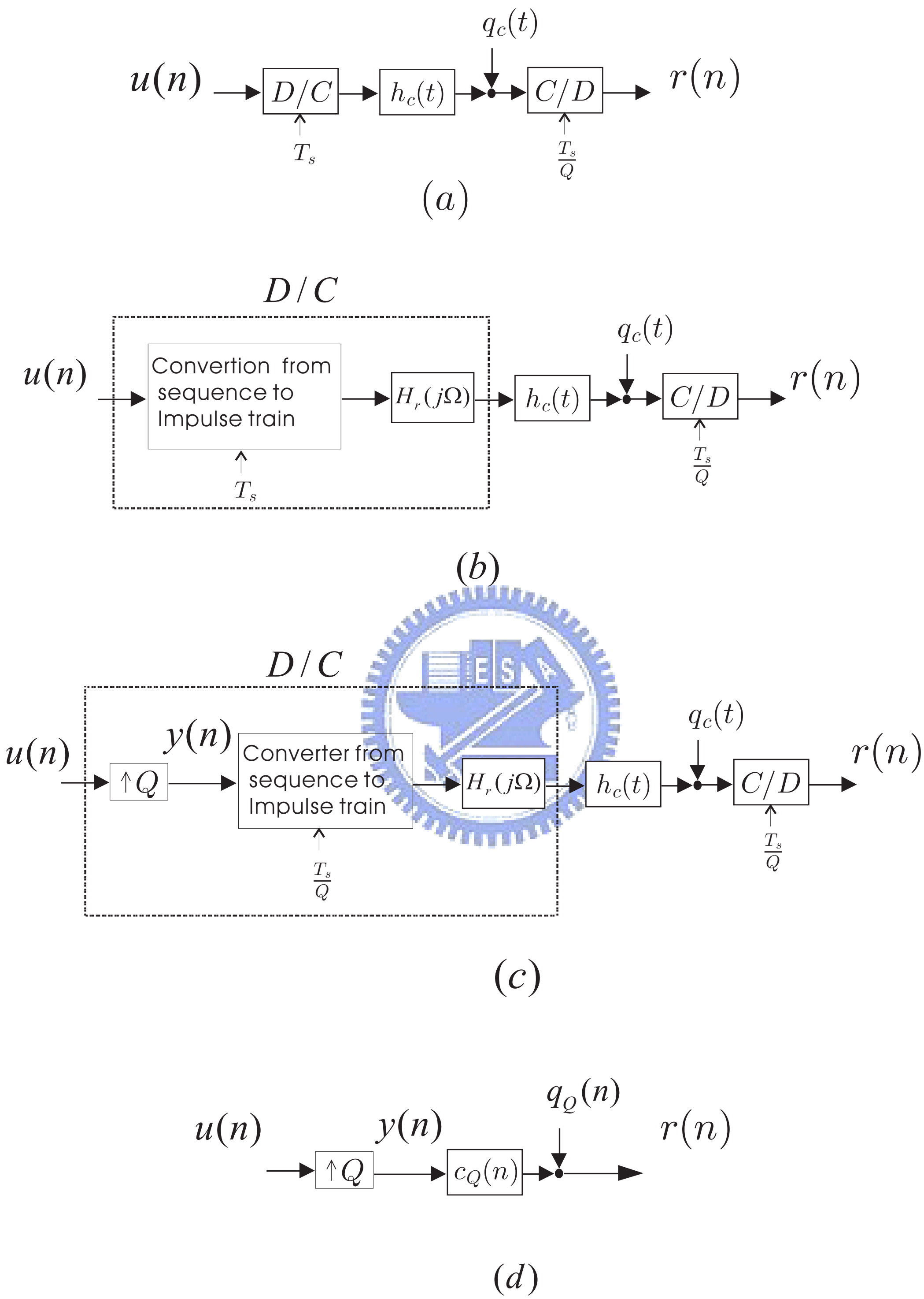


Figure 3.1: Derivation of the equivalent discrete channel model with oversampling

we note that converting a sequence to an impulse train spaced by T_s seconds is equivalent to first upsampling the sequence by Q and converting the resulting sequence to an impulse train spaced by $\frac{T_s}{Q}$ seconds. Using this observation, we can redraw the block diagram in Fig. 3.1(b) as Fig. 3.1(c). The system from $y(n)$ to $r(n)$ in Fig. 3.1(c) is known to be an discrete time LTI system say $C_Q(z)$. The equivalent blockdiagram from $u(n)$ to $r(n)$ is as shown in Fig. 3.1(d). If the continuous-time channel $H_c(j\Omega)$ is bandlimited to $|\Omega| < \frac{\pi}{T_s}$, the impulse response $c_Q(n)$ is related to $h_c(t)$ in a sample manner, i.e., $c_Q(n) = T_s h_c(n\frac{T_s}{Q})$. Then the equivalent block diagram of the DMT system with oversampling is shown as Fig. 3.2(a) and can be redrawn in filterbank form as Fig. 3.2(b).

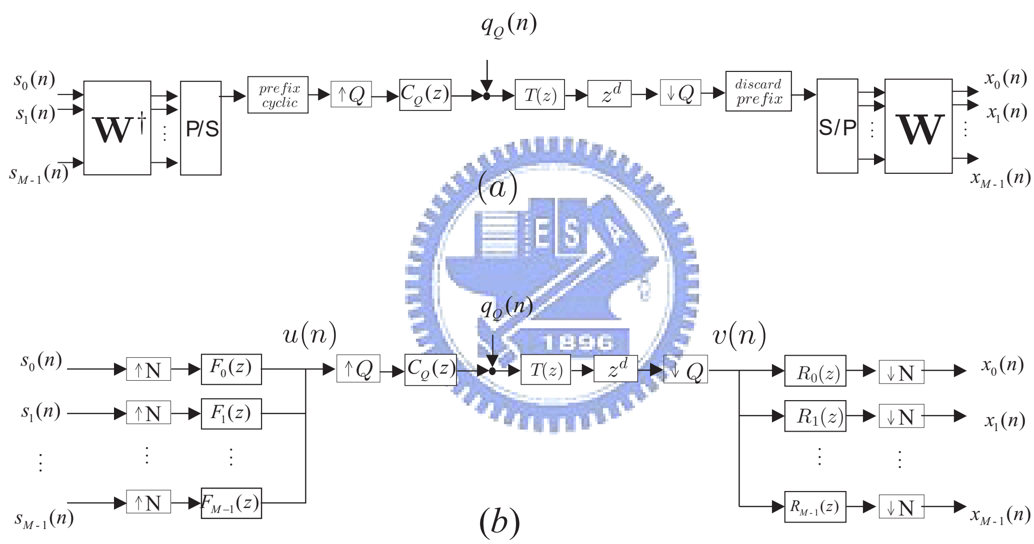


Figure 3.2: DMT system model with oversampling

3.2 Generation of Noise for the oversampling case

In this section we will generate white noise and crosstalk noise, including Far-end crosstalk noise (FEXT) and Near-end crosstalk noise (NEXT), for the VDSL system in the oversampling case. The FEXT and NEXT noise can be generated using the same step.

3.2.1 White noise

We assume the continuous-time white noise PSD is -170dBm/Hz in VDSL system, and it is measured across a terminated resistor $R_v = 100$Ohm. As -170dBm/Hz translates to 10^{-20} watt/Hz. The continuous-time white noise PSD is

$$S_{q_c}(j\Omega) = 10^{\frac{-170}{10}} \cdot 10^{-3} \cdot 100 = 10^{-18}(V^2). \tag{3.1}$$

We assume the frequency response of anti-alising filter before C/D converter $|H_a(j\Omega)| = 1, \forall \mid \Omega \mid \leq \frac{2\pi}{T_s}$. However, the transmitted symbols are bandlimited over $\{\frac{\pi}{T_s} \sim -\frac{\pi}{T_s}\}$ due to oversampling, the anti-alising filter can be choosen as $|H_a(j\Omega)| = 1, \forall \mid \Omega \mid \leq \frac{\pi}{T_s}$. The discrete white noise PSD can be written as

$$S_{q_c}(e^{j\omega}) = 10^{-18} \cdot \frac{1}{T_s} = -107\ (dB) \tag{3.2}$$

where $\frac{1}{T_s} = 17.664$MHz.

3.2.2 Crosstalk noise

Crosstalk noise results from the coupling of adjacent loops within the same cable bundle and can be modeled as two terms, FEXT and NEXT, shown in Fig. 3.3. Near-end crosstalk (NEXT) is simply defined as the unwanted signal coupling from a near-end transmitter into a pair measured at the same end. In the case of Fig. 3.3, the signal is transmitted on the A2. The coupled power is measured at the same end on the A1. Far-end crosstalk (FEXT) is defined as a measure of the unwanted signal coupling from a transmitter at the near-end into a neighboring pair measured at the far-end. Fig. 3.3 is the illustrated example. The signal is transmitted on the A2 and the curve line indicates the FEXT coupling.

The FEXT noise generator is shown in Fig. 3.4(a). The PSD of noise generator $g(t)$ is

$$S_g(j\Omega) = S_x(j\Omega) \cdot 10^{0.8} \tag{3.3}$$

where $S_x(j\Omega)$ is the PSD of transmitted power at downstream. The PSD of noise generator $g(t)$ shown in Fig. 3.4(a) is the $8dB$ addition of transmitted power [2] and the addition of 8dB approximates the power generated by the sum of 20

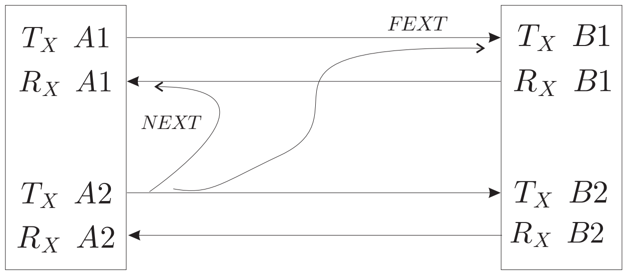


Figure 3.3: Illustration of FEXT and NEXT noise.

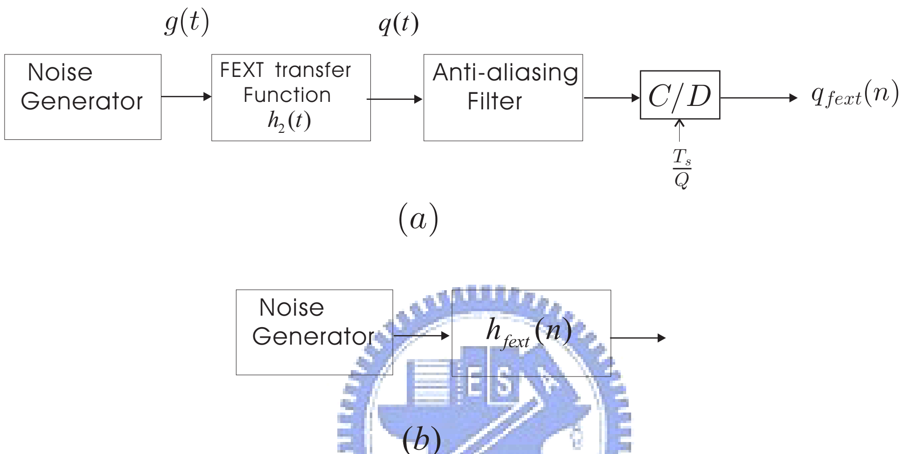


Figure 3.4: (a) Continuous-time FEXT noise generator; (b) discrete time approximation of (a).

VDSL systems operating in a multi-pair cable. The square of magnitude response of FEXT noise transfer function is

$$|H_2(f, L_{loop})|^2 = |H(f, L_{loop})|^2 \cdot K_{fext} \cdot (\frac{1}{49})^{0.6} \cdot L_{loop} \cdot f^2 \tag{3.4}$$

where $|H(f, L_{loop})|$ is the magnitude response of the loop insertion gain transfer function, L_{loop} is loop length in feet, f is frequency in Hertz, and $K_{fext} = 2.44 \cdot 10^{-22}$ is the crosstalk coupling coefficient for category-5 twisted pair. The PSD of discrete FEXT noise will be

$$S_{q_{fext}}(e^{j\omega}) = \frac{Q}{T_s} \cdot S_q(j\frac{Q\omega}{T_s}) \ , \ |\omega| \leq \pi. \tag{3.5}$$

Having obtained $S_{q_{fext}}(e^{j\omega})$, we can use it to generate FEXT noise in discrete time. For example, we can take p-points IDFT of $\sqrt{S_{q_{fext}}(e^{j\frac{2\pi}{p}k})}$ to obtain an FIR filter $h_{fext'}(n)$ of length p. The length p is usually chosen to be long enough so that then is negligible time domain aliasing in taking IDFT. To reduce the length of the transfer function, we can apply a shorter window, e.g. Hanning window. The FIR filter $h_{fext}(n)$ will be

$$h_{fext}(n) = w_{han}(n) \cdot h_{fext'}(n) \tag{3.6}$$

where

$$w_{han}(n) = \begin{cases} 0.5 - 0.5 \cdot cos(2\pi n/257), & 0 \le n \le 257 \\ 0, & otherwise \end{cases}. \tag{3.7}$$

To generate the FEXT noise, we can generate a sequential white Gaussian noise with unit variance to pass through the filter $h_{fext}(n)$. Then the statistic of the output signal of the filter $h_{fext}(n)$ in Fig. 3.4(b) will be a close approximation of that generated in (a).

Fig. 3.5 gives an example of FEXT noise generation. In our simulation, the symbol block size M is 4096, CP length L is 320, and the sampling frequency is 17.664MHz. Suppose downstream transmission over VDSL loop 1 is considered. Fig. 3.5(a) is the PSD of the noise generator $g(t)$. The magnitude response of FEXT transfer function $|H_2(f, L_{loop})|^2$ is shown in Fig. 3.5(b). Fig. 3.5(c) is the PSD of the output of the FEXT transfer function $h_2(t)$ in Fig. 3.4(a), $q(t)$. At the output of the C/D converter, the PSD of discrete FEXT noise $S_{q_{fext}}(e^{j\omega})$ is shown in Fig. 3.5(d). We choose $p = 4096$, the impulse response of $h_{fext'}(n)$ is shown in Fig. 3.5(e). We apply a Hanning window of length 258 on $h_{fext'}(n)$ to obtain $h_{fext}(n) = w_{han}(n) \cdot h_{fext'}(n)$. The plot of $h_{fext}(n)$ is given in Fig. 3.5(f). The magnitude response of $h_{fext}(n)$ filter is shown in Fig. 3.5(g). It is a close approximation of that given in Fig. 3.5(d).

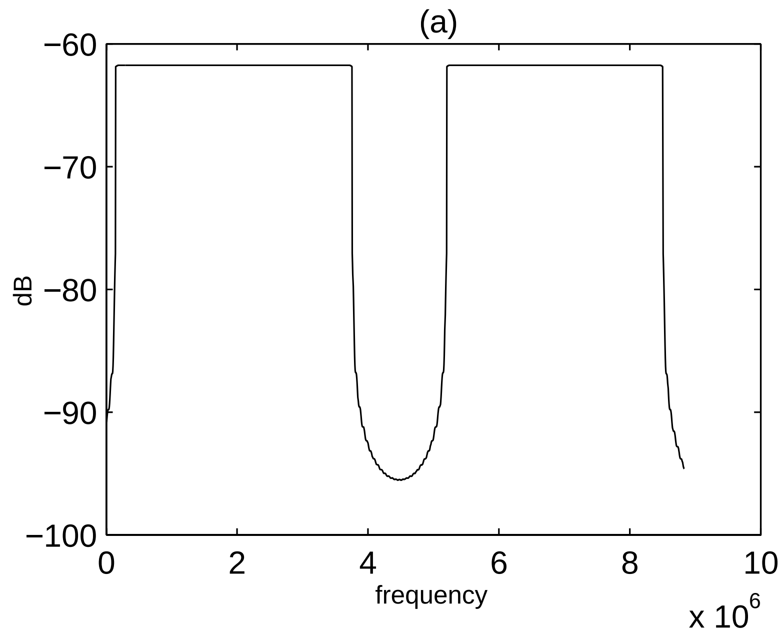
The generation of NEXT noise is similar to that of FEXT noise. For the NEXT noise, the PSD of $g(t)$ is

$$S_g(j\Omega) = S_{x'}(j\Omega) \cdot 10^{0.8} \tag{3.8}$$

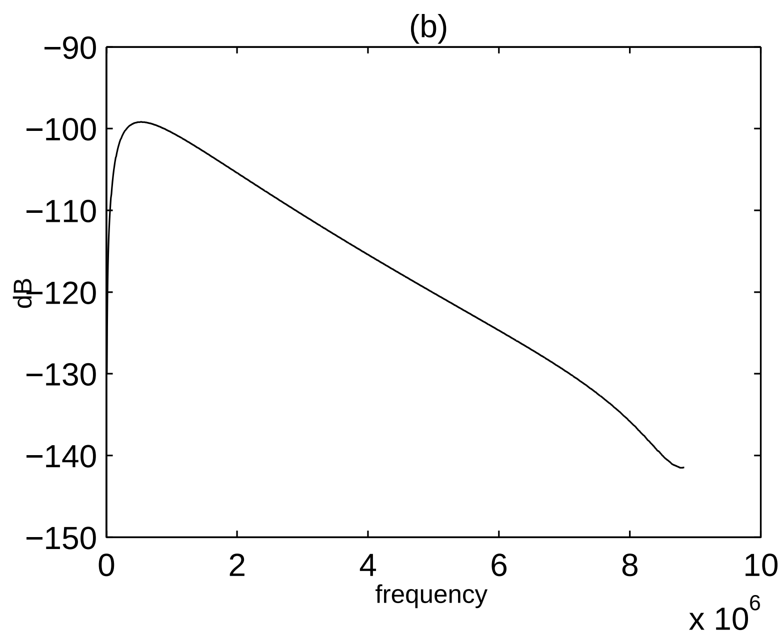
where $S_{x'}(j\Omega)$ is the PSD of the signal transmitted in the opposite direction. The square of NEXT noise transfer function is

$$|H_{next}(f, L_{loop})|^2 = K_{next} \cdot (\frac{1}{49})^{0.6} \cdot f^{1.5} \cdot [1 - |H(f, L_{loop})|^4] \qquad (3.9)$$

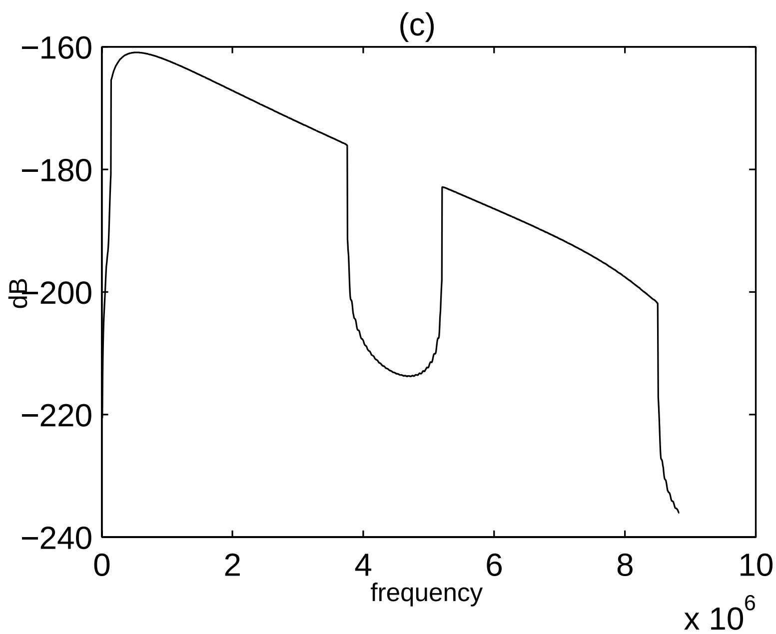
where $|H(f, L_{loop})|$ is the magnitude of loop insertion gain transfer function, L_{loop} is loop length in feet, f is frequency in Hertz, and $K_{next} = 3.30 \cdot 10^{-16}$ is the crosstalk coupling coefficient for category-5 twisted pair. The Fig. 3.5(h) shows the magnitude response of $h_{next}(n)$ filter.

