

# Modeling Orbit Dynamics of FORMOSAT-3/COSMIC Satellites for Recovery of Temporal Gravity Variations

Cheinway Hwang, Ting-Jung Lin, Tzu-Pang Tseng, and Benjamin Fong Chao

**Abstract**—The precise GPS high-low tracking data from the joint Taiwan–USA mission FORMOSAT-3/COSMIC (COSMIC) can be used for gravity recovery. The current orbital accuracy of COSMIC kinematic orbit is 2 cm and is better than 1 cm for 60-s normal points. We model the perturbing forces acting on the COSMIC spacecraft based on standard models of orbit dynamics. The major tool for the numerical work of force modeling is NASA Goddard's GEODYN II software. Considering that COSMIC spacecrafts are not equipped with accelerometers, the accelerations due to atmospheric drag, solar radiation pressure, and other minor surface forces are modeled by estimating relevant parameters over one orbital period from COSMIC's kinematic and reduced dynamic orbits. We carry out experimental solutions of time-varying geopotential coefficients using one month of COSMIC kinematic orbits (August 2006). With the nongravity origin forces properly modeled by GEODYN II, residual orbital perturbations (difference between kinematic and reference orbits) are assumed to be linear functions of time-varying geopotential coefficients and are used as observations to estimate the latter. Both COSMIC and combined COSMIC and GRACE gravity solutions are computed. The COSMIC solution shows some well-known temporal gravity signatures but contains artifacts. The combined COSMIC and GRACE solution enhances some local temporal gravity signatures in the GRACE solution.

**Index Terms**—FORMOSAT-3/COSMIC (COSMIC), GPS, GRACE, kinematic orbit, temporal gravity.

## I. INTRODUCTION

**L**AUNCHED in April 2006, the FORMOSAT-3/COSMIC (COSMIC) is a joint Taiwan–USA satellite mission. The acronym COSMIC stands for Constellation Observing System for Meteorology, Ionosphere, and Climate and will be used hereafter to represent FORMOSAT-3/COSMIC. In addition to its primary objectives of meteorological and ionospheric

research, the data from this mission can be useful for such geodetic applications as orbital science of low-Earth orbiters (LEOs), observation of the Earth's gravity field, and improvement of GPS satellite orbits, as shown in [1]–[3]. Such geodetic applications of COSMIC rely on the data from the GPS precise orbit determination (POD) receivers on the six COSMIC satellites. The GPS POD payloads are detailed on the Web page of the National Space Organization (NSPO) of Taiwan (<http://www.nspo.org.tw/>). Using the same POD procedure as in [4] and [5], the current estimates of orbit accuracy of the COSMIC kinematic orbits are at the centimeter and subcentimeter levels at the 5- and 60-s sampling intervals, respectively (see also [6] and [7]). For gravity recovery, the geometrically determined COSMIC kinematic orbits are functions of orbit dynamic parameters, including geopotential coefficients, and, thus, can be regarded as observations in the least square estimation of these parameters. In view of the large number of satellites (six), varying altitudes of 500–800 km, simple satellite geometry (Section II-B), and an expected life span of more than five years, the GPS data of COSMIC are particularly suitable for temporal gravity recovery.

For each COSMIC satellite, there are two POD antennas sharing one GPS receiver POD +  $X$  and POD –  $X$  (Fig. 1). There are 16 channels for a receiver, of which four are used for ionospheric occultation and 12 for POD. Considering the hardware design, the POD antenna in the flight direction (+ $X$ ) will view more GPS satellites (typically seven) than the other. Unlike GRACE, the six spacecrafts of COSMIC do not have an onboard accelerometer to measure surface forces (nongravitational forces) in order to isolate the gravitational signals. Instead, such surface forces, as well as other perturbing forces acting on the six COSMIC satellites, will be modeled using standard orbit dynamics such as those detailed in [8] and [9]. GRACE is a NASA–German Deutschen Zentrum für Luft-und Raumfahrt mission to recover time-varying gravity variations; a summary of achievements and techniques of the GRACE mission can be found at <http://www.csr.utexas.edu/grace/>. In order to validate the force models for COSMIC satellites, we will experiment with the recovery of temporal gravity variations using one month of COSMIC GPS data (six satellites). GRACE-derived gravity fields will be used as *a priori* information to aid the gravity recovery. The COSMIC-derived gravity will be compared with that of GRACE. Following the convention of NSPO, the six spacecrafts of COSMIC will be denoted as FM1, . . . , FM6.

