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Fuzzy Credibility Relation Method for Multiple Criteria Decision-Making Problems

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ABSTRACT

This paper deals with the problem of ranking alternatives under multiple criteria. A fuzzy credibility relation (FCR) method is proposed. Owing to vague concepts represented in decision data, in this study the rating of each alternative and the weight of each criterion are expressed in fuzzy numbers. Then we define the concordance, discordance, and support indices. By aggregating the concordance index and support index, a fuzzy credibility relation is calculated to represent the intensity of the preferences of one alternative over another. Finally, according to the fuzzy credibility relation, the ranking order of all alternatives can be determined. A numerical example is solved to highlight the procedure of the FCR method at the end of this paper. ©*Elsevier Science Inc. 1997*

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1. INTRODUCTION

In general, a multiple criteria decision-making (MCDM) problem can be concisely expressed in matrix format as

$$\mathbf{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \end{array}, \qquad \text{(A)}$$

$$\mathbf{W} = \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix},$$

where $A_1, A_2, \ldots, A_m$ are possible alternatives, $C_1, C_2, \ldots, C_n$ are criteria with which performances of alternatives are measured, x_{ij} is the rating of alternative A_i with respect to criterion C_j, and w_j is the weight of criterion C_j. For a crisp MCDM problem, the ratings x_{ij} of alternative A_i and the weights w_j of the criteria are given as real numbers.

In general, the methods to solve the crisp MCDM problems can be classified into three categories: aggregation into a unique criterion, outranking methods, and interactive methods [8, 20]. The outranking method was initially suggested by Benayoun et al. [2] and was improved by Roy [16, 17]. However, this method ignores much vital information present in the concordance and discordance matrices and at times leads to wrong decisions [18]. Owing to the fact that an outranking relation is inherently fuzzy in nature, it is undesirable to determine the outranking relation with a crisp relation. Therefore, many authors [4, 13, 15, 19] considered the uncertainty and fuzziness of decision data and proposed different types of preference functions to deal with the strength of the outranking relation.

In the outranking methods mentioned above, the ratings x_{ij} and the weights w_j are given by crisp numbers, but under many conditions, crisp data are inadequate to model real-life situations. In addition, in order to deal with the uncertainty and fuzziness of decision data, some outranking methods mentioned above defined some threshold values and functions to determine the strength of the outranking relations between alternatives. However, in fact using decision-makers makes it difficult to determine these threshold values and functions, and the final solution will be influenced by these threshold values; the resulting preference functions might be unacceptable or unrealistic in some applications [10, 20, 21].

To consider the vague concepts expressed in decision data and avoid the final solution influenced by the threshold values, the use of fuzzy numbers is an adequate means to model uncertainty arising from imprecision in human behavior or incomplete knowledge about the external environment [1]. Therefore, in this study the ratings x_{ij} and the weights w_j are given as trapezoidal fuzzy numbers. Then we retain all the vital information and develop a fuzzy creditability relation (FCR) method to avoid subjective determination of the threshold values.

The organization of this paper is as follows. First, we introduce the basic definitions and notations. Next we define the concordance and support indices to derive the fuzzy credibility relation, and propose a fuzzy credibility relation (FCR) method to solve MCDM problems. Then the FCR method is illustrated with an example. Finally, we give some conclusion at the end of this paper.

2. DEFINITIONS AND NOTATIONS

The basic definitions and notations that follow will be used through out the paper unless otherwise stated.

DEFINITION 2.1 [11]. If $\tilde{n}$ is a fuzzy number with membership function $\mu_{\tilde{n}}(x)$ and whose α-cut is denoted by $\tilde{n}^\alpha = \{x\colon \mu_{\tilde{n}}(x) \geqslant \alpha\} = [n_l^\alpha, n_u^\alpha]$ for $\alpha \in [0,1]$, then $[n_l^{\alpha_2}, n_u^{\alpha_2}] \subset [n_l^{\alpha_1}, n_u^{\alpha_1}]$ when $\alpha_1 < \alpha_2$ $\forall \alpha_1, \alpha_2 \in [0,1]$. $\tilde{n}^\alpha$ is the set of α-cuts and n_l^α and n_u^α are the lower and upper bounds of the set of α-cuts, respectively.