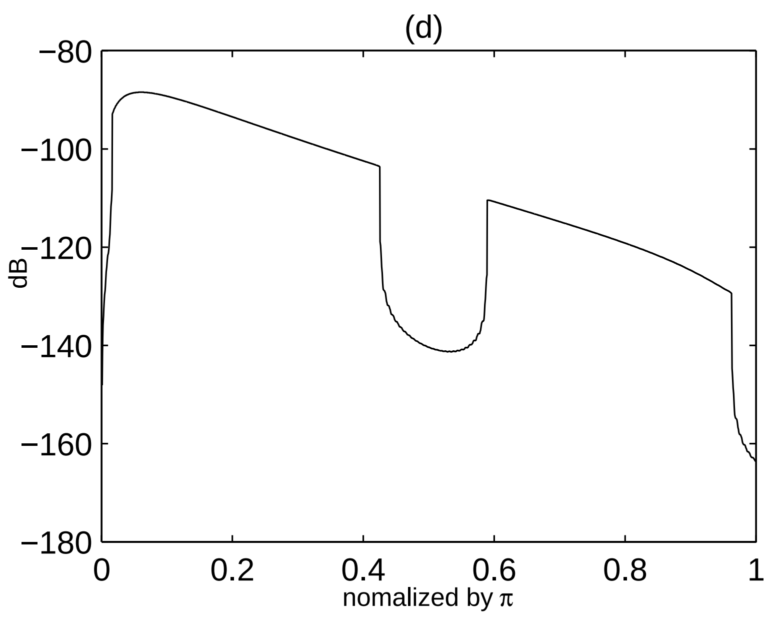


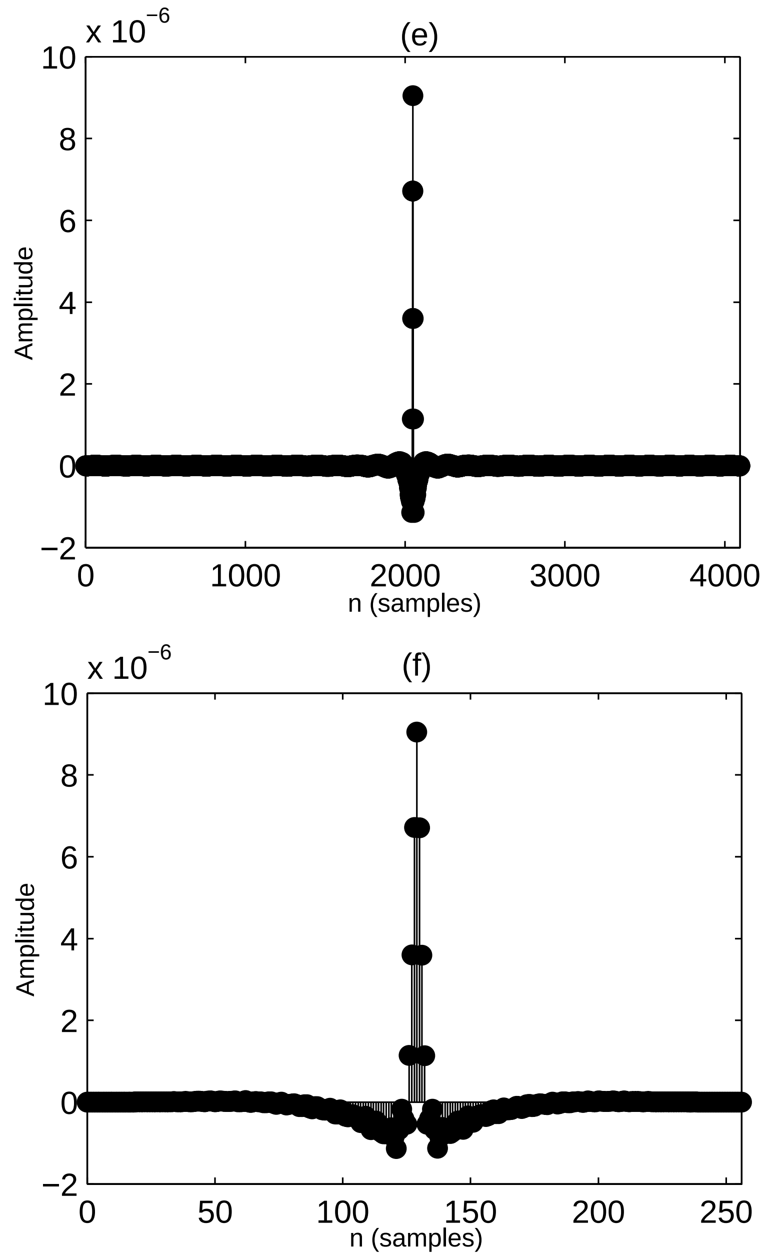
(a)
−60
−70
−80
−90
−100
dB
0
2
4
6
8
10
frequency
x 10^6

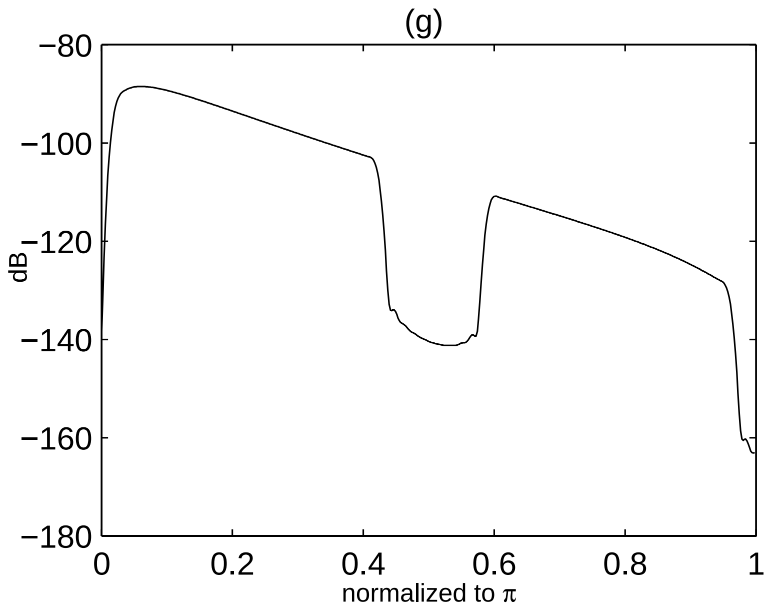


(b)
−90
−100
−110
−120
−130
−140
−150
dB
0
2
4
6
8
10
frequency
x 10^6


(c)
−160
−180
−200
−220
−240
dB
0
2
4
6
8
10
frequency
x 10^6


(d)
−80
−100
−120
−140
−160
−180
dB
0
0.2
0.4
0.6
0.8
1
nomalized by π


x 10^−6
(e)
10
8
6
4
2
0
−2
Amplitude
0
1000
2000
3000
4000
n (samples)
x 10^−6
(f)
10
8
6
4
2
0
−2
Amplitude
0
50
100
150
200
250
n (samples)



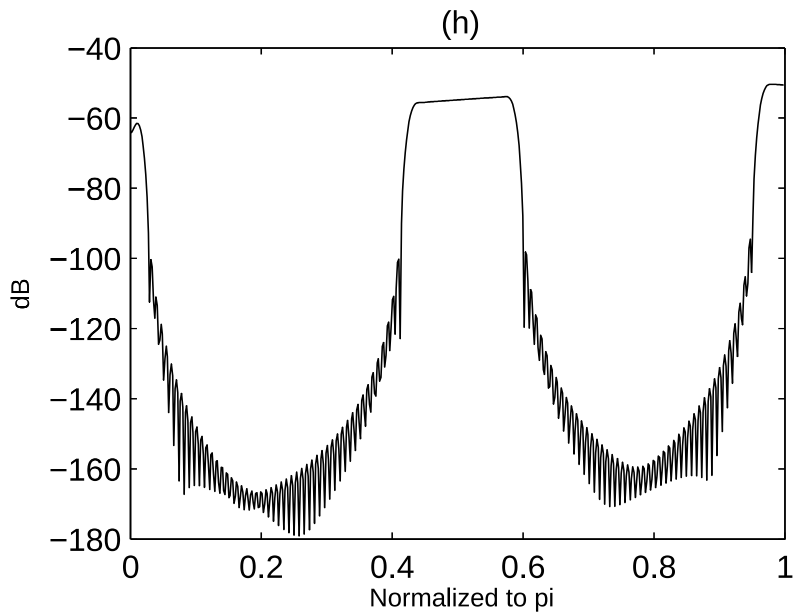


Figure 3.5: (a) PSD of the noise generator, (b) square of the magnitude of FEXT transfer function, (c) PSD of the continuous FEXT noise, (d) PSD of the discrete FEXT noise, (e) impulse response of the $h_{fext'}(n)$, (f) impulse response of the $h_{fext}(n)$, (g) the magnitude response of $h_{fext}(n)$ filter, (h) the magnitude response of $h_{fext}(n)$ filter.

Chapter 4

A survey of TEQ Designs

4.1 Maximum shortening SNR (MSSNR)

The TEQ design for MSSNR method[3] is to minimize the energy of the equivalent channel impulse response h, $h = c * t$, which is out of the target window with length $L+1$ to minimize interference. The largest $L+1$ samples will not necessarily start with the first sample. This delay, d, is normally compensated for at the receiver by delaying the start of the received symbol. The equivalent channel h with length $L_{ch}+T-1$ can be written as

$$
\begin{aligned}
\mathbf{h} &= \begin{pmatrix} h(0) \\ h(1) \\ \vdots \\ h(L_{ch}-1) \\ h(L_{ch}) \\ \vdots \\ h(L_{ch}+T-2) \end{pmatrix} \\
&= \begin{pmatrix} h(0) & 0 & \cdots & & 0 \\ h(1) & h(0) & \ddots & & \\ \vdots & \vdots & & & \\ h(L_{ch}-1) & h(L_{ch}-2) & \cdots & h(L_{ch}-T+1) & h(L_{ch}-T) \\ 0 & h(L_{ch}-1) & \cdots & & h(L_{ch}-T+1) \\ \vdots & \ddots & & & \vdots \\ 0 & \cdots & & 0 & h(L_{ch}-1) \end{pmatrix} \begin{pmatrix} t(0) \\ t(1) \\ \vdots \\ t(T-1) \end{pmatrix} \\
&= \mathbf{Ht}
\end{aligned}
\tag{4.1}
$$

where T is the length of TEQ.

$$
\begin{aligned}
\mathbf{h}_{win} &= \begin{pmatrix} h(d) \\ h(d+1) \\ \vdots \\ h(d+L) \end{pmatrix} \\
&= \begin{pmatrix} h(d) & h(d-1) & \cdots & h(d-T+1 \\ h(d+1) & h(d) & \cdots & h(d-T+2) \\ \vdots & & \ddots & \vdots \\ h(d+L) & h(d+L-1) & \cdots & h(d+L-T+1) \end{pmatrix} \begin{pmatrix} t(0) \\ t(1) \\ \vdots \\ t(T-1) \end{pmatrix} \\
&= \mathbf{H}_{win}\mathbf{t}
\end{aligned}
\tag{4.2}
$$

$$
\begin{aligned}
\mathbf{h}_{wall} &= \begin{pmatrix} h(0) \\ h(d-1) \\ \vdots \\ h(d+L+1) \\ \vdots \\ h(L_{ch}+T-2) \end{pmatrix} \\
&= \begin{pmatrix} h(0) & 0 & \cdots & 0 \\ \vdots & \ddots & & \\ h(d-1) & h(d-2) & \cdots & h(d-T) \\ h(d+L+1) & h(d+L) & \cdots & h(d+L-T+2) \\ \vdots & \ddots & & \\ 0 & \cdots & 0 & h(L_{ch}-1) \end{pmatrix} \begin{pmatrix} t(0) \\ t(1) \\ \vdots \\ t(T-1) \end{pmatrix} \\
&= \mathbf{H}_{wall}\mathbf{t}.
\end{aligned}
\tag{4.3}
$$

Optimum shortening can be expressed as choosing t to minimize $\mathbf{h}_{wall}^{\dagger}\mathbf{h}_{wall}$ while satisfying the constraint $\mathbf{h}_{win}^{\dagger}\mathbf{h}_{win} = 1$. The expressions for the energy outside and inside the window can be written as

$$\mathbf{h}_{wall}^{\dagger}\mathbf{h}_{wall} = \mathbf{t}^{\dagger}\mathbf{H}_{wall}^{\dagger}\mathbf{H}_{wall}\mathbf{t} = \mathbf{t}^{\dagger}\mathbf{A}\mathbf{t} \tag{4.4}$$

$$\mathbf{h}_{win}^{\dagger}\mathbf{h}_{win} = \mathbf{t}^{\dagger}\mathbf{H}_{win}^{\dagger}\mathbf{H}_{win}\mathbf{t} = \mathbf{t}^{\dagger}\mathbf{B}\mathbf{t} \tag{4.5}$$

where $\mathbf{A}$ and $\mathbf{B}$ are symmetric and positive semidefinite matrices. Optimum shortening can be considered as choosing $\mathbf{t}$ to minimize $\mathbf{t}^{\dagger}\mathbf{A}\mathbf{t}$ while satisfying the constraint $\mathbf{t}^{\dagger}\mathbf{B}\mathbf{t} = 1$. The problem becomes to minimize the ratio

$$\phi = \frac{\mathbf{t}^{\dagger}\mathbf{A}\mathbf{t}}{\mathbf{t}^{\dagger}\mathbf{B}\mathbf{t}}. \tag{4.6}$$

If $\mathbf{B}$ is positive definite and can be decomposed using Cholesky Decomposition into $\mathbf{B} = \mathbf{Q}^{\dagger}\mathbf{Q}$. Since $\mathbf{B}$ is full rank, the matrix $\mathbf{Q}^{-1}$ is exists. Let $\mathbf{v} = \mathbf{Q}\mathbf{t}$, the optimize problem becomes

$$\min_{\mathbf{v}} \frac{\mathbf{v}^{\dagger}\mathbf{Q}^{-\dagger}\mathbf{A}\mathbf{Q}^{-1}\mathbf{v}}{\mathbf{v}^{\dagger}\mathbf{v}}. \tag{4.7}$$

The solution of this problem $\mathbf{v}$ is the eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{Q}^{-\dagger}\mathbf{A}\mathbf{Q}^{-1}$. And the optimum TEQ $\mathbf{t}$ can be obtained

$$\mathbf{t} = \mathbf{Q}^{-1}\mathbf{v} \tag{4.8}$$

and the optimum shortening SIR can be expressed as

$$SIR_{opt} = 10log(\frac{\mathbf{t}^{\dagger}\mathbf{B}\mathbf{t}}{\mathbf{t}^{\dagger}\mathbf{A}\mathbf{t}}) = 10log(\frac{1}{\lambda_{min}}). \tag{4.9}$$

where λ_{min} means the minimum eigenvalue of the matrix $\mathbf{Q}^{-\dagger}\mathbf{A}\mathbf{Q}^{-1}$.

4.2 Minimum intersymbol interference (Min-ISI)

Calculating the Maximum Bit Rate method TEQ requires solving a nonlinear optimization problem. In order to use an equalizer in a practical system, we have to avoid nonlinear optimization. In this section, the min-ISI method[9], which can be calculated without using a globally optimal constrained nonlinear optimization solver is introduced.