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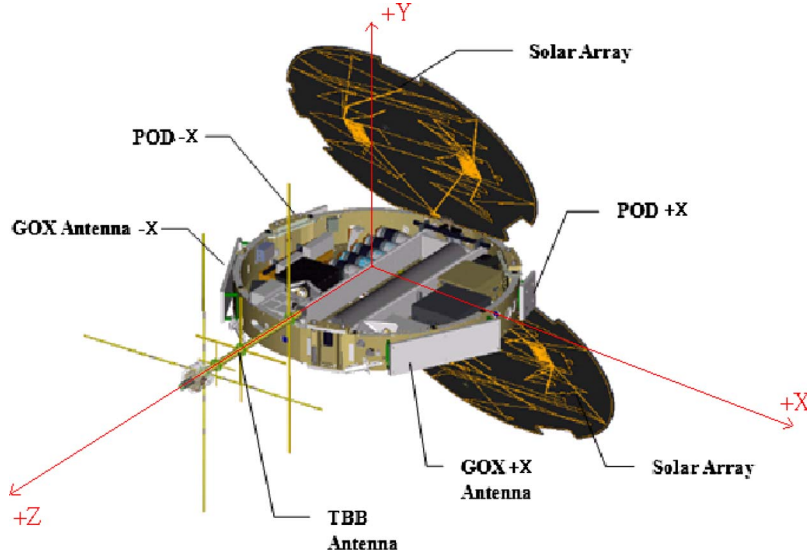


Fig. 1. COSMIC spacecraft, payloads, and spacecraft-fixed coordinate system; the (cylinder) origin is at the center of the main body.

## II. ORBIT DYNAMICS OF COSMIC SATELLITE

### A. Equations of Motion and Forces Acting on Cosmic Satellite

The equations of motion of an artificial Earth satellite, such as a COSMIC LEO, can be expressed as in [9]

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r} + \mathbf{a}_{\text{ns}} + \mathbf{a}_{\text{Pert}} \quad (1)$$

where

- $\mathbf{r}$  vector of satellite coordinates in the inertial frame;
- $\ddot{\mathbf{r}}$  acceleration vector;
- $GM$  Earth's gravitational constant;
- $\mathbf{a}_{\text{ns}}$  acceleration due to Earth's nonsphericity;
- $\mathbf{a}_{\text{Pert}}$  accelerations due to other perturbing forces.

The mathematical theories associated with the accelerations in (1) can be found in a standard textbook of orbit dynamics such as in [8]–[10], and they will not be elaborated here. The accelerations in (1) are associated with certain parameters that are adopted from existing values or estimated by satellite tracking data. For gravity recovery using COSMIC GPS data, some basics about orbit dynamics and the Earth's gravity field are given here for a convenient discussion of the results given later. First, the potential due to the Earth (called geopotential) at the satellite position  $V$  is commonly expressed in a spherical harmonic expansion in [11] as

$$\begin{aligned} V(r, \phi, \lambda) &= \frac{GM}{r} + \frac{GM}{r} \sum_{n=1}^K \left(\frac{a_e}{r}\right)^n \\ &\quad \times \sum_{m=0}^n (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin \phi) \\ &= \frac{GM}{r} + V_{\text{ns}} \end{aligned} \quad (2)$$

where  $(r, \phi, \lambda)$  are the spherical coordinates (radial distance, geocentric latitude, and longitude),  $a_e$  is the semimajor axis of a reference ellipsoid,  $K$  is the maximum degree of expansion

depending on the satellite altitude,  $(\bar{C}_{nm}, \bar{S}_{nm})$  are geopotential coefficients, and  $\bar{P}_{nm}$  is the fully normalized associated Legendre function of degree  $n$  and order  $m$ . The acceleration of a LEO due to the geopotential in (2) is then

$$\mathbf{a}_{\text{earth}} = \frac{\partial V}{\partial \mathbf{r}} = -\frac{GM}{r^3}\mathbf{r} + \mathbf{a}_{\text{ns}}. \quad (3)$$

Thus,  $\mathbf{a}_{\text{earth}}$  accounts for the first two terms in (1). The first term in (1) is called the point mass effect of the Earth and is more than 1000 times larger than any other acceleration in (1). By gravity recovery, we mean estimating the geopotential coefficients  $(\bar{C}_{nm}, \bar{S}_{nm})$ . In fact, due to mass redistribution in the Earth system, the geopotential is time dependent. It is convenient to divide the geopotential into a static (time-average) and a time-varying part. This is equivalent to dividing the coefficients in (2) into a static and a time-varying part as

$$\begin{aligned} \bar{C}_{nm}(t) &= \bar{C}_{nm}^0 + \bar{J}_{nm}(t) \\ \bar{S}_{nm}(t) &= \bar{S}_{nm}^0 + \bar{K}_{nm}(t) \end{aligned} \quad (4)$$

where  $t$  is time. Thus, by recovering the temporal gravity, we mean estimating the time-varying coefficients  $(\bar{J}_{nm}(t), \bar{K}_{nm}(t))$ .

The perturbing forces (accelerations) in (1) can be classified into gravitational and surface forces (or nongravitational forces). The gravitational forces include the Earth's nonsphericity (3),  $N$ -body, solid Earth tide, ocean tide, and relativistic effect. The surface forces include atmospheric drag, solar radiation pressure, and the Earth's radiation pressure. General or empirical accelerations are used to absorb the mis modeled and unmodeled gravitational and surface forces. Without an accelerometer on the COSMIC spacecraft, it is important to properly model the surface forces. In Sections II-B, we describe certain parameters relating to atmospheric drag and solar radiation pressure.