DEFINITION 2.2 [5]. $\tilde{n} = (n_1, n_2, n_3, n_4)$ denotes a trapezoidal fuzzy number if its membership function $\mu_{\tilde{n}}(x)$ is defined as

$$\mu_{\tilde{n}}(x) = \begin{cases} 0, & x \leqslant n_1, \\ \dfrac{x-n_1}{n_2-n_1}, & n_1 \leqslant x \leqslant n_2, \\ 1, & n_2 \leqslant x \leqslant n_3, \\ \dfrac{x-n_4}{n_3-n_4}, & n_3 \leqslant x \leqslant n_4, \\ 0, & x \geqslant n_4. \end{cases} \tag{1}$$

DEFINITION 2.3 [11]. If $\tilde{n}$ is a trapezoidal fuzzy number and $n_l^\alpha > 0$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a positive trapezoidal fuzzy number (PTFN).

Given any two positive trapezoidal fuzzy numbers $\tilde{m} = (m_1, m_2, m_3, m_4)$ and $\tilde{n} = (n_1, n_2, n_3, n_4)$, and a real number $r > 0$, we know that [11]

$$\tilde{m}(+)\tilde{n} = (m_1 + n_1, m_2 + n_2, m_3 + n_3, m_4 + n_4), \tag{2}$$

$$\tilde{m}(-)\tilde{n} = (m_1 - n_4, m_2 - n_3, m_3 - n_2, m_4 - n_1), \tag{3}$$

$$\tilde{m}(\cdot)\tilde{n} = (m_1 \cdot n_1, m_2 \cdot n_2, m_3 \cdot n_3, m_4 \cdot n_4), \tag{4}$$

$$\tilde{m}(:)\tilde{n} = \left(\frac{m_1}{n_4}, \frac{m_2}{n_3}, \frac{m_3}{n_2}, \frac{m_4}{n_1}\right), \tag{5}$$

$$(\tilde{m})^{-1} = \left(\frac{1}{m_4}, \frac{1}{m_3}, \frac{1}{m_2}, \frac{1}{m_1}\right), \tag{6}$$

$$\tilde{m}(\cdot)r = (m_1 \cdot r, m_2 \cdot r, m_3 \cdot r, m_4 \cdot r), \tag{7}$$

$$\tilde{m}(:)r = \left(\frac{m_1}{r}, \frac{m_2}{r}, \frac{m_3}{r}, \frac{m_4}{r}\right). \tag{8}$$

DEFINITION 2.4 [14]. If $\tilde{n}$ is a trapezoidal fuzzy number, and $n_l^\alpha > 0$ and $n_u^\alpha \leqslant 1$ for $\alpha \in [0,1]$, then $\tilde{n}$ is called a normalized positive trapezoidal fuzzy number.

DEFINITION 2.5 [5]. $\tilde{\mathbf{A}}$ is called a fuzzy matrix if there exists at least an entry in $\tilde{\mathbf{A}}$ is a fuzzy number.

3. CONCORDANCE AND DISCORDANCE ANALYSIS

In this study, we consider the following decision matrix $\tilde{\mathbf{D}}$ by modifying matrix $\mathbf{D}$:

$$\tilde{\mathbf{D}} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{11} & \cdots & \tilde{x}_{11} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix},$$

$$\tilde{\mathbf{W}} = \begin{bmatrix} \tilde{w}_1 & \tilde{w}_2 & \cdots & \tilde{w}_n \end{bmatrix},$$

where $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$, $j = 1, 2, \ldots, n$, are positive trapezoidal fuzzy numbers. If

$a_{ij} = b_{ij} = c_{ij} = d_{ij}$, then the rating $\tilde{x}_{ij}$ is a crisp value. $\tilde{\mathbf{W}}$ is a normalized fuzzy criterion weight vector which can be determined by Hsu and Chen's method [9].