The output of TEQ at receiver in DMT system can be expressed as

$$y_k = h_k * t_k * x_k + t_k * n_k$$

where x_k is transmitted signal, h_k is the discrete channel impulse response, n_k is discrete additive noise and t_k is TEQ impulse response. Notice that the transmitted signal consists of symbols of N points separated by cyclic prefix of length ν. Therefore, the ISI can be removed if the equalizer can successfully shorten the channel to $\nu+1$ samples. With this insight, we formulate the following windowing function g_k to isolate the desired pare of h_k

$$g_k = \begin{cases} 1, & \Delta \leq k \leq \Delta + \nu \\ 0, & otherwise \end{cases} \tag{4.10}$$

where Δ is a synchronization delay. With the windowing function, we can separate the signal, interference and noise components of the received signal as

$$y_k = h_k^{signal} * x_k + h_k^{ISI} * x_k + t_k * n_k \tag{4.11}$$

where $h_k^{signal} = g_k(h_k * t_k)$ and $h_k^{ISI} = (1 - g_k)(h_k * t_k)$. The decomposition in **??** enables the following definition of the SNR in each subchannel,

$$SNR_i = \frac{|H_i^{signal}|^2 S_{x,i}}{|H_i^{ISI}|^2 S_{x,i} + |W_i|^2 S_{n,i}} \tag{4.12}$$

where S_x is the N-point power spectrum of x_k (without the cyclic prefix) and S_n is the N-point power spectrum of n_k. In matrix-vector notation,

$$\begin{array}{ccc} H_i^{signal} & = & \mathbf{q}_i^{\dagger}\mathbf{GHt} \\ H_i^{ISI} & = & \mathbf{q}_i^{\dagger}\mathbf{DHt} \\ W_i & = & \mathbf{q}_i^{\dagger}\mathbf{Ft} \end{array} \tag{4.13}$$

where

$$\begin{array}{ccc} \mathbf{t} & = & \begin{pmatrix} w_0 & w_1 & \cdots & w_{N_w-1} \end{pmatrix}^{\top} \\ \mathbf{H} & = & \begin{pmatrix} h_0 & h_{-1} & \cdots & h_{-(N_w-1)} \\ h_1 & h_0 & \cdots & h_{-(N_w-2)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-1} & h_{N-2} & \cdots & h_{-(N-N_w)} \end{pmatrix} \\ \mathbf{G} & = & diag\begin{pmatrix} g_0 & g_1 & \cdots & g_{N-1} \end{pmatrix}^{\top} \\ \mathbf{D} & = & \mathbf{I} - \mathbf{G} \\ \mathbf{q}_i & = & \begin{pmatrix} 1 & e^{j\frac{2\pi i}{N}} & \cdots & e^{j\frac{2\pi i(N-1)}{N}} \end{pmatrix}^{\top}. \end{array} \tag{4.14}$$

So the SNR in subchannel can be written as

$$SNR_i = \frac{|\mathbf{q}_i^{\dagger}\mathbf{GHt}|^2 S_{x,i}}{|\mathbf{q}_i^{\dagger}\mathbf{DHt}|^2 S_{x,i} + |\mathbf{q}_i^{\dagger}\mathbf{Ft}|^2 S_{n,i}}. \tag{4.15}$$

Since the power term is always nonnegative, minimizing the distortion power in each subchannel (the denominator of SNR_i in 4.15) is equivalent to minimizing the sum of the distortion power of all subchannel, which can be written as

$$\mathbf{t}^{\top}\mathbf{H}^{\top}\mathbf{D}^{\top}\sum_{i\in\Phi}(\mathbf{q}_i\frac{S_{x,i}}{S_{n,i}}\mathbf{q}_i^{\dagger})\mathbf{DHt} = \mathbf{t}^{\top}\mathbf{At}. \tag{4.16}$$

To prevent minimization of the signal power, we constrain the signal path impulse response energy to one:

$$\|H_i^{signal}\|^2 = \mathbf{t}^{\top}\mathbf{H}^{\top}\mathbf{G}^{\top}\mathbf{GHt} = \mathbf{t}^{\top}\mathbf{Bt} = 1. \tag{4.17}$$

Finally, the optimization problem for minimum ISI becomes

$$\min_{\mathbf{t}} \mathbf{t}^\top \mathbf{A}\mathbf{t} \quad s.t.\ \mathbf{t}^\top \mathbf{B}\mathbf{t} = 1. \tag{4.18}$$

If $\mathbf{B}$ is positive definite and can be decomposed using Cholesky Decomposition into $\mathbf{B} = \mathbf{Q}^\dagger \mathbf{Q}$. Since $\mathbf{B}$ is full rank, the matrix $\mathbf{Q}^{-1}$ is exists. Let $\mathbf{v} = \mathbf{Q}\mathbf{t}$, the optimize problem becomes

$$\min_{\mathbf{v}} \frac{\mathbf{v}^\dagger \mathbf{Q}^{-\dagger}\mathbf{A}\mathbf{Q}^{-1}\mathbf{v}}{\mathbf{v}^\dagger \mathbf{v}}. \tag{4.19}$$

the solution $\mathbf{v}$ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{Q}^{-\dagger}\mathbf{A}\mathbf{Q}^{-1}$. And the optimum TEQ is obtained as

$$\mathbf{t} = \mathbf{Q}^{-1}\mathbf{v} \quad . \tag{4.20}$$

4.3 Per-tone equalization design (PTEQ)

In TEQ design of PTEQ method is a filterbank approach to the design of TEQ for maximizing the bit rate. The optimum solution for multiple TEQs is given in closed form and it can serve as a theoretical upper bound for all other TEQs. Let N_c and N_t be respectively the order of the channel $c(n)$ and the TEQ $t(n)$. The effect channel becomes $p(n) = c(n) * t(n)$. The receiving filters $H_k(z)$ in Fig. 4.1 are the DEF filters

$$H_k(z) = \sum_{i=L}^{M+L-1} e^{-\frac{j2\pi ki}{M}} z^i$$

where M is the DFT size and L is the cyclic prefix length. And the scalars P_k are given by

$$P_k = C(e^{\frac{j2\pi}{M}k}) T_k(e^{\frac{j2\pi}{M}k}). \tag{4.21}$$

The objectives of TEQ design is that the convolution $c(n) * t_k(n)$ will have most of its energy within a specific window of length L. Impulse responses outside the window will generate interblock ISI. Define the sequence

$$d_n = \begin{cases} 0, & n_w < n \le n_w + L, \\ 1, & 0 \le n \le n_w \ or\ n_w + L \le n \le N_c + N_t \end{cases} \tag{4.22}$$

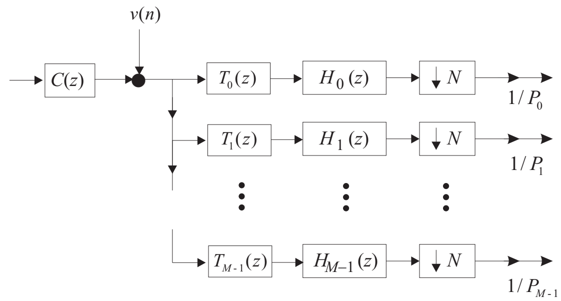


Figure 4.1: DMT receiver with multiple TEQs $T_k(z)$

where n_w is the starting location of the desired window. Then we can describe the ISI term of the kth tone as

$$p_{isi,k}(n) = d(n)(c(n) * t_k(n)). \tag{4.23}$$

From Fig. 4.1, we see that the output error at the kth tone is given by $e_k(n) = (e_{isi,k}(n) + e_{\nu,k}(n))_{\downarrow N}$, $N = M + L$, where

$$\begin{aligned} e_{isi,k}(n) &= h_k(n) * p_{isi,k}(n) * x(n)/P_k \\ e_{\nu,k}(n) &= h_k(n) * t_k(n) * \nu(n)/P_k. \end{aligned} \tag{4.24}$$

As the downsampler $(\bullet)_{\downarrow N}$ does not change the variance, we have

$$\sigma_{e_k}^2 = \sigma_{isi,k}^2 + \sigma_{\nu_k}^2$$

where we have assumed that the signal and noise are uncorrelated.

One can express the error variances using a matrix formulation. Define the vectors

$$\begin{aligned} \mathbf{t}_k &= \begin{pmatrix} t_k(0) & t_k(1) & \cdots & t_k(N_t) \end{pmatrix}^\top \\ \mathbf{w}_k &= \begin{pmatrix} 1 & e^{\frac{j2\pi}{M}k} & \cdots & e^{\frac{j2\pi}{M}kN_t} \end{pmatrix}^\top. \end{aligned}$$

Let $\mathbf{C}$ and $\mathbf{H}_k$ be respectively $(N_c + N_t + 1) \times (N_t + 1)$ and $(M + N_c + N_t) \times (N_c + N_t + 1)$ lower triangular Toeplitz matrices whose first columns are given by

$$\begin{pmatrix} c_0 & c_1 & \cdots & c_{N_c} & 0 & \cdots & 0 \end{pmatrix}^\top$$
$$\begin{pmatrix} e^{\frac{j2\pi}{M}k(M-1)} & \cdots & e^{\frac{j2\pi}{M}k} & 1 & 0 & \cdots & 0 \end{pmatrix}^\top.$$

Let $\mathbf{D}$ be an $(N_c+N_t+1)\times(N_c+N_t+1)$ diagonal matrix with entries $d_{ii}=d(i)$. Using the above definitions, the error variances can be rewritten as :

$$\begin{aligned} \sigma^2_{isi,k} &= \frac{\sigma_x^2 \mathbf{t}_k^\dagger \mathbf{C}_\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{D}\mathbf{C}\mathbf{t}_k}{|C(e^{j2\pi k/M})|^2 \mathbf{t}_k^\dagger \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{t}_k} \\ \sigma^2_{\nu,k} &= \frac{\mathbf{t}_k^\dagger \tilde{\mathbf{H}}_k^\dagger \mathbf{R}_\nu \tilde{\mathbf{H}}_k \mathbf{t}_k}{|C(e^{j2\pi k/M})|^2 \mathbf{t}_k^\dagger \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{t}_k} \end{aligned} \tag{4.25}$$

where R_ν is the $(M+N_t)\times(M+N_t)$ autocorrelation matrix of $\nu(n)$, and $\tilde{\mathbf{H}}_k$ is a lower triangular Toeplitz matrix having the same form as $\mathbf{H}_k$ but with dimensions of $(M+N_t)\times(N_t+1)$.

We have formulated the error variances due to ISI and channel noise. To simplify the notations, we define two $(N_t+1)\times(N_t+1)$ Hermitian matrices:

$$\mathbf{Q}_{isi,k} = \frac{\varepsilon_x \mathbf{C}_\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{D}\mathbf{C}}{|C(e^{j2\pi k/M})|^2}, \mathbf{Q}_{\nu,k} = \frac{\tilde{\mathbf{H}}_k^\dagger \mathbf{R}_\nu \tilde{\mathbf{H}}_k}{|C(e^{j2\pi k/M})|^2}.$$

Note that the matrix $\mathbf{Q}_{isi,k}$ is semi positive definite and the matrix $\mathbf{Q}_{\nu,k}$ is positive definite for all k. Moreover these matrices satisfy

$$\mathbf{Q}_{isi,M-k} = \mathbf{Q}^*_{isi,k}, \mathbf{Q}_{\nu,M-k} = \mathbf{Q}^*_{\nu,k} \tag{4.26}$$

for $k=1,\cdots,\frac{M}{2}-1$. And then the problem of each tone TEQ design cab be obtained by solving

$$\min_{\mathbf{t}_k} \frac{\mathbf{t}_k^\dagger(\mathbf{Q}_{isi,M-k}+\mathbf{Q}_{\nu,k})\mathbf{t}_k}{\mathbf{t}_k^\dagger \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{t}_k}. \tag{4.27}$$

Let $\mathbf{Q}_k^{1/2}$ be the unique positive definite matrix such that $\mathbf{Q}_k^{1/2}\mathbf{Q}_k^{\dagger/2} = (\mathbf{Q}_{isi,k} + \mathbf{Q}_{\nu,k})$. Then by letting $\mathbf{u}_k = \mathbf{Q}_k^{1/2}\mathbf{t}_k$, the problem becomes

$$\max_{\mathbf{u}_k} \frac{\mathbf{u}_k^\dagger \mathbf{Q}_k^{-1/2} \mathbf{w}_k \mathbf{w}_k^\dagger \mathbf{Q}_k^{-\dagger/2} \mathbf{u}_k}{\mathbf{u}_k^\dagger \mathbf{u}_k}. \tag{4.28}$$

The solution $\mathbf{u}_k$ is the eigenvector corresponding to the largest eigenvalue of the matrix $\mathbf{Q}_k^{-1/2}\mathbf{w}_k\mathbf{w}_k^\dagger\mathbf{Q}_k^{-\dagger/2}$. And the each tone optimum TEQ is obtained as

$$\mathbf{t}_k = \mathbf{Q}_k^{-1/2}\mathbf{u}_k \quad . \tag{4.29}$$

4.4 Frequency Domain TEQ Design

Let us consider the filterbank representation of the DMT system shown in Fig. 2.2(c). We can lump together the channel, TEQ and synchronization delay in Fig.2.2(c) and let $H(z) = C_1(z)T(z)z^d$. Assume the length of $C(z)T(z)$ is at most N, then we can write $H(z)$ as

$$H(z) = \sum_{r=-d}^{N-1-d} h_r \cdot z^{-r}.$$

Using the polyphase identity, we know the system from the input symbol $s_k(n)$ to the output symbol $x_i(n)$ is an LTI system. Let the transfer function be $P_{ik}(z)$, then $P_{ik}(z)$ is given by

$$P_{ik}(z) = (F_k(z)R_i(z)H(z))_{\downarrow N} \quad . \tag{4.30}$$

Therefore the system from the input vector $\mathbf{s}(n)$ to output vector $\mathbf{x}(n)$ is an $M \times M$ LTI system $\mathbf{P}(z)$ with $[\mathbf{P}(z)]_{ik} = P_{ik}(z)$. As the length of $H(z)$ is N, then the transfer matrix $\mathbf{P}(z)$ has at most three coefficient matrices. It can be written as

$$\mathbf{P}(z) = \mathbf{P}(-1)z + \mathbf{P}(0) + \mathbf{P}(1)z^{-1}. \tag{4.31}$$

We can write $\mathbf{x}(n)$ as

$$\mathbf{x}(n) = \mathbf{P}(-1)\mathbf{s}(n+1) + \mathbf{P}(0)\mathbf{s}(n) + \mathbf{P}(1)\mathbf{s}(n-1). \tag{4.32}$$

We can see that the elements in $\mathbf{P}(-1)$ represents the interference from the next block $\mathbf{s}(n+1)$; Similarly, $\mathbf{P}(-1)$ represents the interference from the next block $\mathbf{s}(n-1)$; $p_{ik}(1)$ corresponds to the interference from k-th tone to i-th tone of the previous block. On the other hand, the off-diagonal elements of $\mathbf{P}(0)$ represents the interference from the other tones of the same block $\mathbf{s}(n)$. Furthermore, the k-th diagonal element of $\mathbf{P}(0)$ represents the k-th subchannel gain. The diagonal elements of the matrix $\mathbf{P}(0)$ are the M-point DFT of the channel coefficients $\{h_0, h_1, \cdots, h_L\}$.

When the channel length is shorter than the cyclic prefix added at transmitter, there is no interference after removing cyclic prefix. The outputs of DFT matrix at the receiver are the same as the inputs of the transmitter except some scalars, which are the M-pt DFT of the channel. Therefore we consider the interference of a selected set of tones to a chosen set of tones. As the coefficients of $\mathbf{P}(0)$ (off-diagonal), $\mathbf{P}(1)$ and $\mathbf{P}(-1)$ represent interference among tones. Minimizing these coefficients will minimize interference. We will consider interference from a chosen set of tones, source tones, O_s to another set of tones, O_t, target tones. An objective function for this is

$$\phi = \sum_{i \in O_t} \sum_{k \in O_s} (|p_{ik}(0)|^2 + |p_{ik}(-1)|^2 + |p_{ik}(1)|^2). \tag{4.33}$$

To minimize such an objective function, we note that the coefficients of $H(z)$ can be expressed in a matrix form $\mathbf{Ct}$ where $\mathbf{t}$ a is $T \times 1$ column vector consisting of the TEQ coefficients and $\mathbf{C}$ is an $N \times T$ convolution matrix as follow

$$\mathbf{C} = \begin{pmatrix} c_1(0) & 0 & \cdots & 0 \\ c_1(1) & c_1(0) & & \vdots \\ & & \ddots & 0 \\ \vdots & & & c_1(0) \\ c_1(N-T) & & & \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & 0 & c_1(N-T) \end{pmatrix}, \tag{4.34}$$

where $c_1(i)$ is the discrete channel coefficients $c_1(n)$. Then the elements of $\mathbf{P}(-1), \mathbf{P}(0)$ and $\mathbf{P}(1)$ can be expressed in terms of TEQ coefficients as

$$p_{ik}(0) = \mathbf{a}_{ik}^{\dagger}\mathbf{Ct}, \quad p_{ik}(-1) = \mathbf{b}_{ik}^{\dagger}\mathbf{Ct}, \quad p_{ik}(1) = \mathbf{e}_{ik}^{\dagger}\mathbf{Ct}. \tag{4.35}$$

Using (4.35),$|p_{ik}(0)|^2$ can be written as

$$|p_{ik}(0)|^2 = \mathbf{t}^{\dagger}\mathbf{C}^{\dagger}\mathbf{a}_{ik}\mathbf{a}_{ik}^{\dagger}\mathbf{Ct}. \tag{4.36}$$

Similarly, we can also express $|p_{ik}(-1)|^2$ and $|p_{ik}(0)|^2$ as

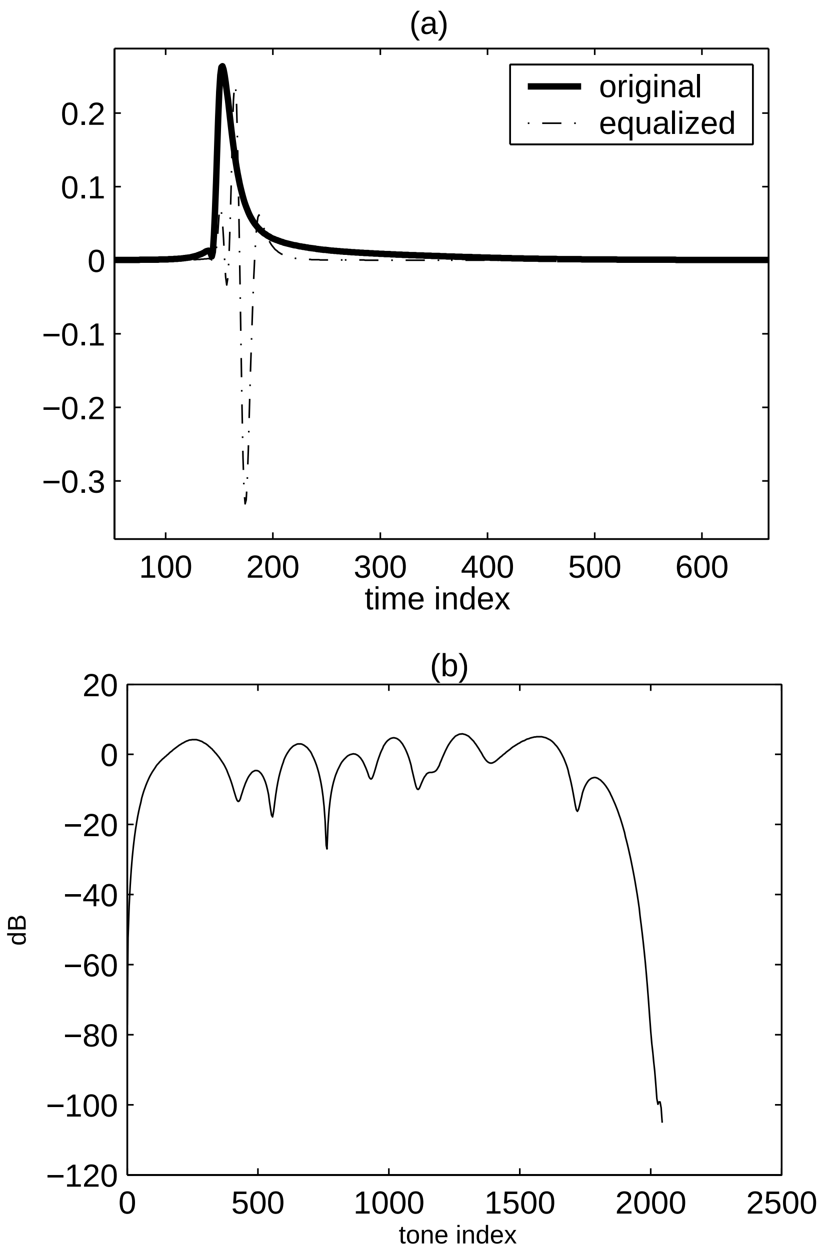
$$|p_{ik}(-1)|^2 = \mathbf{t}^{\dagger}\mathbf{C}^{\dagger}\mathbf{b}_{ik}\mathbf{b}_{ik}^{\dagger}\mathbf{Ct}, \quad |p_{ik}(1)|^2 = \mathbf{t}^{\dagger}\mathbf{C}^{\dagger}\mathbf{e}_{ik}\mathbf{e}_{ik}^{\dagger}\mathbf{Ct}. \tag{4.37}$$

The derivations of $\mathbf{a}_{ik}$, $\mathbf{b}_{ik}$, $\mathbf{e}_{ik}$ will be given in Appendix A. Using the expression of $p_{ik}(0)$, $p_{ik}(1)$, $p_{ik}(-1)$ in (4.36) and (4.37), we have $\phi = \mathbf{t}^{\dagger}\mathbf{St}$, where

$$\mathbf{S} = \mathbf{C}^{\dagger}(\sum_{i \in O_t} \sum_{k \in O_s} (\mathbf{a}_{ik}\mathbf{a}_{ik}^{\dagger} + \mathbf{b}_{ik}\mathbf{b}_{ik}^{\dagger} + \mathbf{e}_{ik}\mathbf{e}_{ik}^{\dagger}))\mathbf{C}. \tag{4.38}$$

Note that ϕ is a quadratic form of the TEQ $\mathbf{t}$. Then the optimum $\mathbf{t}$ that minimizes ϕ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{S}$.