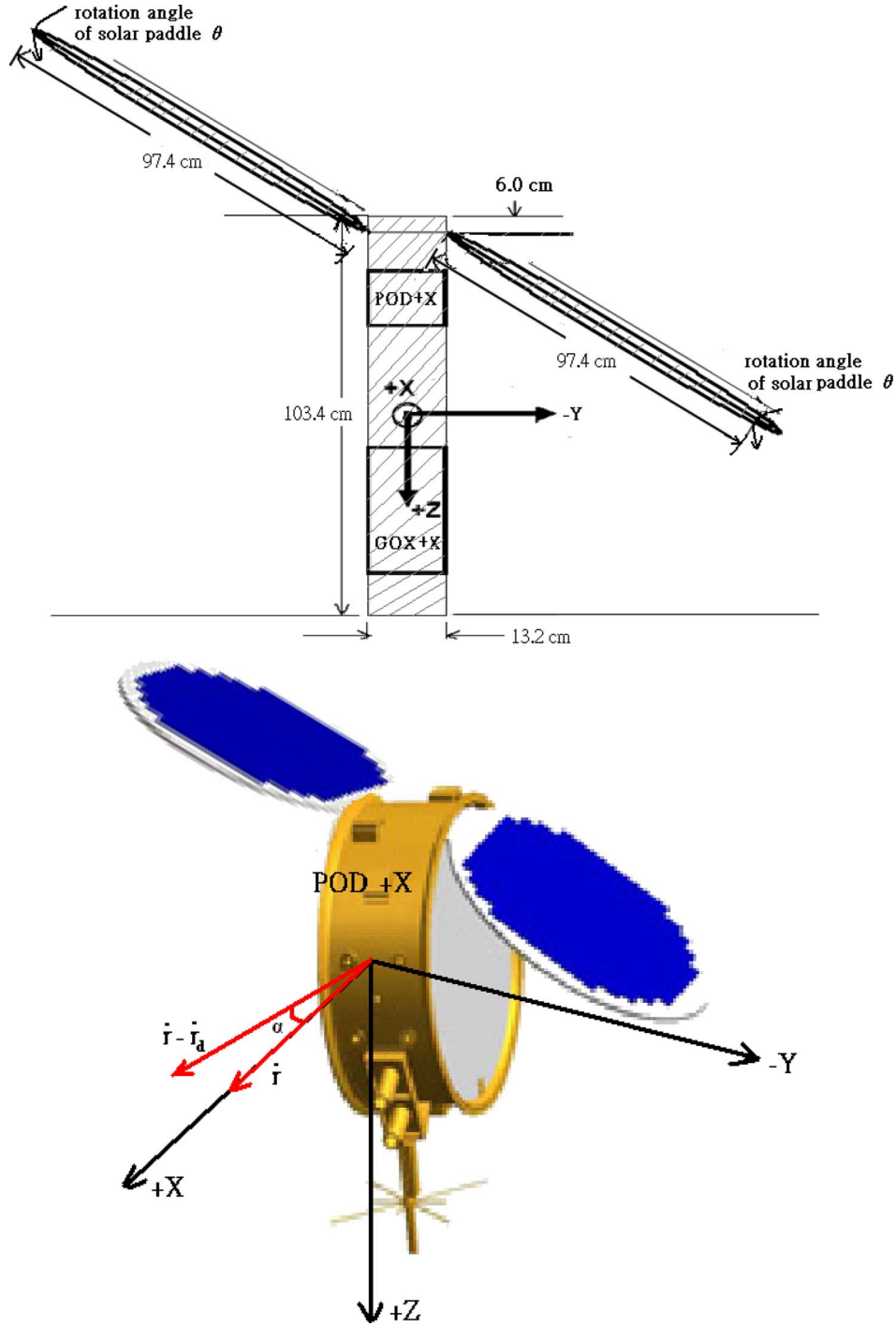


Fig. 2. (Top) Dimensions of the main part and solar panels of a COSMIC LEO, velocity vector  $\dot{\mathbf{r}}$ , and LEO-to-atmosphere vector  $(\dot{\mathbf{r}} - \dot{\mathbf{r}}_d)$ .

### B. Atmospheric Drag and Solar Radiation Effects on Cosmic Satellites

The acceleration vector of a LEO due to atmospheric drag is

$$\mathbf{a}_{\text{drag}} = -\frac{1}{2} C_D \rho \frac{A_d}{m} |\dot{\mathbf{r}} - \dot{\mathbf{r}}_d| (\dot{\mathbf{r}} - \dot{\mathbf{r}}_d) \quad (5)$$

where  $C_D$  is the drag coefficient;  $\rho$  is the atmospheric density;  $A_d$  is the effective area and  $m$  is the mass of the LEO;  $\dot{\mathbf{r}}$ ,  $\dot{\mathbf{r}}_d$  represents the velocity vectors of the LEO and atmosphere in the inertial frame; and  $(\dot{\mathbf{r}} - \dot{\mathbf{r}}_d)$  is the velocity vector of the LEO relative to the atmosphere. For each of the six COSMIC