First, we use the linear scale transformation to transform the various criteria scales into a comparable scale. We obtain the normalized fuzzy decision matrix denoted by $\tilde{\mathbf{R}}$:

$$\tilde{\mathbf{R}} = \left[\tilde{r}_{ij}\right]_{m \times n}, \tag{9}$$

where B and C are the set of *benefit* criteria and *cost* criteria, respectively, and

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}\right), \quad \text{if } j \in B,$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \quad \text{if } j \in C,$$

$$d_j^* = \max_i d_{ij}, \quad \text{if } j \in B,$$

$$a_j^- = \min_i a_{ij}, \quad \text{if } j \in C.$$

The normalization method mentioned above is to preserve the property that the ranges of normalized fuzzy numbers belong to $[0,1]$.

Then we calculate the weighted normalized fuzzy decision matrix as

$$\tilde{\mathbf{V}} = \left[\tilde{v}_{ij}\right]_{m \times n}, \tag{10}$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

After the construction of weighted normalized fuzzy decision matrix $\tilde{\mathbf{V}}$, the pairwise comparison of the preference relationships between the alternatives A_g and A_h can be established as stated in the following section.

3.1. *THE CONCORDANCE SET AND DISCORDANCE SET*

The weighted normalized fuzzy ratings of A_g and A_h ($g, h = 1, 2, \ldots, m$ and $g \neq h$) in $\tilde{\mathbf{V}}$ are denoted as $\tilde{v}_g = [\tilde{v}_{g1}, \tilde{v}_{g2}, \ldots, \tilde{g}_{gn}]$ and $\tilde{v}_h = [\tilde{v}_{h1}, \tilde{v}_{h2}, \ldots, \tilde{v}_{hn}]$, respectively. Then we compare the fuzzy number $\tilde{v}_{gj}$ to

$\tilde{v}_{hj}$. If $\tilde{v}_{gj}$ is larger than (or equal to) $\tilde{v}_{hj}$, we say alternative A_g is at least as good as A_h with respect to the jth criterion. In this way, we partition the criteria $J=\{j|j=1,2,\ldots,n\}$ into concordance set C_{gh} and discordance set D_{gh} designated as

$$C_{gh}=\left\{j|\tilde{v}_{gi}\geqslant\tilde{v}_{hj},\ j=1,2,\ldots,n\right\} \tag{11}$$

and

$$D_{gh}=\left\{j|\tilde{v}_{gj}<\tilde{v}_{hj},\ j=1,2,\ldots,n\right\}. \tag{12}$$

Many authors [3, 6, 7, 12] have been devoted to the investigation with regard to the comparison of fuzzy numbers. One of the useful methods to compare fuzzy numbers was proposed by Lee and Li [12]. It is probably the most logical ranking method [10], which ranks fuzzy numbers based on the fuzzy mean and the fuzzy spread of the fuzzy numbers. In this paper, we use Lee and Li's method to compare the fuzzy numbers to determine the concordance set and the discordance set.

If the discordance set is empty, i.e., $D_{gh}=\varnothing$, then it indicates that A_g fully outranks A_h; otherwise, if the concordance set is empty, i.e., $C_{gh}=\varnothing$, then it indicates that A_g does not outrank A_h absolutely. Practically, the discordance set and the concordance set are usually not empty. In order to determine the degree of "A_g outranks A_h," we must consider the concordance set and the discordance set simultaneously. With respect to criterion j in the concordance set, a larger distance between $\tilde{v}_{gj}$ and $\tilde{v}_{hj}$ indicates a higher concordance degree to say "A_g dominates A_h." In other words, with respect to criterion k in the discordance set, a larger distance between $\tilde{v}_{gk}$ and $\tilde{v}_{hk}$ indicates a higher discordance degree to say "A_g dominates A_h."