We take the VDSL loop7 for example. We use DFT size M is 4096, CP length L is 320 and TEQ taps is 40. The downstream is transmission band. We choose target tone at $\{2000\}$ and source tone at $\{2000-2045\}$ for TEQ design. Fig. 4.2(a) shows the original channel and equalized channel. The SIR of original channel and equalized channel are 35.7dB and 70.8dB. Fig. 4.2(b) shows the magnitude response of TEQ. The zeros of TEQ response is corresponding to the sets, O_t and O_s. Fig. 4.2(c) shows the interference without TEQ and with TEQ.


(a)
original
equalized
0.2
0.1
0
−0.1
−0.2
−0.3
100
200
300
400
500
600
time index
(b)
20
0
−20
−40
−60
−80
−100
−120
dB
0
500
1000
1500
2000
2500
tone index

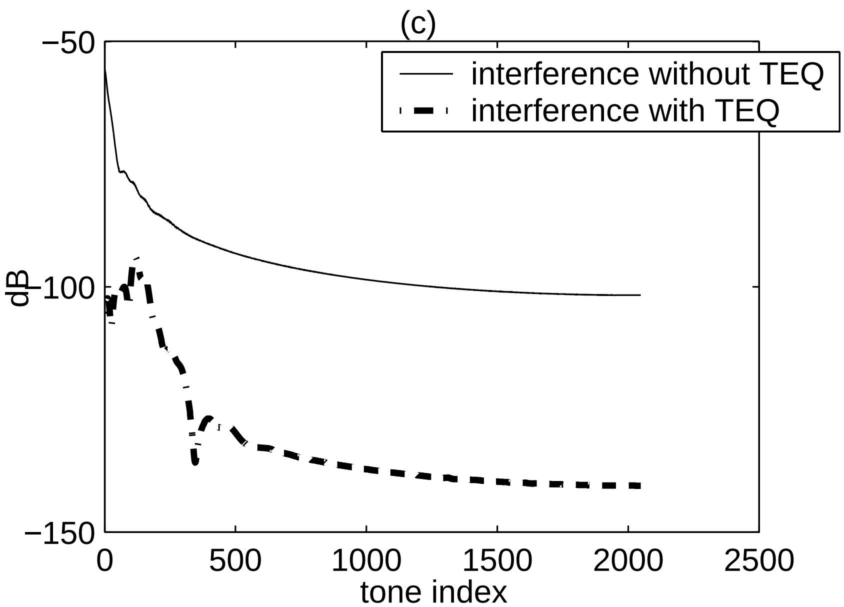


Figure 4.2: TEQ design on $O_t \in \{2000\}, O_s \in \{2000 - 2045\}$. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ, (c) interference with TEQ and without TEQ

Chapter 5

Proposed TEQ Design with oversampling

In this chapter, we will exploit to increase the sampling rate at the receiver in DMT system to design TEQ. We know that the TEQ design without oversampling (Frequency Domain TEQ Design) shown in Section 4.4 has good performance and properties. We can directly control the zeros of TEQ response by choosing target and source tones and shorten the equivalent channel effectively to improve transmission rate. In order to increase freedom for our TEQ design, we increase the sampling rate at receiver in DMT system. For example, if the relation between the TEQ response $T_1(z)$ which is designed by the method without oversampling, and the TEQ response $T_2(z)$, designed by the method with oversampling which oversampling factor is 2, was $T_2(z) = T_1(z^2)$, the $T_1(z)$ is just one choice of $T_2(z)$ shown in Fig. 5.1. To have more choices would get better result possibly.

The equivalent filterbank representation of the DMT system with oversam-

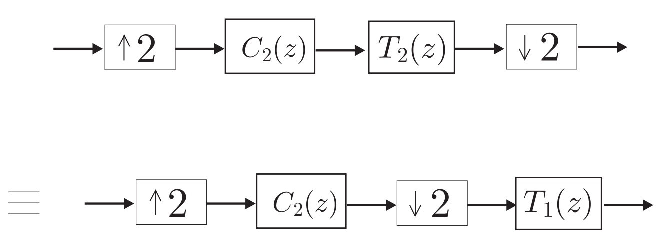


Figure 5.1: Illustration of freedom of proposed TEQ design with oversampling

pling is shown Fig. 3.2(b). In the following derivation, we will consider the case $Q = 2$. The case for more general Q can be obtained in a similar manner. We lump together the channel $C_2(z)$, TEQ $T(z)$, and synchronization delay and call it $H(z)$; $H(z) = C_2(z)T(z)$. Assume the length of $H(z)$ is at most $2N$. Notice that Using the polyphase identity, the system from $u(n)$ to $v(n)$ in Fig. 3.2 is an LTI system with impulse response given by $\psi(n) = h(2n + d)$.

$$\Psi(z) = h_0 z^{\frac{\tilde{d}}{2}} + h_2 z^{\frac{\tilde{d}}{2}-1} + \cdots + h_{2(N-1)} z^{\frac{\tilde{d}}{2}-(N-1)}. \tag{5.1}$$

where $\tilde{d} = d-(d\; mod\; 2)$. Again using the polyphase identity, we know the system from the input symbol $s_k(n)$ to the output symbol $x_i(n)$ is an LTI system. Let the transfer function be $P_{ik}(z)$, then $P_{ik}(z)$ is given by

$$P_{ik}(z) = (F_k(z)R_i(z)\Psi(z))_{\downarrow N} \quad . \tag{5.2}$$

Then the system from the input vector $\mathbf{s}(n)$ to output vector $\mathbf{x}(n)$ is a $M \times M$ LTI system $\mathbf{P}(z)$ with $[\mathbf{P}(z)]_{ik} = P_{ik}(z)$. The receiver output vector $\mathbf{x}(n)$ is related to the transmitter input vector $\mathbf{s}(n)$ by

$$\mathbf{P}(z) = \mathbf{P}(-1)z + \mathbf{P}(0) + \mathbf{P}(1)z^{-1}. \tag{5.3}$$

Using above expression, we can write $\mathbf{x}(n)$ in terms of $\mathbf{s}(n)$ as

$$\begin{aligned}\mathbf{x}(n) = &\underbrace{\begin{bmatrix} a_{00} & \cdots & a_{0,M-1} \\ a_{10} & \cdots & a_{1,M-1} \\ \vdots & \ddots & \vdots \\ a_{M-1,0} & \cdots & a_{M-1,M-1} \end{bmatrix}}_{\mathbf{P}(0)} \mathbf{s}(n) + \underbrace{\begin{bmatrix} b_{00} & \cdots & b_{0,M-1} \\ b_{10} & \cdots & b_{1,M-1} \\ \vdots & \ddots & \vdots \\ b_{M-1,0} & \cdots & b_{M-1,M-1} \end{bmatrix}}_{\mathbf{P}(1)} \mathbf{s}(n-1) \\ &+ \underbrace{\begin{bmatrix} e_{00} & \cdots & e_{0,M-1} \\ e_{10} & \cdots & e_{1,M-1} \\ \vdots & \ddots & \vdots \\ e_{M-1,0} & \cdots & e_{M-1,M-1} \end{bmatrix}}_{\mathbf{P}(-1)} \mathbf{s}(n+1),\end{aligned} \tag{5.4}$$

or to simplify as

$$\mathbf{x}(n) = \mathbf{P}(0)\mathbf{s}(n) + \mathbf{P}(1)\mathbf{s}(n-1) + \mathbf{P}(-1)\mathbf{s}(n+1). \tag{5.5}$$

We can see that the elements in $\mathbf{P}(-1)$ represents the interference from the next block $\mathbf{s}(n+1)$; $p_{ik}(-1)$ represents the interference from k-th tone to i-th tone of respecting the next block. Similarly, $\mathbf{P}(1)$ represents the interference from the previous block $\mathbf{s}(n-1)$; $p_{ik}(1)$ corresponds to the interference from k-th tone to i-th tone of the previous block. On the other hand, the off-diagonal elements of $\mathbf{P}(0)$ represents the interference from the other tones of the same block $\mathbf{s}(n)$. Furthermore, the k-th diagonal element of $\mathbf{P}(0)$ represents the k-th subchannel gain. The diagonal elements of the matrix $\mathbf{P}(0)$ are the M-point DFT of the channel coefficients $\{\psi_0, \psi_1, \cdots, \psi_L\}$.

We consider the interference of a selected set of tones to a chosen set of tones. As the coefficients of $\mathbf{P}(0)$, $\mathbf{P}(1)$ and $\mathbf{P}(-1)$ represent interference among tones. Minimizing these coefficients will minimize interference. We will consider interference from a chosen set of tones, source tones, O_s to another set of tones, O_t, target tones. An objective function for this is

$$\phi = \sum_{i \in O_t} \sum_{k \in O_s} (|p_{ik}(0)|^2 + |p_{ik}(-1)|^2 + |p_{ik}(1)|^2). \tag{5.6}$$

To minimize such an objective function, we note that the coefficients of $\Psi(z)$ can be expressed in a matrix form $\mathbf{C}'\mathbf{t}$ where $\mathbf{t}$ is the $T \times 1$ TEQ vector and $\mathbf{C}'$ is an $2N \times T$ Toeplitz matrix with the first column given by

$$\begin{pmatrix} c_2(0) & c_2(1) & \cdots & c_2(2N-T) & 0 & \cdots & 0 \end{pmatrix}^\top,$$

where $c_2(i)$ are the coefficient of channel $c_2(n)$. Let $\mathbf{C}$ be the $N \times T$ submatrix of $\mathbf{C}'$ obtained by keeping only the even rows of $\mathbf{C}'$ shown as follow

$$\mathbf{C} = \begin{pmatrix} c_2(0) & 0 & 0 & \cdots & 0 \\ c_2(2) & c_2(1) & c_2(0) & & \vdots \\ & & & \ddots & 0 \\ \vdots & \vdots & & & c_2(0) \\ c_2(2N-T) & & & & \\ 0 & & & \ddots & \vdots \\ \vdots & & & \ddots & \\ 0 & 0 & \cdots & 0 & c_2(2N-T) \end{pmatrix}_{N \times T}, \tag{5.7}$$

We can express the coefficients of $\Psi(z)$ as $\psi = \mathbf{C}_2\mathbf{t}$. Then the elements of $\mathbf{P}(-1), \mathbf{P}(0)$ *and* $\mathbf{P}(1)$, can be expressed in terms of TEQ coefficients,

$$p_{ik}(0) = \mathbf{a}_{ik}^{\dagger}\mathbf{C}\mathbf{t}, \quad p_{ik}(-1) = \mathbf{b}_{ik}^{\dagger}\mathbf{C}\mathbf{t}, \quad p_{ik}(1) = \mathbf{e}_{ik}^{\dagger}\mathbf{C}\mathbf{t}. \tag{5.8}$$

Using (5.8), $|p_{ik}(0)|^2$ can be written in terms of TEQ coefficients as

$$|p_{ik}(0)|^2 = \mathbf{t}^{\dagger}\mathbf{C}^{\dagger}\mathbf{a}_{ik}\mathbf{a}_{ik}^{\dagger}\mathbf{C}\mathbf{t}. \tag{5.9}$$

We also can express $|p_{ik}(-1)|^2$ and $|p_{ik}(0)|^2$ in the form of (5.9),

$$|p_{ik}(-1)|^2 = \mathbf{t}^{\dagger}\mathbf{C}^{\dagger}\mathbf{b}_{ik}\mathbf{b}_{ik}^{\dagger}\mathbf{C}\mathbf{t}, \quad |p_{ik}(1)|^2 = \mathbf{t}^{\dagger}\mathbf{C}^{\dagger}\mathbf{e}_{ik}\mathbf{e}_{ik}^{\dagger}\mathbf{C}\mathbf{t}. \tag{5.10}$$

Let S be following matrix,

$$\mathbf{S} = \mathbf{C}^{\dagger}(\sum_{i\in o_t}\sum_{k\in o_s}(\mathbf{a}_{ik}\mathbf{a}_{ik}^{\dagger} + \mathbf{b}_{ik}\mathbf{b}_{ik}^{\dagger} + \mathbf{e}_{ik}\mathbf{e}_{ik}^{\dagger}))\mathbf{C}. \tag{5.11}$$

Then the objective function (5.6) becomes

$$\phi = \mathbf{t}^{\dagger}\mathbf{S}\mathbf{t}, \tag{5.12}$$

which is a quadratic form of $\mathbf{t}$. Then the optimum TEQ $\mathbf{t}$ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{S}$.

Choice of target and source tones. From the next examples, we would observe the zeros of our proposed TEQ response are corresponding to the target and source tones which are chosen. We would choose the target and source tones at null tones, unused tones, to let the transmission bands free from the zeros. The zeros located at transmission bands would reduce transmission rate. Therefore, the transmission bands free from the zeros would improve the transmission rate. In our proposed TEQ design, we can directly control the zeros of TEQ response by choosing target and source tones.

Incorporation of additional frequency criterion. Under some conditions, the magnitude response of equalized channel at transmission band would be too low due to TEQ response and the bit rate becomes worse. Then we modify our objective function by considering to minimize TEQ response at null tones.

We can also explicitly include the frequency domain criterion in the design. The frequency bands that are not used for transmission can be exploited and the frequency response of the TEQ can be further shaped to improve transmission rate. One way to incorporate frequency domain criterion is to minimize the frequency response of the TEQ on null tones,

$$\phi_f = \sum_{k \in O_n} |T(e^{j\frac{2\pi}{M}k})|^2 \tag{5.13}$$

where O_n denotes the set of unused tones. Let $\mathbf{W}_1$ be a submatrix of the $M \times M$ DFT matrix obtain by keeping the first T column. The k-th DFT coefficient of $T(z)$ can be expressed as $\mathbf{w}_k \mathbf{t}$, where $\mathbf{w}_k$ is the k-th row of $\mathbf{W}_1$. Stack all the $\mathbf{w}_k$ with $k \in O_n$ together and call the resulting matrix $|O_n| \times T$ $\mathbf{W}_{O_n}$, where $|O_n|$ denotes the number of elements in the set O_n. Then ϕ_f can be written in terms of $\mathbf{t}$ as

$$\phi_f = \mathbf{t}^\dagger \mathbf{W}_{O_n}^\dagger \mathbf{W}_{O_n} \mathbf{t}. \tag{5.14}$$

An objective function with frequency weighting is

$$\phi' = (1-\alpha)\phi + \alpha\phi_f. \tag{5.15}$$

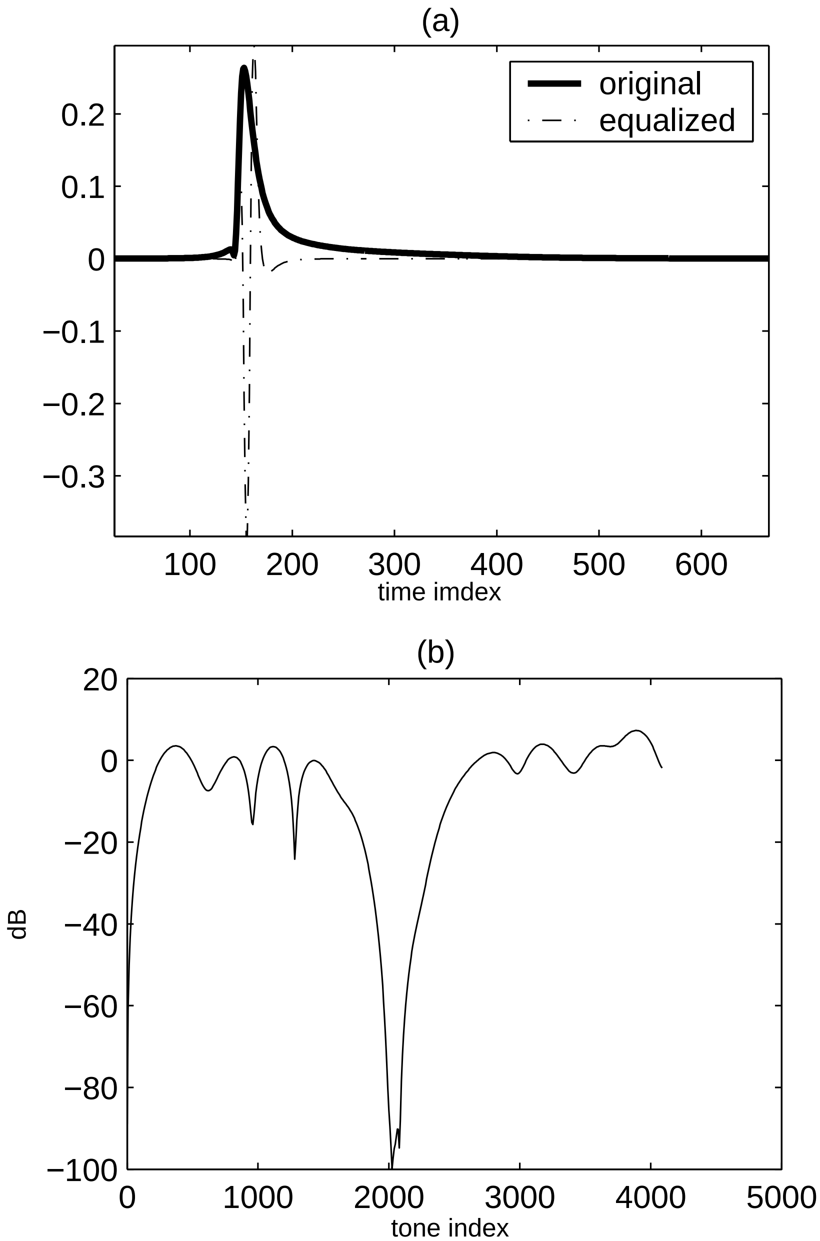
The objective function ϕ' can be written as

$$\phi' = \mathbf{t}^\dagger \underbrace{\left[(1-\alpha)\cdot \mathbf{S} + \alpha \cdot \mathbf{W}_{O_n}^\dagger \mathbf{W}_{O_n}\right]}_{\mathbf{Z}} \mathbf{t}, \tag{5.16}$$

which is also a quadratic form of $\mathbf{t}$. The optimum TEQ $\mathbf{t}$ is the eigenvector corresponding to the smallest eigenvalue of the matrix $\mathbf{Z}$.

The order of the elements of the matrix $\mathbf{W}_{O_n}^\dagger \mathbf{W}_{O_n}$ is around 10^{-12} and that of the matrix $\mathbf{S}$ is around 10. In order to let the weighting constant α be consistent value, we can normalized the matrix $\mathbf{W}_{O_n}^\dagger \mathbf{W}_{O_n}$ with value 10^{-12} and the matrix $\mathbf{S}$ with value 10. After normalization, the weighting value α will be consistent value which is between 0 and 1.