spacecrafts, the mass (with full thrust fuel) has been determined before the launch (April 2006). The remaining thrust fuel during the flight is observed and used to adjust the time-dependent mass after the launch in [6] and [7]. The effective area  $A_d$  is the projected area of the area of the satellite in the flight direction onto a plane perpendicular to the direction  $(\dot{\mathbf{r}} - \dot{\mathbf{r}}_d)$ . A COSMIC spacecraft travels in a manner that the POD + X antenna points to the flight direction (Fig. 2). Thus, the total area in the flight direction is

$$A_T = A_{\text{main}} + A_{\text{panel}} \quad (6)$$

where  $A_{\text{main}}$  is the area of the main body and  $A_{\text{panel}}$  is the area of two solar panels, which are, respectively, computed as

$$\begin{aligned} A_{\text{main}} &= 1.034 \times 0.132 \text{ (in square meters)} \\ A_{\text{panel}} &= 2 \times \pi \left( \frac{0.974}{2} \right)^2 \sin \theta \text{ (in square meters)} \end{aligned} \quad (7)$$

where  $\theta$  is the rotational angle of the solar panel (Fig. 2). Here, we assume that the thickness of the solar panels is negligible. The effective area is then computed by

$$A_d = A_T \cos^{-1} (\dot{\mathbf{r}} \cdot (\dot{\mathbf{r}} - \dot{\mathbf{r}}_d) / |\dot{\mathbf{r}}| |\dot{\mathbf{r}} - \dot{\mathbf{r}}_d|). \quad (8)$$

The velocity vector of the atmosphere is computed as

$$\dot{\mathbf{r}}_d = \begin{bmatrix} -\omega_h y \\ -\omega_h x \\ 0 \end{bmatrix} \quad (9)$$

where  $x$  and  $y$  are the geocentric coordinate components of the LEO in the inertial frame and  $\omega_h$  is the rotational velocity of atmosphere at an altitude of  $h$  computed in [12] as

$$\omega_h = \omega_e (1 - 1.588187 \times 10^{-3} h + 1.88539 \times 10^{-5} h^2 - 5.108229 \times 10^{-8} h^3 + 3.917401 \times 10^{-11} h^4) \quad (10)$$

where  $\omega_e$  is the mean rotational velocity of the Earth ( $7.292115 \times 10^{-5}$  rad/s) and  $h$  is in kilometers.

The acceleration vector due to solar radiation pressure is

$$\mathbf{a}_{\text{srp}} = \nu P_s C_r \frac{A_s}{m} (\text{au})^2 \frac{\mathbf{r} - \mathbf{r}_s}{|\mathbf{r} - \mathbf{r}_s|^3} \quad (11)$$

where  $\nu$  is a shadow function,  $P_s$  is the solar flux at one astronomical unit (au) ( $4.560 \times 10^{-6}$  N/m<sup>2</sup>),  $C_r$  is the reflectivity coefficient,  $A_s$  is the effective area (different from the effective area for atmospheric drag), and  $\mathbf{r}_s$  is the position vector of the sun. The method to compute the effective area for the solar radiation pressure is the same as that used in the atmospheric drag. In this case, the effective area lies in a plane perpendicular to the vector  $(\mathbf{r} - \mathbf{r}_s)$ .  $C_r$  can be expressed as  $(1 + \varepsilon)$ , where  $\varepsilon$  is the reflectivity (from zero to one), which depends on the material of satellite parts. The shadow function depends on the position of LEO;  $\nu = 0$  when the LEO is in the Earth's shadow and  $\nu = 1$  when the LEO is illuminated by the sun. The orbit dynamic modeling software we use (GEODYN II, see Section II-C) is able to determine shadow functions in the cases of umbra and penumbra, based on the ratio of the sunlight received at the LEO location so that, in practice, the shadow function for a COSMIC satellite varies from zero to one.

### C. Use of GEODYN II for Estimating Parameters of Forces

In this paper, we use the NASA Goddard GEODYN II software to model the perturbing forces described earlier. The mathematical models of the force models used in GEODYN II are given in [9], [13], and [14]. GEODYN II has been used extensively for POD/prediction and force modeling in such Earth resource satellites as TOPEX/Poseidon and GRACE. Temporal gravity fields from such satellite tracking data

TABLE I  
STANDARDS FOR THE ORBIT DYNAMICS OF COSMIC SATELLITES

Model/parameter	Standard
Conventional inertial reference frame	J2000
N-body	JPL DE-403
Earth gravity model	GGM02S [17]
Polar motion	IERS standard 2000
Reference ellipsoid	$a_e = 6378136.3$ m, $f = 1/298.257$
GM	$396800.4415$ km <sup>3</sup> s <sup>-2</sup>
Ocean tides	GOT00.2 [18]
Solid Earth tides	IERS standard 2000
Atmosphere density	Mass Spectrometer Incoherent Scatter (MSIS) Empirical Drag Model [19]
Earth radiation pressure	Second-degree zonal spherical harmonic model [20]
Solar radiation pressure	one coefficient every 1.5 hours
Atmosphere drag	one coefficient every 1.5 hours
General accelerations	9 parameters every 1.5 hours