In order to determine the difference between fuzzy numbers, we use the dissemblance index for fuzzy numbers to calculate the distance between fuzzy numbers [11, 22]. The dissemblance index of $\tilde{v}_{gj}$ and $\tilde{v}_{hj}$ is expressed as

$$d\left(\tilde{v}_{gj},\tilde{v}_{hj}\right)=\frac{1}{2}\int_{\alpha=0}^{1}\Delta\left(\tilde{v}_{gj}^{\alpha},\tilde{v}_{hj}^{\alpha}\right)d\alpha, \tag{13}$$

where $\Delta(\tilde{v}_{gj}^{\alpha},\tilde{v}_{hj}^{\alpha})=|v_{gjl}^{\alpha}-v_{hjl}^{\alpha}|+|v_{gju}^{\alpha}-v_{hju}^{\alpha}|$, $\tilde{v}_{gj}^{\alpha}=[\tilde{v}_{gjl}^{\alpha},v_{gju}^{\alpha}]$, and $\tilde{v}_{hj}^{\alpha}=[v_{hjl}^{\alpha},v_{hju}^{\alpha}]$. Referring to the ELECTRE method [2, 10], we define the concordance index CI_{gh} by aggregating the difference of $\tilde{v}_{gj}$ and $\tilde{v}_{hj}$ for all criteria to represent the strength of A_g dominates A_h as

$$\mathrm{CI}_{gh} = \frac{\sum_{j \in C_{gh}} d(\tilde{v}_{gj}, \tilde{v}_{hj})}{\sum_{j \in J} d(\tilde{v}_{gj}, \tilde{v}_{hj})}. \tag{14}$$

Similarly, we also define the discordance index as

$$\mathrm{DI}_{gh} = \frac{\max_{j \in D_{gh}} d(\tilde{v}_{gj}, \tilde{v}_{hj})}{\max_{j \in J} d(\tilde{v}_{gj}, \tilde{v}_{hj})}. \tag{15}$$

However, if we aggregate the discordance index and the concordance index directly with the same procedures as the ELECTRE [10, 17] method, the ELECTRE III [15] method, or Singh's method [18], we will not distinguish the difference between two alternatives effectively in many cases. For example, in two cases we obtain (a) $\mathrm{CI}_{ij} = 0.8$, $\mathrm{DI}_{ij} = 1.0$ and (b) $\mathrm{CI}_{ji} = 0.2$, $\mathrm{DI}_{ji} = 1.0$. Intuitively, A_j is dominated by A_i. However, if we follow the same procedure as in the ELECTRE method with concordance threshold $\bar{c} = 0.8$ and discordance threshold $\bar{d} = 0.2$, then the outranking degree e_{ij} of A_i over A_j is equal to zero and the outranking degree e_{ji} of A_j over A_i is also equal to zero. Because the discordance indices are 1 in situations (a) and (b), the outranking degrees between alternative A_i and A_j are also equal to zero when we adopt the ELECTRE III method. According to the method of Singh et al. [18] we obtain the following results:

(i) $\mathrm{CI}_{ij} = 0.8$, $d'_{ij} = 1 - \mathrm{DI}_{ij} = 0$, $e_{ij} = \min\{\mathrm{CI}_{ij}, d'_{ij}\} = \min(0.8, 0\} = 0$.
(ii) $\mathrm{CI}_{ji} = 0.2$, $d'_{jt} = 1 - \mathrm{DI}_{ji} = 0$, $e_{ji} = \min\{\mathrm{CI}_{ji}, d'_{ji}\} = \min\{0.2, 0\} = 0$.