We take the $Q = 2$ case for example. In simulations, M is 4096, L is 320 and the TEQ taps is 40. The downstream band and VDSL loop7 channel are considered. The channel consists of additive Gaussian noise and crosstalk noise including FEXT and NEXT noise. Fig. 5.2(a) shows the original and equalized channel. We minimize the interference of target tone at $\{2000\}$ from the source tones at $\{2000-2045\}$. The SIR of original and equalized channel are 35.7dB and 75.0dB. The TEQ design with $Q = 2$ for just minimizing interference of one tone from other tones shorten the channel effectively. Fig. 5.2(b) shows the magnitude response of TEQ. The zeros of TEQ response are corresponding to the target and source tones. Fig. 5.2(c) shows the interference without TEQ and with TEQ. The interference with proposed TEQ for $Q = 2$ case is more smaller than that caused by original channel. We minimize the objective function ϕ by choosing the O_t at $\{2000 - 2045\}$ and O_s at $\{950 - 1000\}$. The original and equalized channel are shown in Fig. 5.3(a) and the SIR of equalized channel is 73.7dB. Good shortening for minimizing interference of the set O_t from the set O_s can be achieved. The magnitude response of TEQ is shown in Fig. 5.3(b). The zeros of TEQ response locate at the sets, O_t and O_s which be chosen. Fig. 5.4 shows the TEQ design considering the frequency criterion. The equalized channel for choosing O_t at $\{2000\}$, O_s at $\{2000 - 2045\}$, O_n at $\{2740 - 3140\}$ and $\alpha = 8 \cdot 10^{-16}$ are shown in Fig. 5.4(a) and SIR is 72.8dB. TEQ design for minimizing interference and considering the frequency criterion shorten the channel effectively. Fig. 5.4(b) shows the magnitude response of TEQ. The zeros of TEQ response are also corresponding to the sets, O_t, O_s and O_n. From the above simulations, TEQ design with oversampling keeps the advantage of the method in Section 3.4. It also shorten the channel effectively and can directly control the zeros of TEQ response by choosing the sets, O_t, O_s and O_n. TEQ design with oversampling have more freedom than method without oversampling. From Fig. 4.2(c) and Fig. 5.2(c) and the measure of SIR, we know that TEQ design with oversampling much better shortening than method without oversampling. And the TEQ design method without oversampling just one special case of method with oversampling for $Q = 1$ case. The performance of transmission rate will be shown in next chapter.



(a)
original
equalized
0.2
0.1
0
−0.1
−0.2
−0.3
100
200
300
400
500
600
time imdex
(b)
20
0
−20
−40
−60
−80
−100
dB
0
1000
2000
3000
4000
5000
tone index

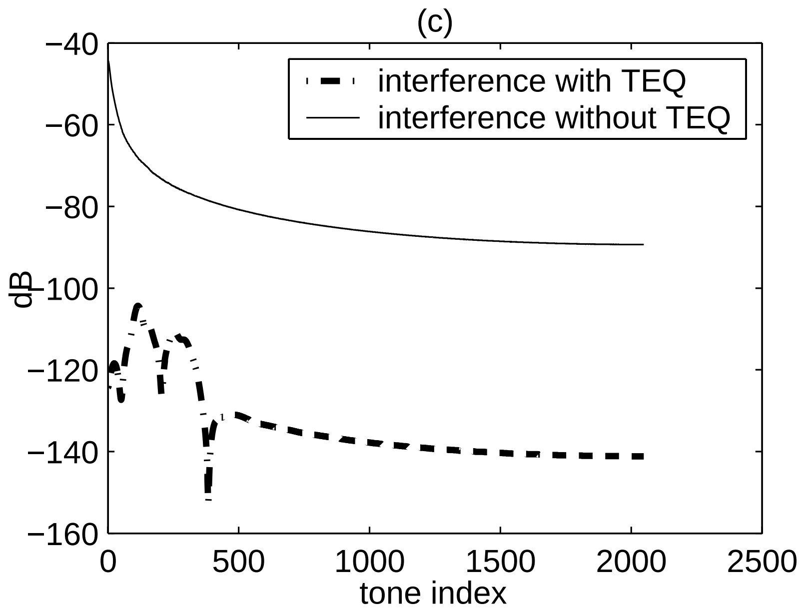
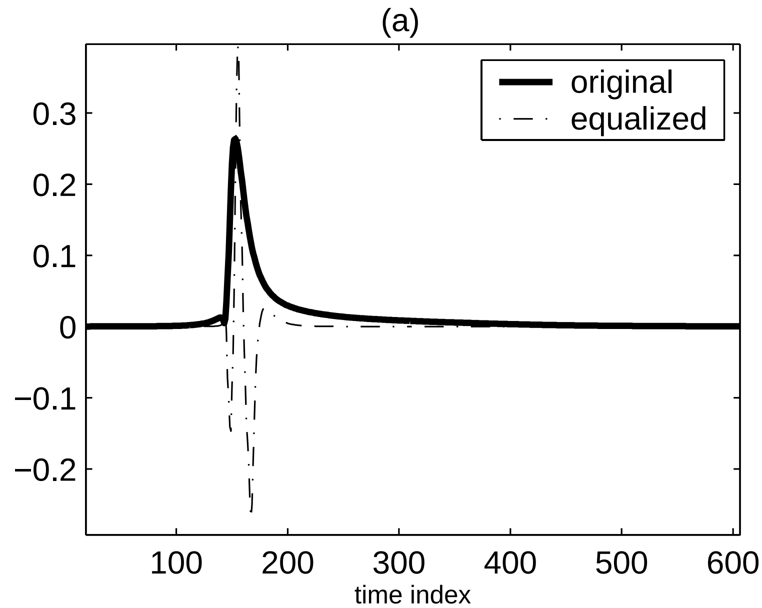


Figure 5.2: TEQ design on $O_t \in \{2000\}, O_s \in \{2000 - 2045\}$. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ, (c) interference with TEQ and without TEQ



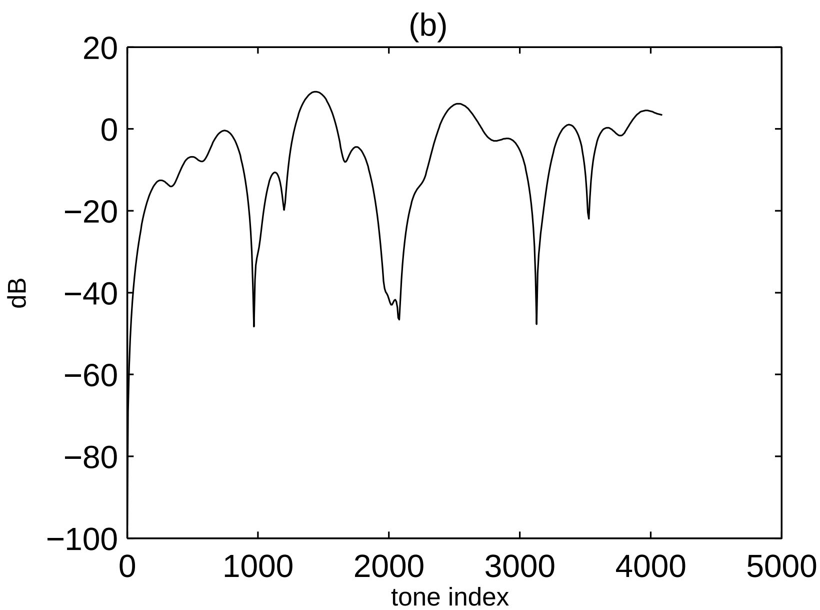


Figure 5.3: TEQ design on $O_t \in \{2000 - 2045\}, O_s \in \{950 - 1000\}$. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ.

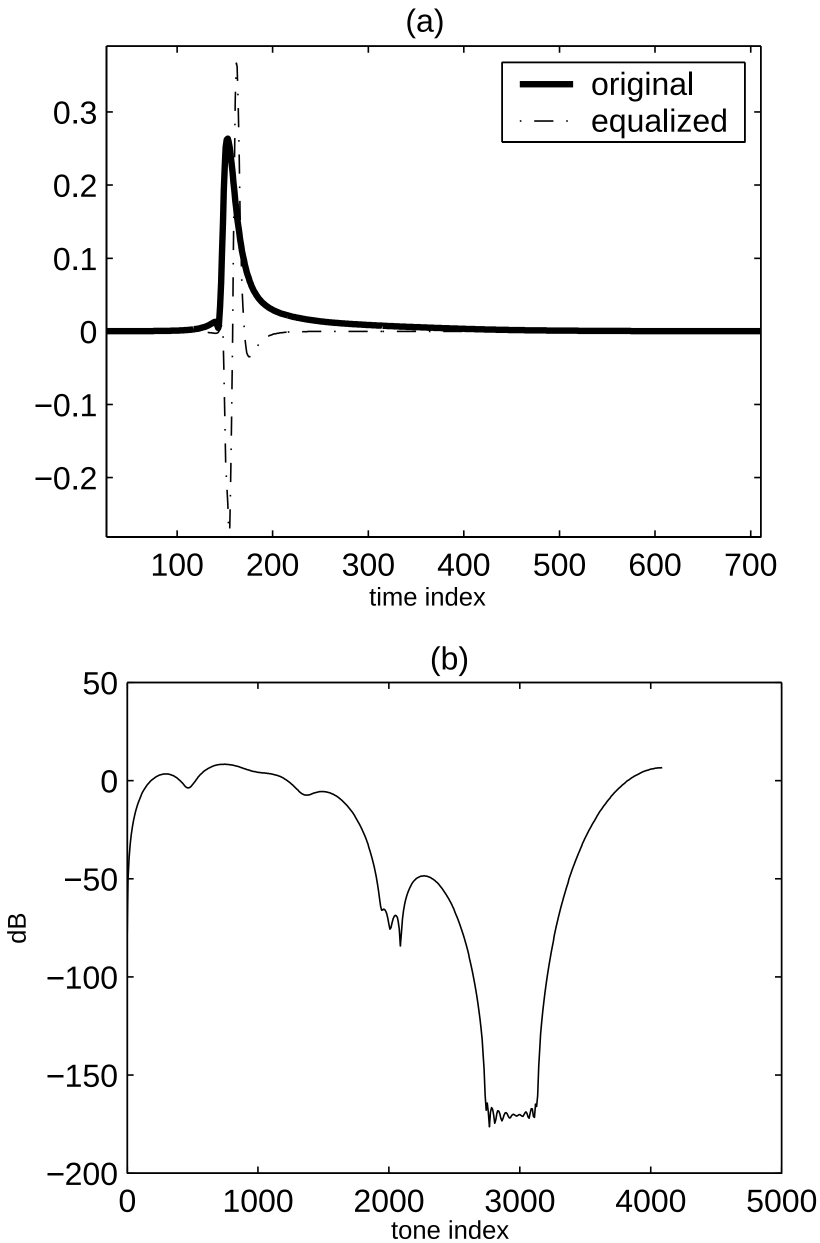


Figure 5.4: TEQ design on $O_t \in \{2000\}, O_s \in \{2000-2045\}, O_N \in \{2740-3140\}$. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ.

Chapter 6

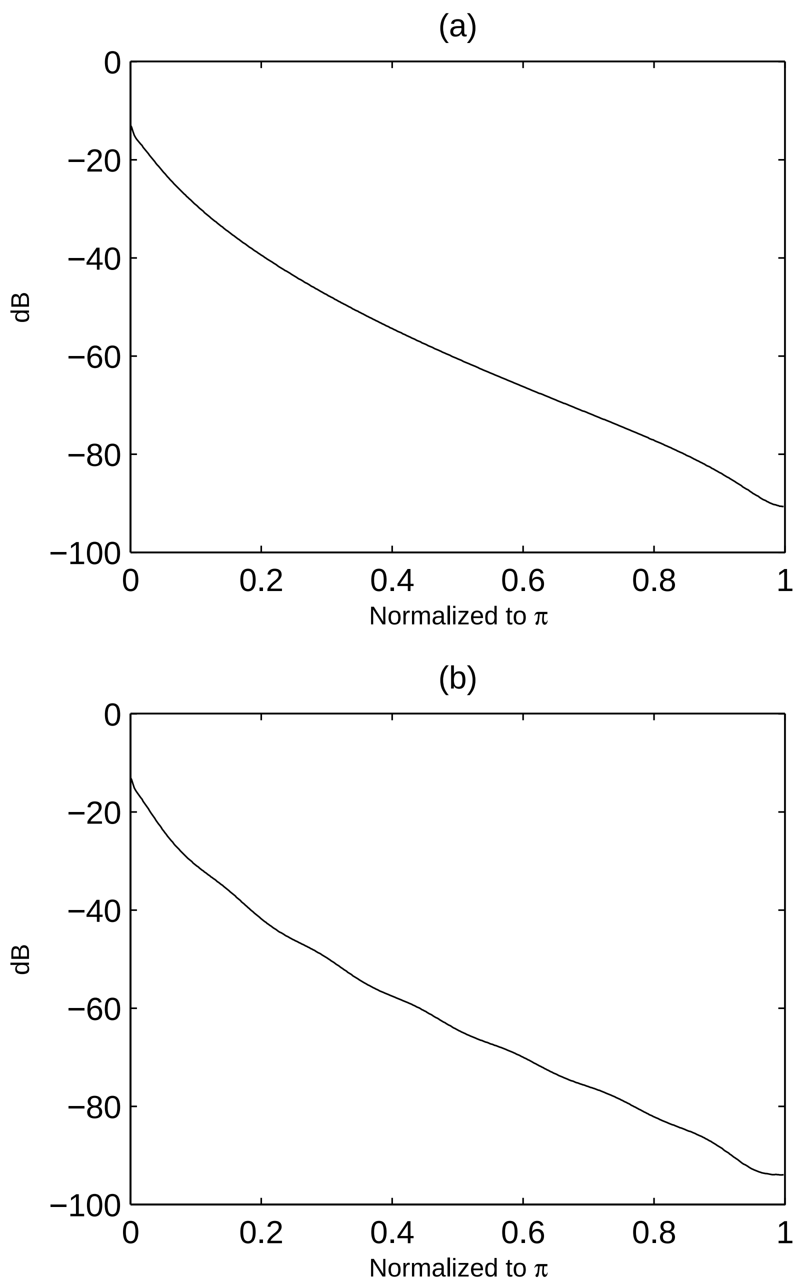
Numerical Simulation

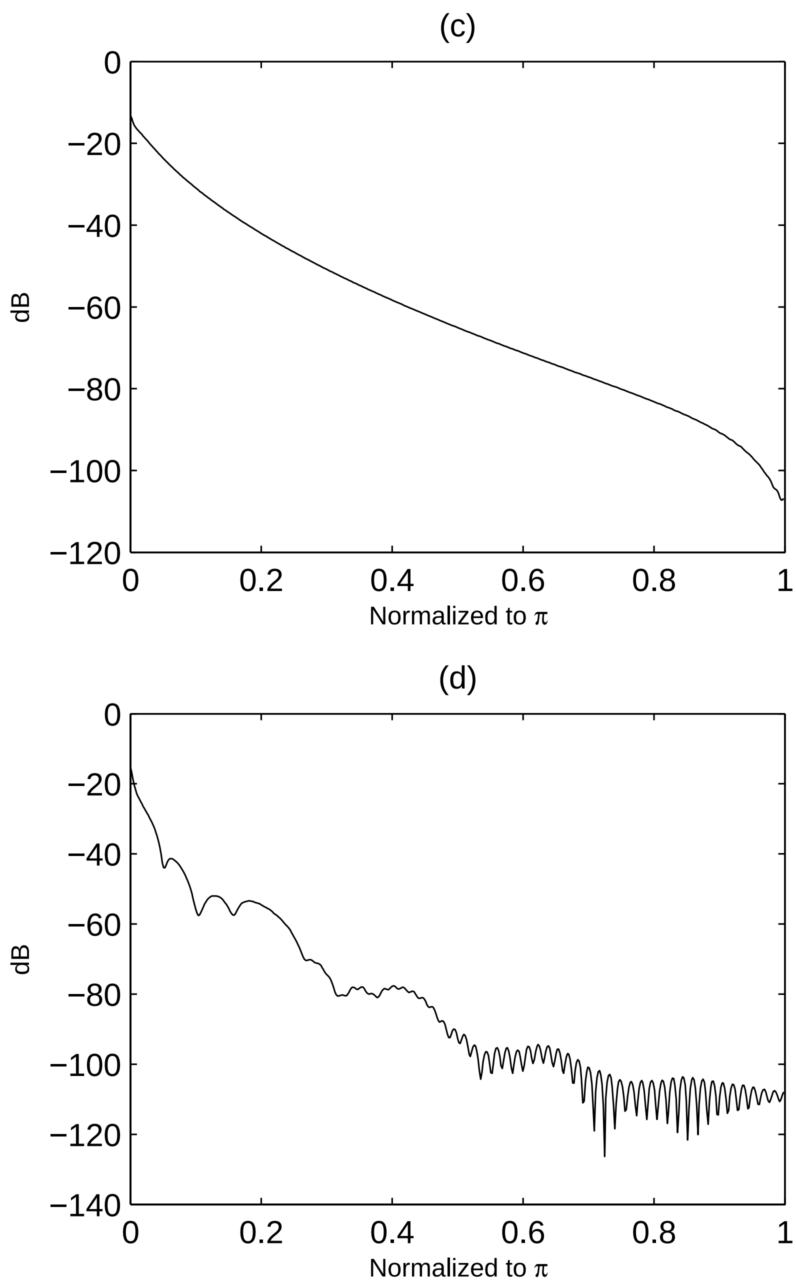
6.1 Simulation Environment

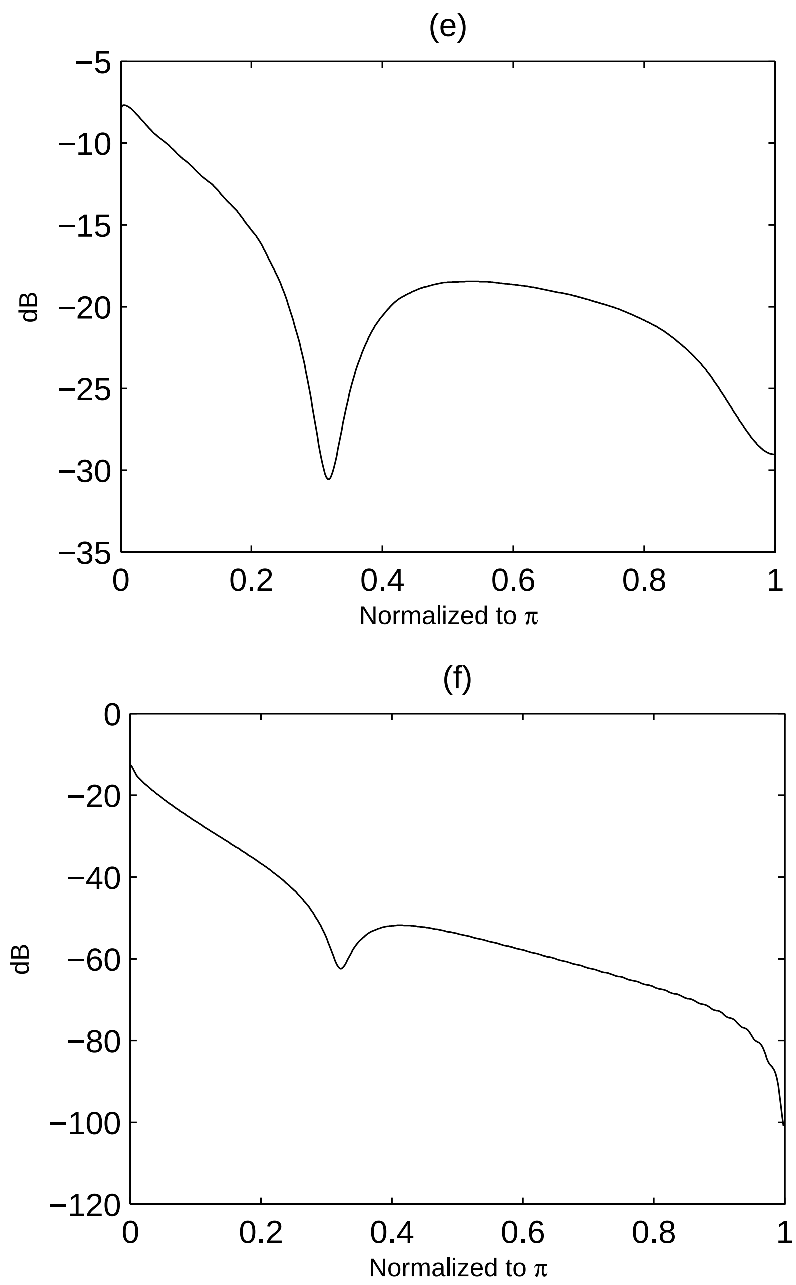
We will use VDSL for our simulations. The DFT size M is 4096 and the length of cyclic prefix L is 320. The TEQ has 40 taps and the sampling frequency $f_s = \frac{1}{T_s}$ is 17.664MHz. We consider downstream transmission and a 4-QAM modulation is used for the DMT symbol. The tones that can be used for downstream transmission are $\{33 - 870\}$ and $\{1206 - 1970\}$; the rest of the tones are not used and on thess tones zeros are sent. The channel noise consists of additive white Gaussian noise (-170dBm/Hz) and crosstalk noise, including far-end crosstalk (FEXT) and near-end crosstalk (NEXT) coming from 20 VDSL disturbers. There are seven types of the VDSL test loops in our simulations listed in Table 6.1 and their frequency response are shown in Fig. 6.1.