as Satellite Laser Ranging and GRACE K-band Inter-satellite Ranging have been derived with GEODYN II, e.g., [15] and [16]. The parameters for the force models of COSMIC are given in Table I. For surface forces, we solve for the atmospheric drag coefficient, and the radiation coefficient and nine empirical coefficients of general acceleration along the radial, along-track, and cross-track directions every 1.5 h (one orbital period) using COSMIC orbits (see Section III-A). In preparation for the execution of GEODYN II, a file containing the ephemeris of the planets and a file containing A1UTC, polar motion, and solar and magnetic fluxes must be made ready. A1UTC is the difference between the atomic time used in GEODYN II (A1) and the universal time (UTC), which is available from the Web site <http://hpiers.obspm.fr/eop-pc/>. The solar and magnetic fluxes are obtained from NOAA's web site <ftp://ftp.ngdc.noaa.gov> under the directory STP/GEOMAGNETIC\_DATA/INDICE. These raw data are processed to produce binary files suitable for input to GEODYN II. Other operational details of GEODYN II can be found in the manuals by Pavlis *et al.* [13]. As an example, Fig. 3 shows the estimated atmospheric drag coefficients and reflectivity coefficients from Days 225 to 232 for FM5. These estimated coefficients vary over time, and the mean values/standard deviations of the drag and reflectivity coefficients are 2.12/0.29 and 1.23/0.30, respectively. The general accelerations for COSMIC (Table I) at an altitude of 520 km are on the order of  $10^{-11}$  ms<sup>-2</sup>.

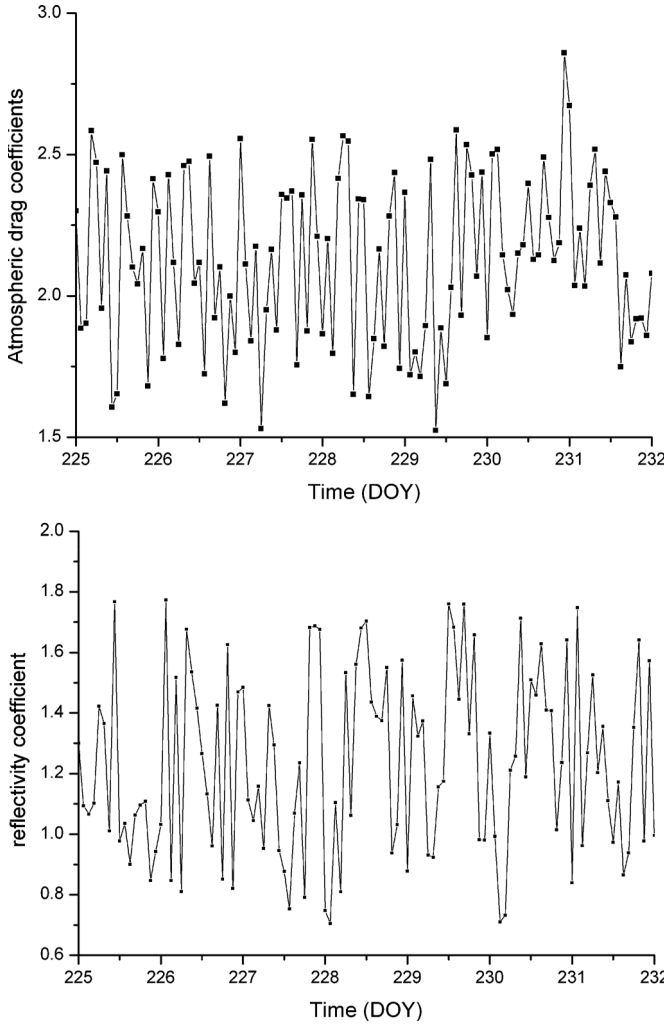


Fig. 3. (Top) Estimated atmospheric drag coefficients and solar reflectivity coefficients of FM5 from Day 225 to 232, in 2006.

### III. EXPERIMENTAL SOLUTION OF TEMPORAL GRAVITY

#### A. Data: Kinematic Orbits of Cosmic and Accuracy Assessment

The kinematic orbits of the six COSMIC satellites used for gravity recovery in this paper are over the time span from August 2 to August 31, 2006. Zero-differenced phase observables were used in determining such kinematic orbits by the software Bernese 5.0. Two approaches are available in Bernese 5.0 for orbit determination using GPS data, which are as follows: the reduced dynamic and kinematic approaches. The reduced dynamic approach estimates orbit-arc-dependent parameters including the initial state vector (six Keplerian elements), nine solar radiation coefficients, and three stochastic pulses every 6 min in the radial, along-track, and cross-track directions. The kinematic approach estimates the kinematic parameters of an orbit arc, including epoch coordinate components, receiver clock errors, and phase ambiguities. In August 2006, the altitudes of the six satellites are 512, 543, 521, 515, 800, and 505 km for FM1, FM2, FM3, FM4, FM5, and FM6, respectively. The inclination angles of all COSMIC orbits are  $72^\circ$ , and the eccentricities are nearly zero.