These methods mentioned above neglect the information provided by the values of concordance indices when the values of discordance indices are equal to 1. Therefore, the contribution of the difference values of concordance indices must be considered. Meanwhile, a higher discordance index indicates a lower outranking degree for one alternative over another. Thus, according to the discordance indices of each pair of alternatives, we transform the discordance index into a support index CI^*_{gh} as

$$\mathrm{CI}^*_{gh} = \frac{\mathrm{DI}_{hg}}{\mathrm{DI}_{gh} + \mathrm{DI}_{hg}}. \tag{16}$$

With the transformation of the discordance index into the support index, the larger value of CI^*_{gh} represents the higher degree of "A_g outranks A_h."

Combining the concordance and support indices, the credibility degree of "A_g outranks A_h" can then be expressed as $e_{gh} = \min\{CI_{gh}, CI^*_{gh}\}$. We call $\mathbf{E} = [e_{gh}]_{m\times m}$ the fuzzy credibility relation matrix.

In the example mentioned above, we obtain the following results:

(i) $CI_{ij} = 0.8$, $CI^*_{ij} = 0.5$, $e_{ij} = \min\{0.8, 0.5\} = 0.5$.
(ii) $CI_{ji} = 0.2$, $CI^*_{ji} = 0.5$, $e_{ji} = \min\{0.2, 0.5\} = 0.2$.

It means that the credibility value of "A_i outranks A_j" is higher than the credibility value of "A_j outranks A_i."

After constructing the fuzzy credibility relation matrix, a ranking procedure is developed to determine the ranking order of each alternative.

3.2. *RANKING PROCEDURE*

According to the fuzzy credibility relation matrix $\mathbf{E}$, the fuzzy strict credibility relation matrix can be defined as

$$\mathbf{E}^s = \left[e^s_{ij}\right]_{m\times m}, \tag{17}$$

where

$$\mu_{E^s}(A_i, A_j) = e^s_{ij} = \begin{cases} e_{ij} - e_{ji}, & \text{when } e_{ij} \geqslant e_{ji}, \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

The value of e^s_{ij} indicates the degree of strict dominance of alternative A_i over alternative A_j. Then, using the fuzzy strict credibility relation matrix $E^s = [e^s_{ij}]_{m\times m}$, the nondominated degree of each alternative A_i ($i = 1, 2, \ldots, m$) can be defined as

$$\mu^{ND}(A_i) = \min_{\substack{A_j \in \Omega \\ A_j \neq A_i}} \left\{1 - \mu_{E^s}(A_j, A_i)\right\} = 1 - \max_{\substack{A_j \in \Omega \\ A_j \neq A_i}} \mu_{E^s}(A_j, A_i), \tag{19}$$

where $\Omega = \{A_1, A_2, \ldots, A_m\}$.

A large value of $\mu^{ND}(A_i)$ indicates that the alternative A_i has a higher nondominated degree than others. Then we can use the $\mu^{ND}(A_i)$ values to rank a set of alternatives. The ranking procedure is described as follows:

Step 1. Set $K=0$ and $\Omega=\{A_1, A_2, \ldots, A_m\}$.

Step 2. Select the alternatives which have the highest nondominated degree, say A_h, $A_h=\max_i\{\mu^{ND}(A_i)\}$. The ranking for A_h is $r(A_h)=K+1$.

Step 3. Delete the alternatives A_h from Ω, that is, $\Omega=\Omega\backslash A_h$. The corresponding row and column of A_h are deleted from the fuzzy strict credibility relation matrix.

Step 4. Recalculate the nondominated degree for each alternative A_i, $A_i \in \Omega$. If $\Omega=\varnothing$, then stop. Otherwise, set $K=K+1$ and return to step 2.

4. NUMERICAL EXAMPLE

A hypothetical example is designed to demonstrate the computational process of this fuzzy credibility relation (FCR) method. Suppose that a manufacturing company desires to select a suitable city for establishing a new factory. After preliminary screening, three candidates A_1, A_2, and A_3 remain for further evaluation. The company considers five criteria to select the most suitable candidate:

(1) land cost (C_1)
(2) transportation distance (C_2)
(3) numbers of satellite factory (C_3)
(4) human resource (C_4)
(5) the flexibility of government policy (C_5)

The benefit and cost criteria sets are $B=\{3,4,5\}$ and $C=\{1,2\}$, respectively.