Loop	Length(feet)
VDSL1L	4500
VDSL2L	4500
VDSL3L	4500
VDSL4L	4500
VDSL5	950
VDSL6	3250
VDSL7	4900

Table 6.1: VDSL test loop length


(a)
0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π
(b)
0
−20
−40
−60
−80
−100
dB
0
0.2
0.4
0.6
0.8
1
Normalized to π


(c)
dB
0
−20
−40
−60
−80
−100
−120
0 0.2 0.4 0.6 0.8 1
Normalized to π
(d)
dB
0
−20
−40
−60
−80
−100
−120
−140
0 0.2 0.4 0.6 0.8 1
Normalized to π


(e)
dB
−5
−10
−15
−20
−25
−30
−35
0
0.2
0.4
0.6
0.8
1
Normalized to π
(f)
dB
0
−20
−40
−60
−80
−100
−120
0
0.2
0.4
0.6
0.8
1
Normalized to π

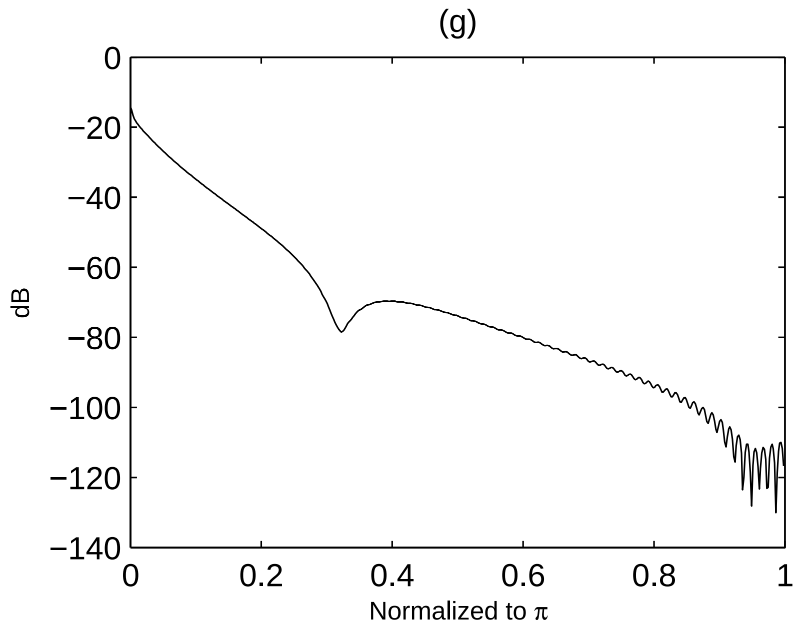


Figure 6.1: Frequency response of 7 VDSL test loops. (a) VDSL 1L, (b) VDSL 2L, (c) VDSL 3L, (d) VDSL 4L, (e) VDSL 5, (f) VDSL 6, (g) VDSL 7.

6.2 Performance Measure

The number of bits achieved for i-th tone is :

$$b_i = \lfloor log_2(1 + \frac{SNR_i}{\Gamma}) \rfloor$$

Γ represents the gap corresponding to the symbol error rate. In our simulations, $P_e = 10^{-5}$ and $\Gamma = 4.7863$ are considered. SNR_i is the signal to interference and noise ratio of i-th tone. In VDSL specification, the max number of bits on each tone is 15 and the transmission rate is

$$\frac{1}{NT_s} \sum_{i=0}^{\frac{M}{2}-1} b_i$$

where $M = 4096$, $N = M + L = 4416$, and $f_s = \frac{1}{T_s} = 17.664$MHz.

The measure of SIR shows the channel shortening effect. The measure of SIR is defined as

$$SIR = \max_d \frac{\sum_{i=d}^{d+L} |h_i|^2}{\sum_{i=0, i \neq (d,\cdots,d+L)}^{L_{ch}} |h_i|^2}$$

where d is synchronization delay, L_{ch} is the length of equalized channel.

6.3 Simulation Results

We use the VDSL7 test loops for simulations. In this section, we will show four examples for using proposed TEQ design. We choose the target tones from the tones that are not used for downstream transmission, tones with indexes ≥ 2000 and source tones at $\{950 - 1000\}$. In order to reduce the simulation complexity, we choose target tones at $\{2000, 2004, \cdots, 2044\}$ with tone decimation by 4.

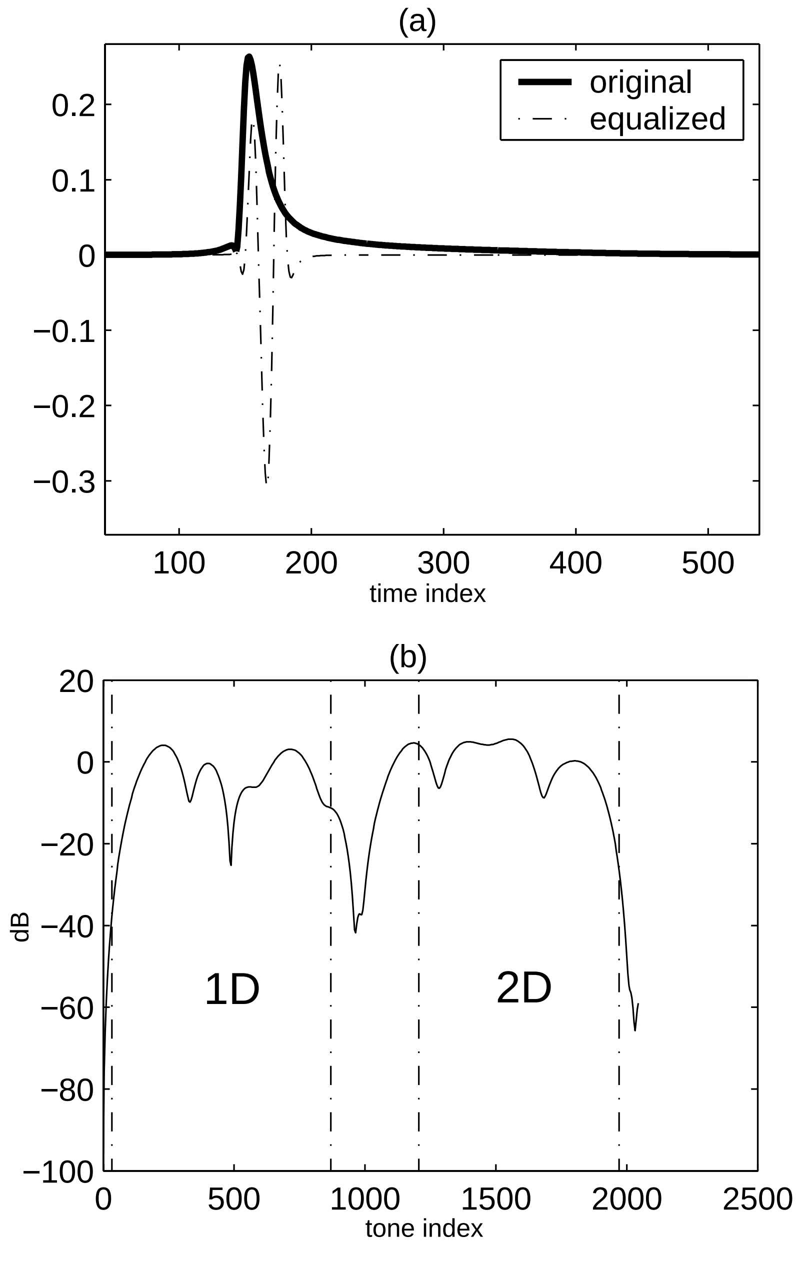
Example 1 (Q=1) Fig. 6.2(a) shows the original channel and the equalized channel. The SIR of the original channel and equalized channel are 35.7 dB and 74 dB. Fig. 6.2(b) shows the magnitude response of TEQ. We can see that the TEQ is more attenuated in the frequency bands correspondind to the target and source tones. The magnitude response of equalized channel and original channel is shown in Fig. 6.2(c). Fig. 6.2(d) shows the bit allocation after channel shortening. The transmission rate in this simulation is 42.3 Mbits/sec.

Example 2 (Q=2) TEQ design for $Q = 2$ case is shown in Fig. 6.3. Fig. 6.3(a) shows the original channel and the equalized channel. The SIR of the equalized channel is 76.5 dB. The magnitude response of the TEQ is shown in Fig. 6.3(b). The zeros of TEQ response locate at the unused tones that be chosen. We can find that the TEQ response is like to be compressed due to oversampling. Then the magnitude response of equalized and original channel is shown in Fig. 6.3(c). Fig. 6.3(d) shows the bit allocation. The transmission rate is 55.9 Mbits/sec. The transmission rate using TEQ design with oversampling is better than that without oversampling.

Example 3 (Q=1 with additional frequency criterion incorporated) In this example, the TEQ is designed with the frequency criterion incorporated. Fig. 6.4(a) shows the impulse response of the original channel and the equalized channel. The SIR of the equalized channel 73.8 dB. Comparing Fig. 6.4(b) and Fig. 6.2(b), the TEQ is flatter in the transmission bands and even more attenuated in null tones due to the additional frequency criterion. The magnitude response of TEQ is shown in Fig. 6.4(b). The magnitude response of the equalized channel is shown in Fig. 6.4(c). Fig. 6.4(d) shows the bit allocation. The transmission rate in this simulation is 48.2 Mbits/sec. The transmission rate with frequency weighting is better than that without frequency weighting.

Example 4 (Q=2 with additional frequency criterion incorporated) TEQ design for $Q = 2$ case with considering frequency criterion is shown in Fig. 6.5. Fig. 6.5(a) shows the original channel and the equalized channel. The SIR of the equalized channel is 73.9 dB. The magnitude response of TEQ is shown in Fig. 6.5(b). The magnitude response of equalized and original channel is shown in Fig. 6.5(c). Fig. 6.5(d) shows the bit allocation after channel shortening. The transmission rate is 58.8 Mbits/sec. The transmission rate using TEQ design with oversampling and frequency weighting is the best.

The above examples shows that our proposed TEQ design shorten the original channel effectively. Good channel shortening can be achieved. And the transmission band can free from the zeros due to the TEQ magnitude response by choosing target tones, source tones, and frequency weighting tone index. We can find the $Q = 2$ case have better channel shortening property and transmission rate than $Q = 1$ case. We also find the case incorporation of additional frequency criterion have much better than without considering frequency criterion.


(a)
original
equalized
0.2
0.1
0
−0.1
−0.2
−0.3
100
200
300
400
500
time index
(b)
20
0
−20
−40
−60
−80
−100
dB
1D
2D
0
500
1000
1500
2000
2500
tone index

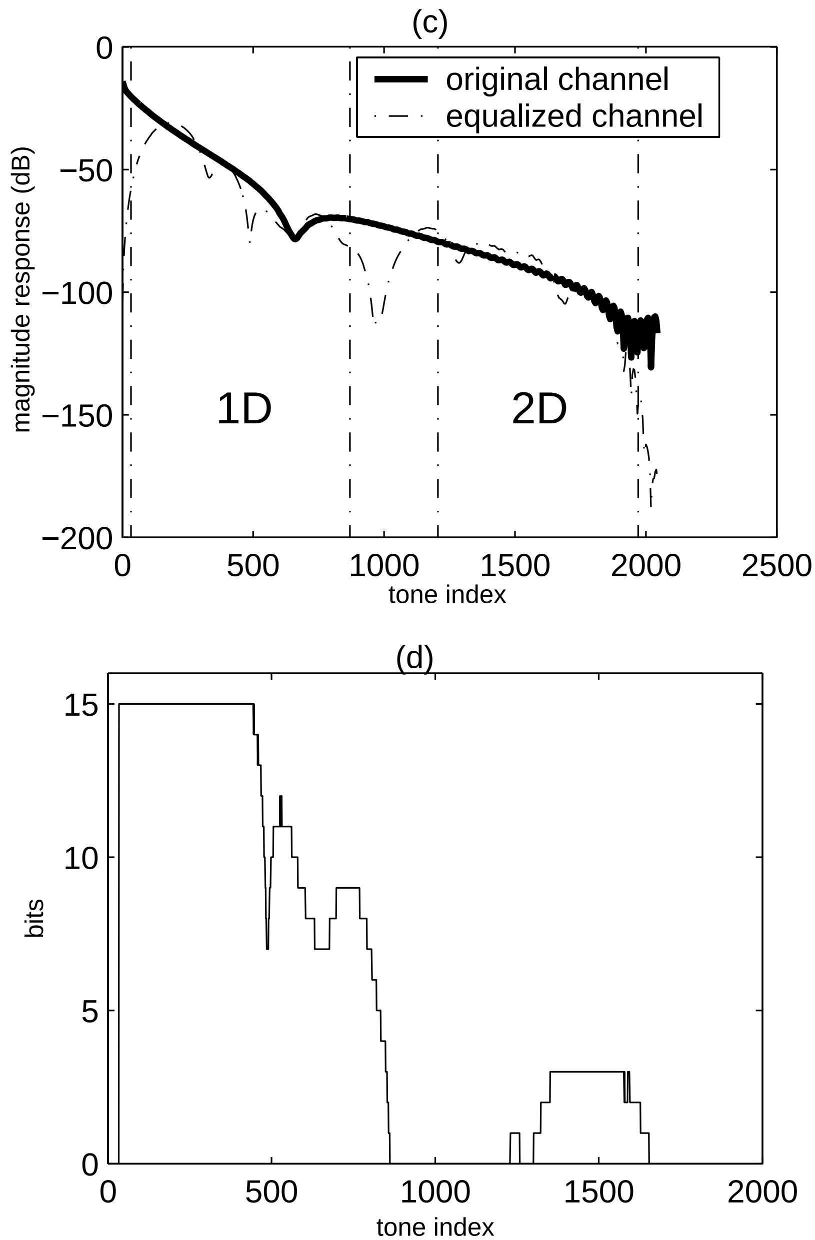
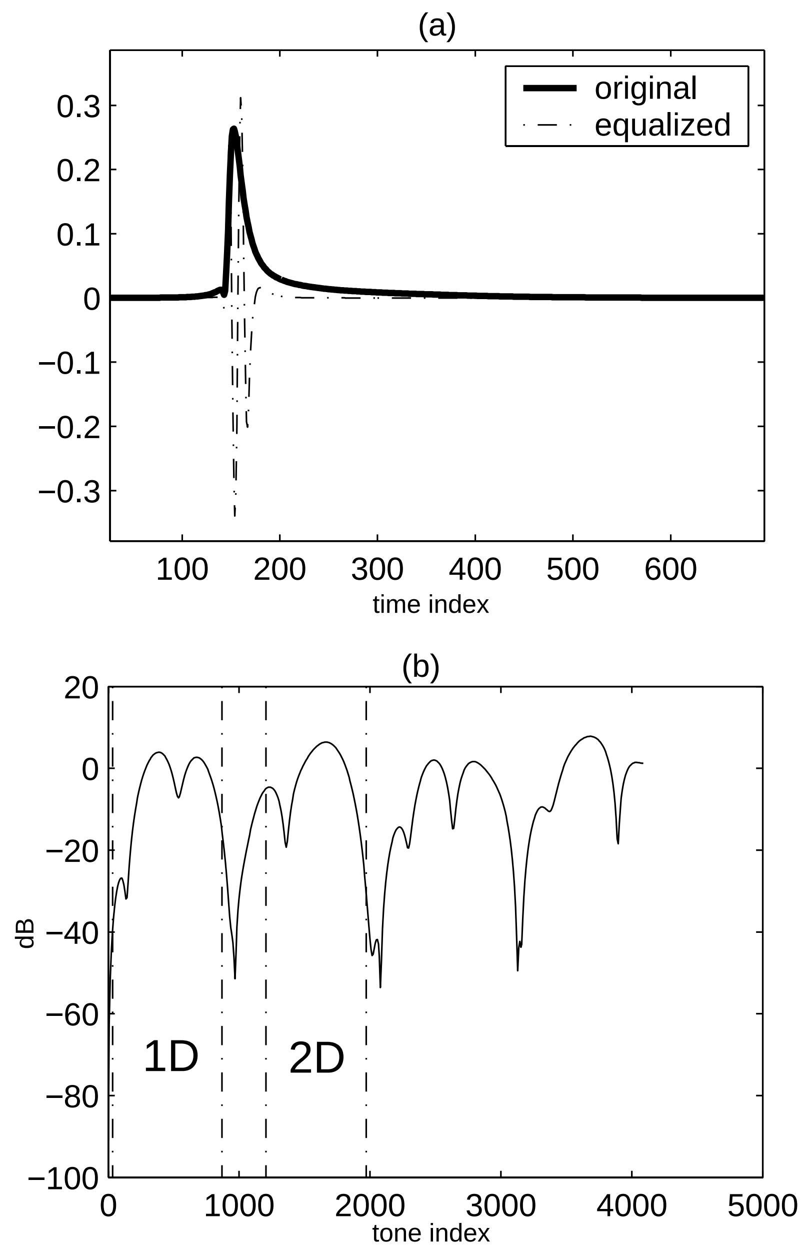


Figure 6.2: TEQ design for $Q = 1$ case. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ, (c) magnitude response of original and equalized channel, (d) bit allocations.