The six COSMIC satellites will be raised to an altitude of 800 km at the final phase (about 13 months after the launch). We find that the quality of GPS data depends on the quality of satellite attitude data. At an altitude of 800 km, typical standard errors of attitude measurements are  $0.5^\circ$  and  $3^\circ$  over the equator and the polar regions, respectively, and are larger at a lower altitude. When the attitude of a satellite is poorly determined, the uncertainties in the estimated GPS phase ambiguities are relatively large, leading to degraded orbital accuracy.

To reduce noises and data volume, the original 5-s (0.2 Hz) kinematic orbits can be resampled using an algorithm similar to that used in the normal point reduction of satellite laser ranging as in [8]. In this paper, we adopt the algorithm used by the International Laser Ranging Service (<http://ilrs.gsfc.nasa.gov/>) with modification for COSMIC. Specifically, we use the following steps to generate normal point kinematic orbits.

- 1) Use the dynamic orbit as the reference orbit to generate differenced orbits. A differenced orbit component is

$$p_i = x_i^k - x_i^r, \quad i = 1, 2, 3 \quad (12)$$

where  $x_i^k$  and  $x_i^r$  are components of the kinematic and dynamic orbits, respectively.

- 2) Remove large outliers in the kinematic orbit, which will not be used in the subsequent computations. A large outlier is defined as  $|p_i| \geq 20$  cm.
- 3) Within a bin (a window containing many differenced orbits), the differenced orbits are fitted by a polynomial in time using least squares. The polynomial is called the trend function  $f(t)$ .
- 4) For each orbit component, compute the residuals at the times of observations as

$$\nu_i = p_i - f(t_i). \quad (13)$$

- 5) Compute the root-mean-square value (rms) of the residuals. Identify outliers using a rejection level of 2.5 times of rms, and neglect these outliers in step 3) of the next iteration.
- 6) Repeat steps 3)–5) until no outlier is found.
- 7) Divide the accepted residuals into bins starting from 0h UTC.
- 8) Compute the mean value  $\bar{\nu}_m$  and the mean time of the accepted residuals within each bin. The number of accepted residuals within the bin  $m$  is denoted as  $n_m$ .
- 9) For each orbit component, locate the kinematic orbit  $x_m^k$  and its residual  $\nu_m$ , whose observation time  $t_m$  is nearest to the mean time of the accepted residuals in the bin  $m$ .
- 10) Compute the normal point kinematic orbit as

$$NP_m = x_m^k - \nu_m + \bar{\nu}_m. \quad (14)$$

- 11) Compute the standard error of normal points as (if  $n_m = 1$ , this bin is neglected)

$$\sigma_m = \sqrt{\frac{\sum_{j=1}^{n_m} \nu_j^2}{n_m(n_m - 1)}}. \quad (15)$$

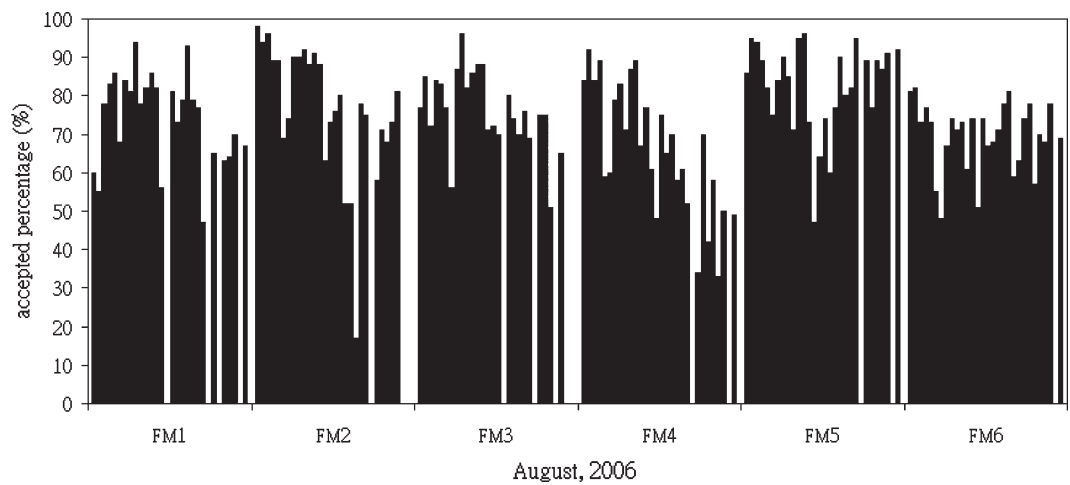


Fig. 4. Percentages of acceptance of kinematic orbits for normal point computations.