Now we apply the fuzzy credibility relation (FCR) method to solve this problem. The computational procedure is summarized as follows:

Step 1. The fuzzy decision matrix and the normalized fuzzy weight of each criterion are given as

$$\tilde{\mathbf{D}}=\begin{bmatrix} 4.5 & 55 & 100 & (3,5,6,7) & (4,5,6,8) \\ 5.5 & 70 & 60 & (6,7,8,9) & (4,4,5,5,7) \\ 6.0 & 60 & 120 & (4,5,6,7) & (6,7,8,9) \end{bmatrix},$$

$$\tilde{w}_1=(0.5,0.6,0.8,1.0),$$

$$\tilde{w}_2=(0.35,0.5,0.6,0.75),$$

$$\tilde{w}_3=(0.35,0.4,0.55,0.7),$$

$$\tilde{W}_4 = (0.4, 0.55, 0.7, 0.8),$$

$$\tilde{w}_5 = (0.4, 0.5, 0.6, 0.7).$$

Step 2. The normalized fuzzy decision matrix is calculated as

$$\tilde{\mathbf{R}} = \begin{bmatrix} 1.0 & 1.0 & 0.83 & (0.33, 0.56, 0.67, 0.78) & (0.44, 0.56, 0.67, 0.89) \\ 0.82 & 0.79 & 0.5 & (0.67, 0.78, 0.89, 1.0) & (0.44, 0.5, 0.56, 0.78) \\ 0.75 & 0.92 & 1.0 & (0.44, 0.56, 0.67, 0.78) & (0.67, 0.78, 0.89, 1.0) \end{bmatrix}.$$

Step 3. The weighted normalized fuzzy decision matrix is calculated as

$$\tilde{\mathbf{V}} = \begin{bmatrix} (0.5, 0.6, 0.8, 1.0) & (0.35, 0.5, 0.6, 0.75) & (0.29, 0.33, 0.46, 0.58) & (0.13, 0.31, 0.47, 0.62) & (0.18, 0.28, 0.4, 0.62) \\ (0.41, 0.49, 0.66, 0.82) & (0.28, 0.4, 0.47, 0.59) & (0.12, 0.2, 0.28, 0.56) & (0.27, 0.43, 0.62, 0.8) & (0.18, 0.25, 0.34, 0.55) \\ (0.38, 0.45, 0.60, 0.75) & (0.32, 0.46, 0.55, 0.69) & (0.35, 0.4, 0.55, 0.7) & (0.18, 0.31, 0.47, 0.62) & (0.27, 0.39, 0.53, 0.7) \end{bmatrix}.$$

Step 4. The concordance and discordance sets are determined, respectively, as

$$C_{12} = \{1,2,3,5\}, \quad C_{13} = \{1,2\}, \quad C_{21} = \{4,5\}, \quad C_{23} = \{1,4\},$$

$$C_{31} = \{3,4,5\}, \quad C_{32} = \{2,3,5\}, \quad D_{12} = \{4\}, \quad D_{13} = \{3,4,5\},$$

$$D_{21} = \{1,2,3\}, \quad D_{23} = \{2,3,5\}, \quad D_{31} = \{1,2\}, \quad D_{32} = \{1,4\}.$$

Step 5. The distances of each pair of alternatives with respect to each criterion are computed as shown in Table 1. According to the results of Table 1, three matrices CI, DI, and CI* defined in (14), (15), and (16), respectively, can be shown as

$$\mathrm{CI} = \begin{bmatrix} - & 0.74 & 0.53 \\ 0.26 & - & 0.31 \\ 0.47 & 0.69 & - \end{bmatrix},$$