(a)
original
equalized
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
100
200
300
400
500
600
time index
(b)
20
0
−20
−40
−60
−80
−100
dB
1D
2D
0
1000
2000
3000
4000
5000
tone index

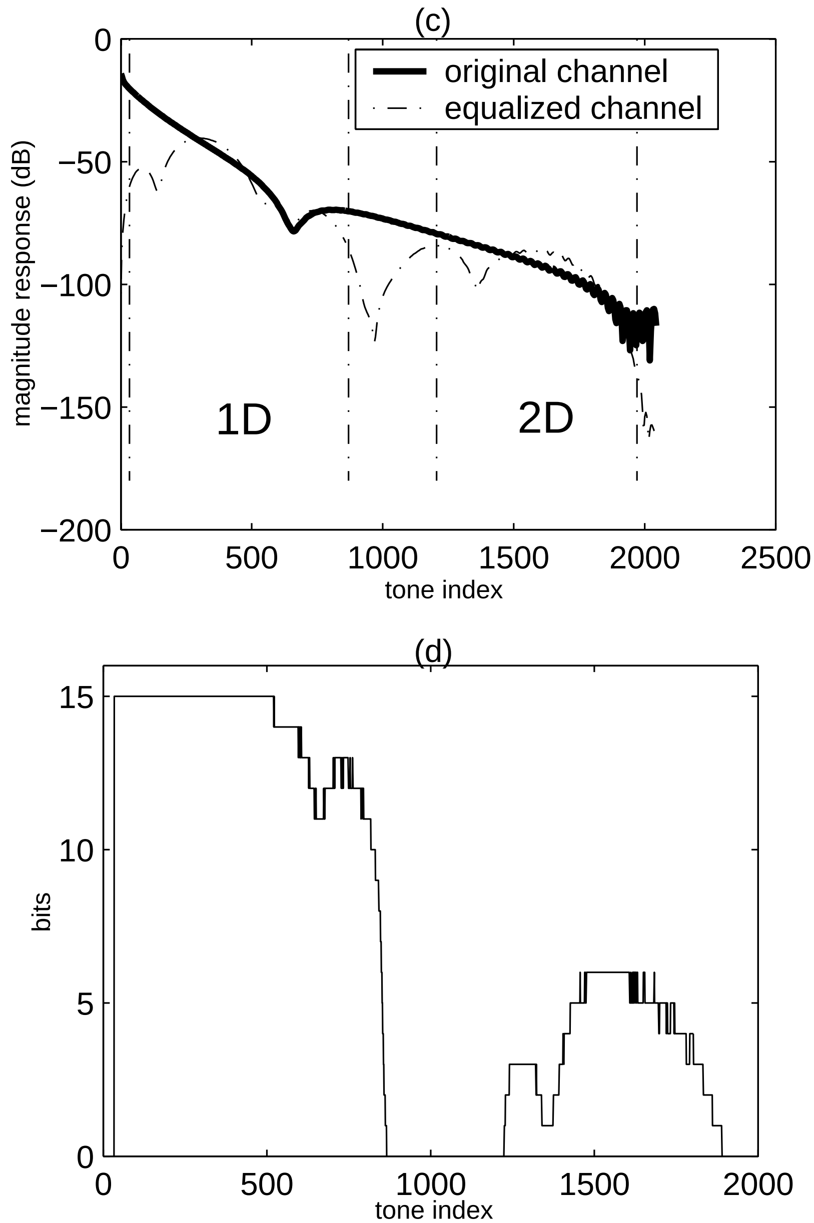
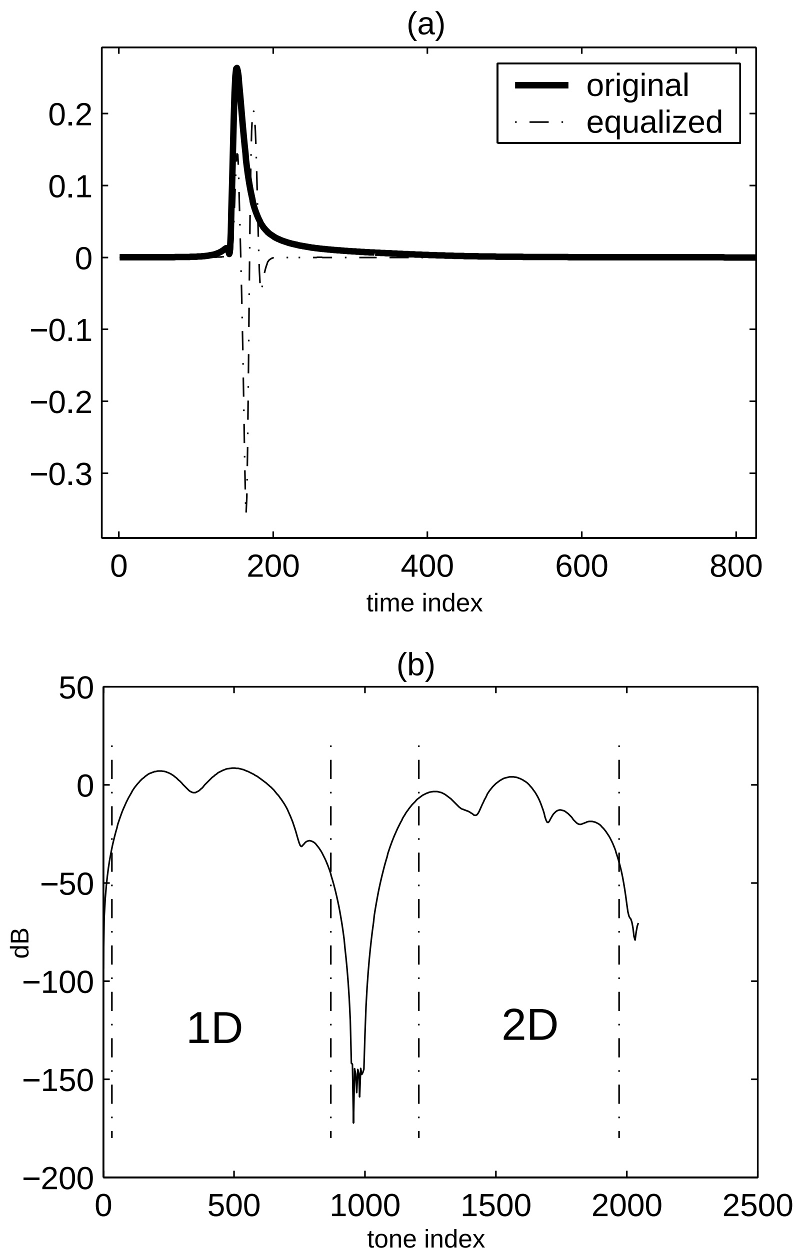


Figure 6.3: TEQ design for $Q = 2$ case. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ, (c) magnitude response of original and equalized channel, (d) bit allocations.


(a)
original
equalized
0.2
0.1
0
−0.1
−0.2
−0.3
0
200
400
600
800
time index
(b)
50
0
−50
−100
−150
−200
dB
1D
2D
0
500
1000
1500
2000
2500
tone index

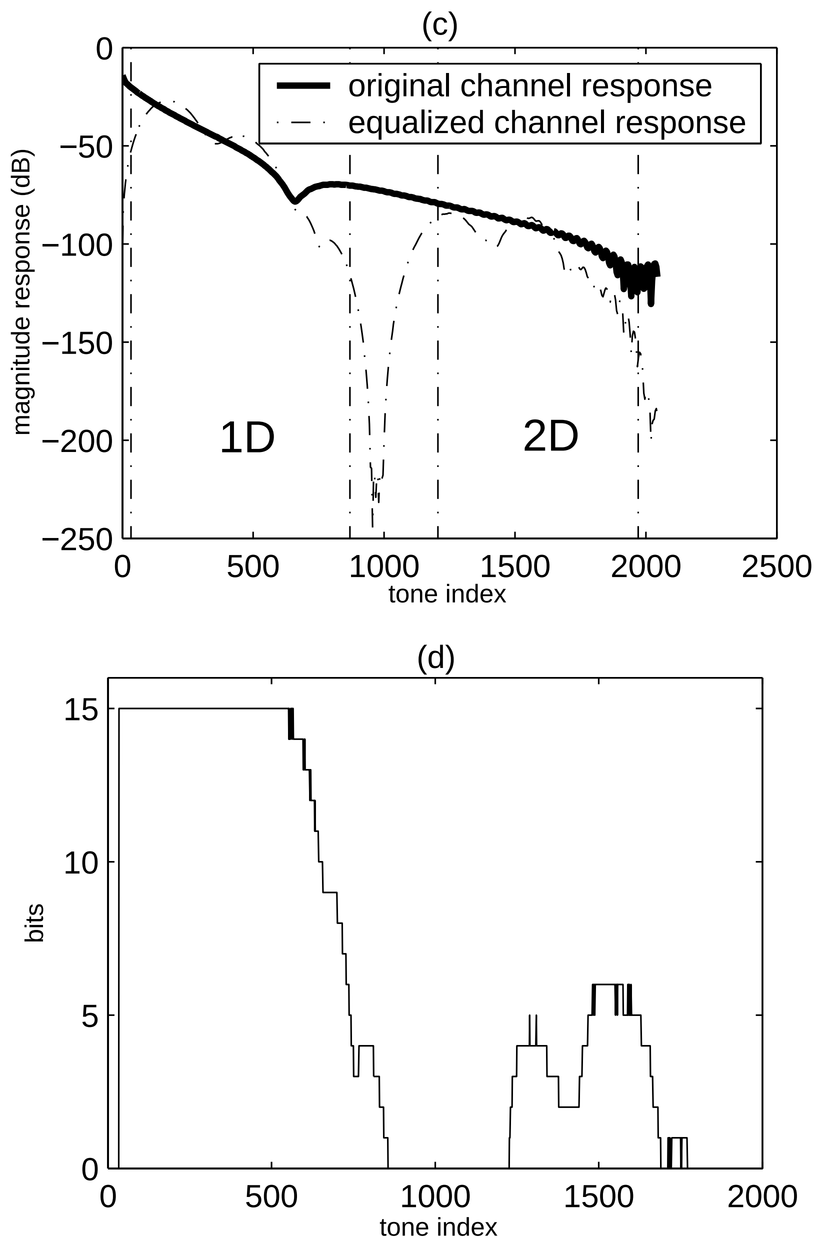
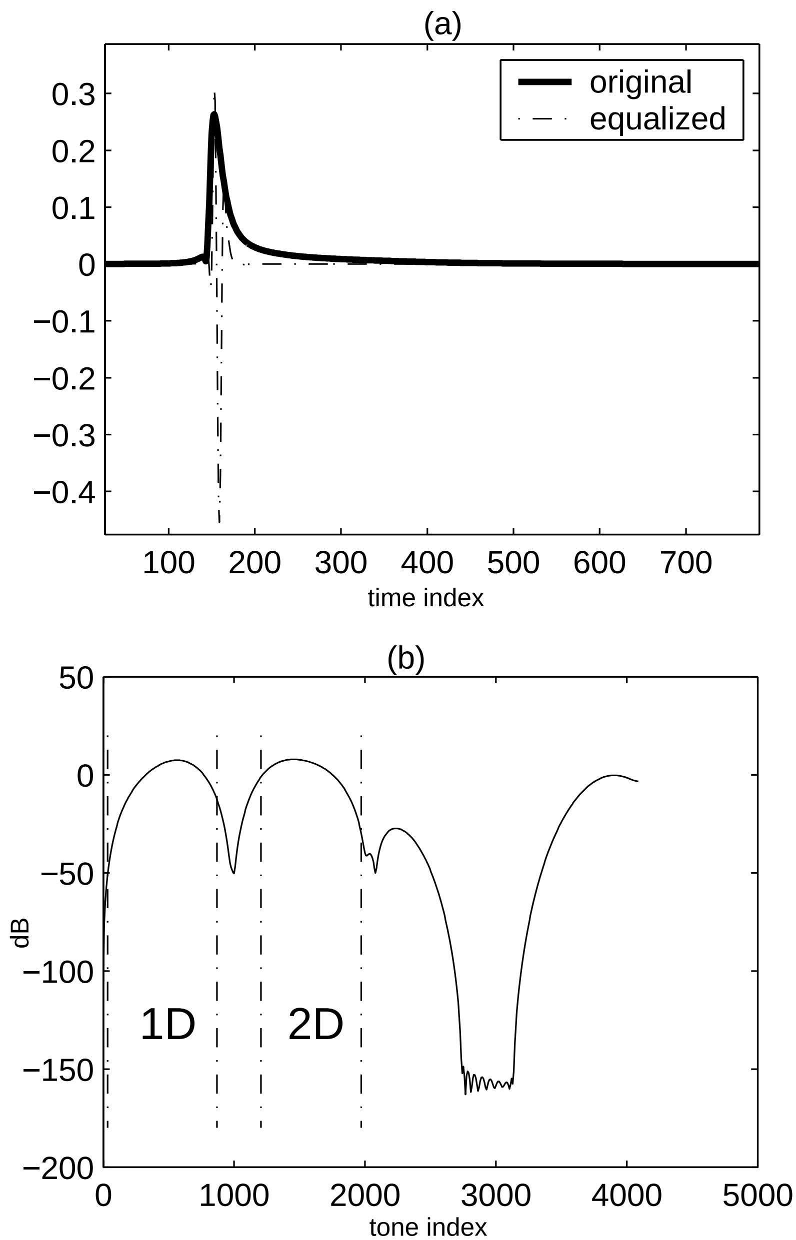


Figure 6.4: TEQ design for $Q = 1$ case incorporation of additional frequency criterion. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ, (c) magnitude response of original and equalized channel, (d) bit allocations.


(a)
original
equalized
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
100
200
300
400
500
600
700
time index
(b)
50
0
−50
−100
−150
−200
dB
1D
2D
0
1000
2000
3000
4000
5000
tone index

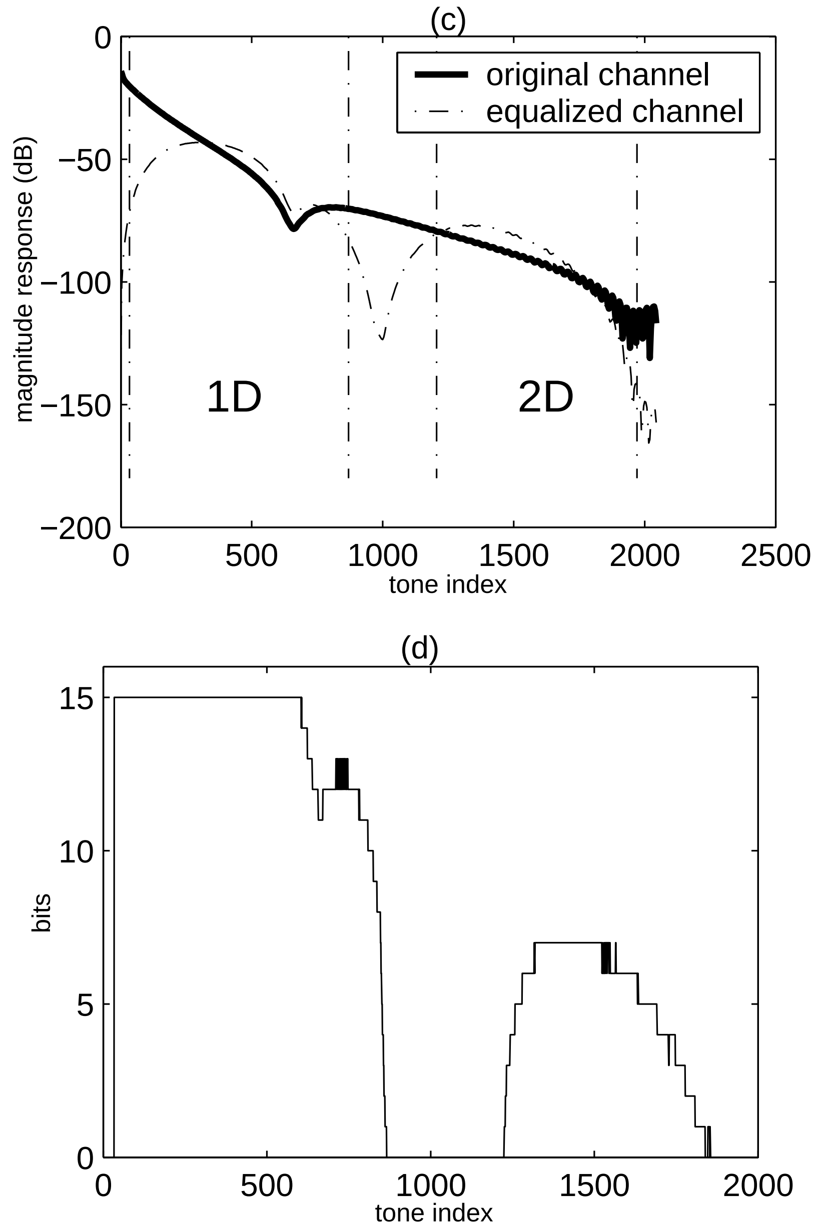


Figure 6.5: TEQ design for $Q = 2$ case incorporation of additional frequency criterion. (a) Impulse response of original and equalized channel, (b) magnitude response of TEQ, (c) magnitude response of original and equalized channel, (d) bit allocations.

6.3.1 Transmission Rate Comparisons

The comparison of transmission rates of TEQ design with oversampling ($Q = 2$), TEQ design without oversampling ($Q = 1$), MSSNR, MERRY, Minimum-ISI and Per-tone equalization methods are listed in Table 6.2. In our proposed TEQ design, $Q = 2$ case have much better transmission rate than $Q = 1$ case. However, from the table we also observe that our proposed TEQ design outperform MSSNR, MERRY, and Min-ISI methods in transmission rates and close to the Per-tone equalization method. The Table 6.3 shows the transmission rate comparisons of that, -140dBm white Gaussian noise is considered. And Table 6.4 shows the transmission rate comparisons with frequency criterion and without it.