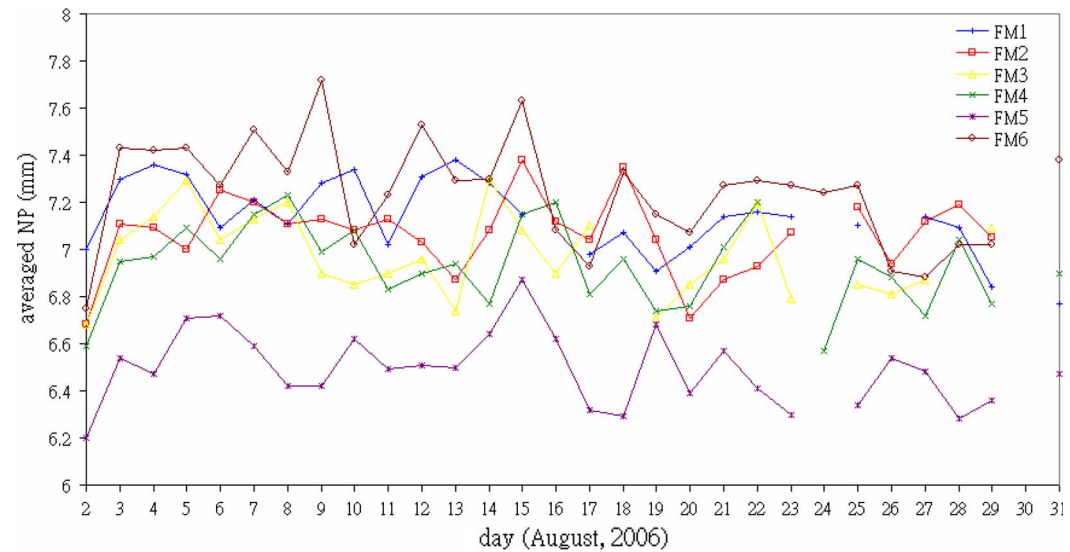


Fig. 5. Standard errors of normal point kinematic orbits in August 2006.

In Step 2), 20 cm is an empirical value. The bin size can be adjusted according to the desired spatial resolution of gravity solution and data compression ratio. The degree of the fitted polynomial increases with the bin size. For a 1-min bin, a 2° polynomial is found to be optimal. Statistically, the standard errors of normal points will be smaller than those of raw orbits.

The normal point kinematic orbits are actually used for gravity recovery. Fig. 4 shows the percentages of accepted 5-s kinematic orbits in August 2006 after removing the outliers. The average percentages of acceptance are 74.3%, 76.5%, 73.4%, 66.0%, 82.5%, and 69.5% for FM1, FM2, FM3, FM4, FM5, and FM6, respectively. In most cases, rejected data are due to bad attitude control and/or poor clock resolution. FM5 has the largest percentage of acceptance due to its 800-km altitude, where the attitude control is better than that of the other five spacecrafts. Fig. 5 shows daily standard errors of the 1-min normal orbits, which will be used as data weights in the gravity recovery. On average, the standard error of the normal point orbits is 7 mm, compared with the 2-cm orbit error for

TABLE II  
STATISTICS OF STANDARD ERRORS OF  
NORMAL POINT KINEMATIC ORBITS

	MAX. (mm)	MEAN (mm)	MIN. (mm)
FM1	7.38	7.13	6.77
FM2	7.38	7.06	6.68
FM3	7.30	6.98	6.68
FM4	7.23	6.93	6.57
FM5	6.87	6.49	6.20
FM6	7.72	7.24	6.75

the raw 5-s orbits. Table II shows the statistics of the standard errors for the six COSMIC satellites in August 2006. Again, FM5 has the least standard error of all satellites in the normal point data.



### B. Method of Gravity Recovery

For gravity recovery, the observables are now normal point kinematic orbits of COSMIC at a 1-min interval. Considering that the focus of this paper is on temporal gravity variation, the unknowns are time-varying geopotential coefficients ( $\bar{J}_{nm}(t), \bar{K}_{nm}(t)$ ) in (4). Based on the linear orbital perturbation theories of Kaula [21] and Hwang [2], the mathematical and stochastic models for the observable–unknown relationship can be expressed as

$$\Delta \mathbf{r}_{\text{obs}} = \mathbf{r}_{\text{kin}} - \mathbf{r}_{\text{ref}} = \Delta \mathbf{r}_1(\boldsymbol{\beta}) + \Delta \mathbf{r}_2(\boldsymbol{\varepsilon}) + \mathbf{e} \quad (16)$$

where the vector  $\Delta \mathbf{r}_{\text{obs}}$  contains residual orbits in the radial, along-track, and cross-track directions; vectors  $\mathbf{r}_{\text{kin}}$  and  $\mathbf{r}_{\text{ref}}$  contain “observed” kinematic orbits and reference orbits, respectively; and vectors  $\boldsymbol{\beta}$ ,  $\boldsymbol{\varepsilon}$ , and  $\mathbf{e}$  contain time-varying geopotential coefficients, empirical parameters, and random errors of kinematic orbits, respectively. The assumption in the model of (16) is that the nongravity origin forces in the reference dynamic orbits from GEODYN II have been removed. The term  $\Delta \mathbf{r}_1$  is the design matrix composed of the time-varying geopotential coefficients. The term  $\Delta \mathbf{r}_2$  is used to compensate for the deficiency of the linear orbital perturbation and usually contains periodic functions with periods of one and two cycles per revolution, plus functions representing resonant effects. Specifically, for each of the radial, along-track, and cross-track residual orbit components, we use the following empirical model for  $\Delta \mathbf{r}_2$ :

$$\begin{aligned} \Delta r_i = & a_0 + a_1 \cos u + a_2 \sin u + a_3 t \cos u \\ & + a_4 t \sin u + a_5 t^2 + a_6 t^2 + a_7 t \sin 2u \\ & + a_8 t \cos 2u + a_9 \cos 2u + a_{10} \sin 2u \end{aligned} \quad (17)$$

where  $i = 1, 2$ , and  $3$  (three orbit components),  $u$  is the argument of latitude, and  $t$  is time relative to an initial epoch.