TABLE 1

The Distance Measurements

Distance	C_1	C_2	C_3	C_4	C_5	Max	Sum
(A_1, A_2)	0.13	0.12	0.13	0.15	0.04	0.15	0.57
(A_1, A_2)	0.18	0.05	0.09	0.01	0.10	0.18	0.43
(A_2, A_3)	0.05	0.07	0.21	0.14	0.14	0.21	0.61

$$\mathrm{DI} = \begin{bmatrix} - & 1.0 & 0.56 \\ 0.87 & - & 1.0 \\ 1.0 & 0.67 & - \end{bmatrix},$$

$$\mathrm{CI}^* = \begin{bmatrix} - & 0.47 & 0.64 \\ 0.53 & - & 0.40 \\ 0.36 & 0.60 & - \end{bmatrix}.$$

In this case, using the method of Singh et al. [18], we obtain the outranking degrees $e_{12} = 0$ and $e_{23} = 0$. However, the concordance degree for alternative A_1 over A_2 ($\mathrm{CI}_{12} = 0.74$) is larger than the concordance degree for alternative A_2 over A_3 ($\mathrm{CI}_{23} = 0.31$). Intuitively considering concordance and discordance indices simultaneously, the credibility degree e_{12} should be larger than e_{23}. Thus, this method shows an unacceptable result.

Step 6. Construct the fuzzy credibility relation matrix as

$$\mathbf{E} = \begin{bmatrix} - & 0.47 & 0.53 \\ 0.26 & - & 0.31 \\ 0.36 & 0.60 & - \end{bmatrix}.$$

Step 7. Construct the fuzzy strict credibility relation matrix as

$$\mathbf{E}^s = \begin{bmatrix} - & 0.21 & 0.17 \\ 0 & - & 0 \\ 0 & 0.29 & - \end{bmatrix}.$$

Step 8. Compute the nondominated degree of each alternative A_i $(i = 1, 2, 3)$ as

$$\mu^{\mathrm{ND}}(A_1) = 1.0,$$

$$\mu^{\mathrm{ND}}(A_2) = 0.71,$$

$$\mu^{\mathrm{ND}}(A_3) = 0.83.$$

Step 9. The alternative A_1 has the highest nondominated degree and set $r(A_1) = 1$.

Step 10. Delete the alternative A_1 from the fuzzy strict credibility relation matrix.

Step 11. After deleting the alternative A_1, the new fuzzy strict credibility relation matrix is

$$\mathbf{E}^s = \begin{bmatrix} — & 0 \\ 0.29 & — \end{bmatrix}.$$

The nondominated degrees of alternatives A_2 and A_3 are 0.71 and 1.0, respectively. Therefore, $r(A_3) = 2$ and $r(A_2) = 3$.

5. CONCLUSION

In general, multicriteria problems adhere to uncertain and imprecise data, and fuzzy set theory is adequate to deal with it. In this paper, a fuzzy credibility relation (FCR) method based on the fuzzy ratings and fuzzy weights is proposed to solve fuzzy MCDM problems. Decision-makers are difficult to determine the threshold values. Meantime, the final solution is often influenced by the threshold values. Therefore, the FCR method considers fuzzy assessment data instead of threshold values to model the uncertainty arising from imprecision in human behavior or incomplete knowledge about the external environment.

In this paper, a support index is determined by transformation of the discordance index, which indicates the outranking degree for one alternative over another from the viewpoint of the discordance set. By aggregating the concordance and support indices, denoted by credibility degree, the intensity of the preferences of one alternative over another is effectively represented. Through constructing the fuzzy credibility relation matrix, a systematic and objective procedure is proposed to rank a finite set of alternatives in the FCR method. This method provides a stepwise way to produce the ranking order of each alternative.

The framework of FCR provided in this paper can be easily extended to the analysis of other problems such as project management, selection of a site for an industry, and many other areas of management decision problems.

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