Loop	$Q = 1$	$Q = 2$	MSSNR	Min.ISI	PTEQ
VDSL1L	68.92	74.06	51.33	59.58	77.11
VDSL2L	66.57	72.75	40.07	51.44	73.51
VDSL3L	68.98	70.55	49.01	52.94	72.28
VDSL4L	37.20	38.16	35.55	12.91	48.47
VDSL5	93.68	93.77	80.14	93.91	93.93
VDSL6	78.31	83.22	66.63	66.56	83.42
VDSL7	48.20	58.83	38.98	40.01	60.78

Table 6.2: Bit rate (Mbits/sec) on VDSL loops (AWGN:-170dBm)

Loop	$Q = 1$	$Q = 2$	MSSNR	Min.ISI	PTEQ
VDSL1L	47.46	52.90	41.17	47.27	59.30
VDSL2L	43.50	47.54	30.66	41.77	53.92
VDSL3L	42.94	46.68	37.72	42.50	52.29
VDSL4L	20.92	21.13	20.80	11.32	34.12
VDSL5	93.67	93.70	79.64	93.80	93.87
VDSL6	54.49	58.68	53.90	53.86	69.05
VDSL7	31.96	34.12	26.82	31.63	39.96

Table 6.3: Bit rate (Mbits/sec) on VDSL loops (AWGN:-140dBm)

Loop	$Q = 1$ (w)	$Q = 1$ (w/o)	$Q = 2$ (w)	$Q = 2$ (w/o)
VDSL1L	68.92	68.19	74.06	74.00
VDSL2L	66.57	58.07	72.75	65.07
VDSL3L	68.98	62.06	70.55	69.80
VDSL4L	37.20	36.97	38.16	37.18
VDSL5	93.68	93.47	93.77	93.36
VDSL6	78.31	74.72	83.22	80.68
VDSL7	48.20	42.31	58.83	55.90

Table 6.4: Bit rate (Mbits/sec) on VDSL loops with and without frequency criterion (AWGN:-170dBm).

6.3.2 SIR Comparisons

The comparison of SIR of TEQ design with oversampling ($Q = 2$), TEQ design without oversampling ($Q = 1$), MSSNR, MERRY, and Minimum-ISI methods are listed in Table 6.5. The SIR is good measure for channel shortening effect. From the table, we observe that our proposed TEQ design shorten the channel effectively.

Loop	$Q = 1$	$Q = 2$	MSSNR	Min.ISI
VDSL1L	82.62	79.8	128.5	82.0
VDSL2L	72.84	71.4	121.5	91.1
VDSL3L	89.64	90.4	123.3	72.1
VDSL4L	56.03	61.0	101.4	52.9
VDSL5	147.66	162.2	169.0	102.9
VDSL6	93.99	96.3	122.9	85.4
VDSL7	73.80	74.0	102.0	59.3

Table 6.5: SIR Measure (dB) on VDSL loops

Conclusion

In this thesis, We have proposed a new frequency domain based TEQ design to increase design freedom by using oversampling at receiver in DMT system. We consider not only minimizing the ISI but frequency criterion corresponding to the magnitude response of TEQ to design TEQ. The objective function can be simplified as a quadratic form of TEQ coefficients. And we can directly control the zeros of TEQ response by choosing the target tones, source tones and frequency weighting. It is a good advantage of our proposed TEQ design. The transmission band would free from the zeros using our proposed TEQ design and then the better transmission rate would be achieved. In our proposed TEQ design, the channel can be shortened effectively and we have much better transmission rate than many TEQ design methods, and very close to the per-tone equalization method.

Appendix A

Derivation of $a_{Q,ik}$, $b_{Q,ik}$, and $e_{Q,ik}$

We will derive the elements of $b_{Q,ik}$, $e_{Q,ik}$ and $a_{Q,ik}, k \neq i$, in this Appendix for $Q = 2$ case. The case for more general Q can be obtained in a similar manner. Suppose the synchronization delay is d, and then between the decimator output $v(z)$ and the expander input $u(z)$ shown in Fig. 3.2.(b) can be written by using the polyphase identity as

$$v(z) = (C_2(z)T(z)z^d)_{\downarrow 2} \cdot u(z) \tag{A.1}$$

Then we define $H(z) = C(z)T(z)$ with length QN. The system from $u(n)$ to $v(n)$ is

$$\Psi(z) = h_0 z^{\frac{\tilde{d}}{2}} + h_2 z^{\frac{\tilde{d}}{2}-1} + \cdots + h_{2(N-1)} z^{\frac{\tilde{d}}{2}-N-1}. \tag{A.2}$$

where $\tilde{d} = d - (d \ mod \ 2)$. We can write $\Psi(z) = \sum_{r=0}^{N-1} h_{2r} \cdot z^{\frac{\tilde{d}}{2}-r}$. The k-th transmitting filter is $F_k(z) = \frac{1}{\sqrt{M}} \sum_{i=0}^{N-1} W_M^{-k(M-L+i)} \cdot z^{-i}$ and n-th receiving filter is $R_i(z) = \frac{1}{\sqrt{M}} \sum_{l=L}^{N-1} W_M^{i(l-L)} \cdot z^l$. Then we use the polyphase identity and the elements of transfer matrix $\mathbf{P}_2(z)$ can be written as

$$\begin{aligned} P_{2,ik}(z) &= (F_k(z)\Psi(z)R_i(z))_{\downarrow N} \\ &= (\frac{1}{M} \sum_{n=0}^{N-1} \sum_{l=L}^{N-1} \sum_{r=0}^{\frac{N}{2}-1} W_M^{[-k(i-L)+i(l-L)]} \cdot h_{2r} \cdot z^{l-n+\frac{\tilde{d}}{2}-r})_{\downarrow N} \quad . \end{aligned} \tag{A.3}$$

The equation (A.3) can be written as

$$P_{2,ik}(z) = \sum_{n=0}^{N-1} (\frac{1}{M} \sum_{l=L}^{N-1} \sum_{r=0}^{\frac{N}{2}-1} W_M^{[-k(i-L)+i(l-L)]} \cdot h_{2r} \cdot z^{l-n+\frac{\tilde{d}}{2}-r})_{\downarrow N} \quad . \tag{A.4}$$

The off diagonal elements,$p_{2,ik}(0)$, of matrix $\mathbf{P}_2(0)$ caused by the interference of the i-th tone from the k-th tone at the same symbol block is the constant part of the transfer function $P_{2,ik}(z)$ and can be written as

$$p_{2,ik}(0) = \sum_{n=0}^{N-1} [g_a]_{ik}^{(n)} \tag{A.5}$$

Because $p_{2,ik}(0)$ is the constant part of $P_{ik}(z)$, we know that the order of z in (A.4) is zero before decimation by N, that is

$$l - n + \frac{\tilde{d}}{2} - r = 0 \tag{A.6}$$

Then we first discuss the (A.5) from index n,

$$\begin{aligned}
& n = 0, \qquad then\ r = l + \tfrac{\tilde{d}}{2} \\
& \Longrightarrow [g_a]_{ik}^{(0)} = \tfrac{1}{M} \sum_{l=L}^{N-1} W_M^{[kL+i(l-L)]} \cdot h_{2l+\tilde{d}} \\
& \quad \vdots \\
& n = L + \tfrac{\tilde{d}}{2}, \quad then\ r = l - L \\
& \Longrightarrow [g_a]_{ik}^{(L+\frac{\tilde{d}}{2})} = \tfrac{1}{M} \sum_{l=L}^{N-1} W_M^{[-k(\frac{\tilde{d}}{2})+i(l-L)]} \cdot h_{2l-2L} \\
& \quad \vdots \\
& n = N-1, \ then\ r = l - (N-1) + \tfrac{\tilde{d}}{2} \\
& \Longrightarrow [g_a]_{ik}^{(N-1)} = \tfrac{1}{M} \sum_{l=L}^{N-1} W_M^{[k+i(l-L)]} \cdot h_{2l-2(N-1)+\tilde{d}}
\end{aligned} \tag{A.7}$$

However, $p_{2,ik}(0)$ can be expressed in matrix form as follow,

$$\begin{aligned}
p_{2,ik}(0) \quad &= \mathbf{1} \cdot \begin{pmatrix}
0 & \cdots & 0 & W_M^{kL} & \cdots & W_M^{kL+i(N-\frac{\tilde{d}}{2}-1-L)} \\
\vdots & 0 & W_M^{-k(L+1)} & & \cdots & \vdots \\
0 & & & & & \\
W_M^{-k\frac{\tilde{d}}{2}} & \cdots & \cdots & & & 0 \\
\vdots & \vdots & \vdots & & 0 & \vdots \\
W_M^{[k-i(1+\frac{\tilde{d}}{2})]} & \cdots & W_M^{k-n} & 0 & \cdots\cdots & 0
\end{pmatrix} \cdot \widetilde{\mathbf{h}} \\
&= \mathbf{a}_{2,ik}^{\dagger} \cdot \widetilde{\mathbf{h}}
\end{aligned} \tag{A.8}$$

where

$$\mathbf{1} = (\,1 \quad 1 \quad \cdots \quad 1\,)_{1\times N} \quad , \quad \widetilde{\mathbf{h}} = \begin{pmatrix} h_0 \\ h_2 \\ \\ \vdots \\ \\ h_{2N-2} \end{pmatrix} \tag{A.9}$$

where $\mathbf{1}$ is a $1\times N$ with all one elements row vector, $N\times 1$ column vector $\widetilde{\mathbf{h}}$ consists of the equalized channel coefficients h_i and $\mathbf{a}_{2,ik}$ is a $N \times 1$ column vector. The elements of $\mathbf{a}_{2,ik}$ are

$$[\mathbf{a}_{2,ik}]_u = \begin{cases} \frac{1}{M} \sum\limits_{q=L+\frac{\tilde{d}}{2}-u}^{N-1} W_M^{[-k(q-L)+i(q-L-\frac{\tilde{d}}{2}+u)]}, & 0 \le u \le \frac{\tilde{d}}{2}-1 \\ 0, & \frac{\tilde{d}}{2} \le u \le L+\frac{\tilde{d}}{2} \\ \frac{1}{M} \sum\limits_{q=0}^{N-u+\frac{\tilde{d}}{2}-1} W_M^{[-k(q-L)+i(u-\frac{\tilde{d}}{2}+q-L)]}, & L+\frac{\tilde{d}}{2}+1 \le u \le N-1 \end{cases} \tag{A.10}$$

the interference of i-th tone from the k-th tone at the same symbol block can be written as,

$$p_{2,ik}(0) = \mathbf{a}_{2,ik}^{\dagger} \cdot \widetilde{\mathbf{h}} \tag{A.11}$$

The elements,$p_{2,ik}(-1)$, of the matrix $\mathbf{P}(-1)$ caused by the interference of the next symbol block is the advance part of $P_{2,ik}(z)$ and can be written as

$$p_{2,ik}(-1) = \sum_{n=0}^{N-1} [g_b]_{ik}^{(n)} \tag{A.12}$$

The $p_{2,ik}(-1)$ is the advance part of $P_{2,ik}(z)$, and then the order of z in (A.4) is N before decimation by N, that is ,

$$l - n + \frac{\tilde{d}}{2} - r = N \tag{A.13}$$

Then we discuss the (A.12) from index n,

$$
\begin{aligned}
& n=0, \qquad \mathit{then}\ r = l+\tfrac{\tilde{d}}{2}-N \\
& \Longrightarrow [g_b]_{nk}^{(0)} = \tfrac{1}{M}\sum_{l=L}^{N-1} W_M^{[kL+i(l-L)]}\cdot h_{2l+\tilde{d}-2N} \\
& \quad \vdots \\
& n=\tfrac{\tilde{d}}{2}-1, \quad \mathit{then}\ r = l-N+1 \\
& \Longrightarrow [g_b]_{nk}^{(\frac{\tilde{d}}{2}-1)} = \tfrac{1}{M}\sum_{l=L}^{N-1} W_M^{[-k(-L+\frac{\tilde{d}}{2}-1)+i(l-L)]}\cdot h_{2l-2N+2} \\
& \quad \vdots \\
& n=N-1, \ \mathit{then}\ r = l+\tfrac{\tilde{d}}{2}-2N+1 \\
& \Longrightarrow [g_b]_{nk}^{(N-1)} = \tfrac{1}{M}\sum_{l=L}^{N-1} W_M^{[k+i(l-L)]}\cdot h_{2l-4N+\tilde{d}+2}
\end{aligned}
\tag{A.14}
$$

However, $p_{2,ik}(-1)$ can be expressed in matrix form as follow,

$$
\begin{aligned}
p_{2,ik}(-1) &= \mathbf{1}\cdot\mathbf{B}\cdot\widetilde{\mathbf{h}} \\
&= \mathbf{b}_{2,ik}^{\dagger}\cdot\widetilde{\mathbf{h}}
\end{aligned}
\tag{A.15}
$$

where

$$
\mathbf{B} = \begin{pmatrix}
W_M^{[kL+i(-\frac{\tilde{d}}{2})]} & W_M^{[kL+i(-\frac{\tilde{d}}{2}+1)]} & \cdots & W_M^{[kL+i(-\frac{\tilde{d}}{2}+1)]} & 0 & \cdots & 0 \\
W_M^{[-k(-L+1)+i(-\frac{\tilde{d}}{2}+1)]} & \cdots & W_M^{[-k(-L+1)+i(-\frac{\tilde{d}}{2}+1)]} & 0 & \cdots & & \vdots \\
\vdots & & & & & & \\
W_M^{[-k(-L+\frac{\tilde{d}}{2}-1)-i]} & 0 & \cdots & \vdots & & & 0 \\
0 & \vdots & \vdots & & & & \vdots \\
0 & \cdots & 0 & 0 & \cdots & & 0
\end{pmatrix}
\tag{A.16}
$$

is an $N\times N$ matrix and the $\mathbf{b}_{2,ik}$ is a $N\times 1$ column vector, its elements are

$$
[\mathbf{b}_{2,ik}]_u = \begin{cases} \frac{1}{M}\sum\limits_{q=0}^{\frac{\tilde{d}}{2}-u-1} W_M^{[-k(q-L)+i(q-\frac{\tilde{d}}{2}+u)]}, & 0\le u\le \frac{\tilde{d}}{2}-1 \\ 0, & \mathit{otherwise} \end{cases}
\tag{A.17}
$$

the interference of i-th tone from the k-th tone due to the next symbol block can be written as,

$$
p_{2,ik}(-1) = \mathbf{b}_{2,ik}^{\dagger}\cdot\widetilde{\mathbf{h}}
\tag{A.18}
$$

The elements,$p_{2,ik}(1)$, of the matrix $\mathbf{P}(1)$ caused by the interference due to the previous symbol block is the delay part of $P_{2,ik}(z)$ and can be written as

$$
p_{2,ik}(1) = \sum_{n=0}^{N-1}[g_e]_{ik}^{(n)}
\tag{A.19}
$$

The $p_{2,ik}(1)$ is the delay part of $P_{2,ik}(z)$, and then the order of z in (A.4) is $-N$ before decimation by N, that is,

$$l - n + \frac{\tilde{d}}{2} - r = -N \tag{A.20}$$

Then we discuss the (A.19) from index n,

$$\begin{aligned}
& n = 0, \qquad \mathit{then}\ r = l + \tfrac{\tilde{d}}{2} + N \\
& \Longrightarrow [g_e]_{ik}^{(0)} = \tfrac{1}{M} \sum_{l=L}^{N-1} W_M^{[kL+i(l-L)]} \cdot h_{2l+\tilde{d}+2N} \\
& \quad \vdots \\
& n = N-1, \mathit{then}\ \ r = l + \tfrac{\tilde{d}}{2} + 1 \\
& \Longrightarrow [g_e]_{nk}^{(N-1)} = \tfrac{1}{M} \sum_{l=L}^{N-1} W_M^{[k+i(l-L)]} \cdot h_{2l+\tilde{d}+2}
\end{aligned} \tag{A.21}$$

However, $p_{2,ik}(1)$ can be expressed in matrix form as follow,

$$\begin{aligned}
p_{2,ik}(1) \quad &= \mathbf{1} \cdot \begin{pmatrix}
0 & & \cdots & 0 & & \cdots & 0 \\
0 & & \cdots & 0 & & \cdots & 0 \\
 & & \ddots & & & 0 & W_M^{-k(N+\frac{\tilde{d}}{2}-\frac{M}{2}+1)} \\
\vdots & & & & & & \vdots \\
 & & & 0 & & & \\
0 & \cdots & 0 & W_M^{k} & \cdots & & W_M^{[k+i(\frac{M-\tilde{d}}{2}-L-2)]}
\end{pmatrix} \cdot \widetilde{\mathbf{h}} \\
&= \mathbf{e}_{2,ik}^{\dagger} \cdot \widetilde{\mathbf{h}}
\end{aligned} \tag{A.22}$$

where $\mathbf{e}_{2,ik}$ is a $N \times 1$ column vector and its elements are

$$[\mathbf{e}_{2,ik}]_u = \begin{cases} \frac{1}{M} \sum\limits_{q=0}^{u-L-\frac{\tilde{d}}{2}-1} W_M^{[k(q+1)+i(u-L-\frac{\tilde{d}}{2}-1-q)]}, & L + \frac{\tilde{d}}{2} + 1 \leq u \leq N-1 \\ 0, & \mathit{otherwise} \end{cases} \tag{A.23}$$

the interference of i-th tone from the k-th tone due to the previous symbol block can be written as,

$$p_{2,ik}(1) = \mathbf{e}_{2,ik}^{\dagger} \cdot \widetilde{\mathbf{h}}. \tag{A.24}$$

General form. For the convenience of derivation, we discuss the case of $Q = 2$. The proposed TEQ design can be generalized for that the sampling rate at receiver is Q times of it at transmitter. The elements of $\mathbf{a}_{Q,ik}$, $\mathbf{b}_{Q,ik}$, and $\mathbf{e}_{Q,ik}$

can be written in a general form as follow:

$$[\mathbf{a}_{Q,ik}]_u = \begin{cases} \frac{1}{M} \sum\limits_{q=L+\frac{\tilde{d}}{Q}-u}^{N-1} W_M^{[-k(q-L)+i(q-L-\frac{\tilde{d}}{Q}+u)]}, & 0 \le u \le \frac{\tilde{d}}{Q}-1 \\ 0, & \frac{\tilde{d}}{Q} \le u \le L+\frac{\tilde{d}}{Q} \\ \frac{1}{M} \sum\limits_{q=0}^{N-u+\frac{\tilde{d}}{Q}-1} W_M^{[-k(q-L)+i(u-\frac{\tilde{d}}{Q}+q-L)]}, & L+\frac{\tilde{d}}{Q}+1 \le u \le N-1 \end{cases} \tag{A.25}$$

$$[\mathbf{b}_{Q,ik}]_u = \begin{cases} \frac{1}{M} \sum\limits_{q=0}^{\frac{\tilde{d}}{Q}-u-1} W_M^{[-k(q-L)+i(q-\frac{\tilde{d}}{Q}+u)]}, & 0 \le u \le \frac{\tilde{d}}{Q}-1 \\ 0, & otherwise \end{cases} \tag{A.26}$$

$$[\mathbf{e}_{Q,ik}]_u = \begin{cases} \frac{1}{M} \sum\limits_{q=0}^{u-L-\frac{\tilde{d}}{Q}-1} W_M^{[k(q+1)+i(u-L-\frac{\tilde{d}}{Q}-1-q)]}, & L+\frac{\tilde{d}}{Q}+1 \le u \le N-1 \\ 0, & otherwise \end{cases} \tag{A.27}$$

where $\tilde{d} = d - y$, $y = d \pmod{Q}$. And the column vector $\tilde{\mathbf{h}}$ becomes $\tilde{\mathbf{h}}_Q = (\, h_y \quad h_{y+Q} \quad \cdots \quad h_{y+Q\cdot(N-1)}\,)$ is a $(N-1)\times 1$ column vector.

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