With GEODYN II, the reference orbits of COSMIC are determined by numerically integrating the equations of motion that take into account all perturbing forces acting on COSMIC satellites (see Section II-A and Table I). As stated in Table I, the coefficients for atmospheric drag, solar radiations, and general accelerations are determined every 1.5 h from the COSMIC kinematic orbits, and then, these estimated coefficients are used in the integration of the equations of motion. The GRACE-derived gravity model GGM02S, as in [17], is used as the Earth’s static gravity model, which contains coefficients ( $\bar{C}_{nm}^0, \bar{S}_{nm}^0$ ) in (4). If the reference orbits are generated using an optimal static gravity model such as GGM02S and all other perturbing forces in (1) are properly modeled, we can assume that the residual orbits in  $\Delta \mathbf{r}_{\text{obs}}$  are linear functions of time-varying geopotential coefficients, as expressed in (16).

For parameter estimation, (16) is transformed to the matrix representation

$$\mathbf{L} = \mathbf{A}\mathbf{X} - \mathbf{V} \quad (18)$$

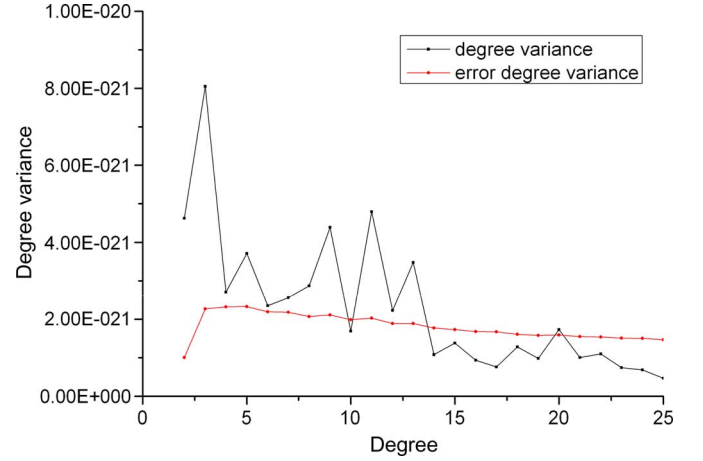


Fig. 6. Degree variance and formal error degree variances of time-varying geopotential coefficients from the COSMIC solution.

where  $\mathbf{A}$  is the design matrix containing the partials of residual orbit components with respect to time-varying geopotential coefficients and empirical parameters and where vectors  $\mathbf{V}$ ,  $\mathbf{X}$ , and  $\mathbf{L}$  contain random errors, unknowns (geopotential coefficients and empirical parameters), and observations (residual orbits), respectively. The needed partials for  $\mathbf{A}$  are detailed in [2] and will not be repeated here. Given *a priori* values of the unknowns and the associated weight matrix  $\mathbf{P}_\mathbf{X}$ , the least square solution of  $\mathbf{X}$  is

$$\mathbf{X} = (\mathbf{A}^T \mathbf{P}_\mathbf{A} + \mathbf{P}_\mathbf{X})^{-1} \mathbf{A}^T \mathbf{P}_\mathbf{L} \quad (19)$$

where  $\mathbf{P}$  is the weight matrix containing inverses of the squared standard errors (Fig. 4 and Table II). Considering that COSMIC is not in a polar orbit, it is necessary to use  $\mathbf{P}_\mathbf{X}$  to stabilize the estimation of  $\mathbf{X}$ . For the geopotential coefficient part of  $\mathbf{P}_\mathbf{X}$ , it is a diagonal matrix containing the inverted variances of time-varying geopotential coefficients. The variances were computed as follows. The geopotential coefficients of GGM02S were subtracted from the monthly coefficients of GRACE gravity models over the period July 2003–August 2006 to obtain monthly time-varying coefficients. The GGM02S and monthly GRACE gravity models are available at the Web site of the Center for Space Research, The University of Texas at Austin (<http://www.csr.utexas.edu/grace>). The degree variances of the monthly time-varying coefficients were computed, and the average degree variances were determined. The average degree variances were then least squares fit to a model whose expression is similar to that of the Kaula rule in [21], i.e.,  $\alpha n^{-\beta}$ , where  $n$  is the spherical harmonic degree (2). The result shows that the average degree variance follows:

$$\bar{\sigma}_n^2 = \frac{1}{2n+1} \sum_{m=0}^n \left( \bar{J}_{nm}^2 + \bar{K}_{nm}^2 \right) \approx 5.65 \times 10^{-22} n^{-1.5} \quad (20)$$

where  $\bar{J}_{nm}, \bar{K}_{nm}$  are coefficients given in (4). A diagonal element of  $\mathbf{P}_\mathbf{X}$  corresponding to any geopotential coefficient of

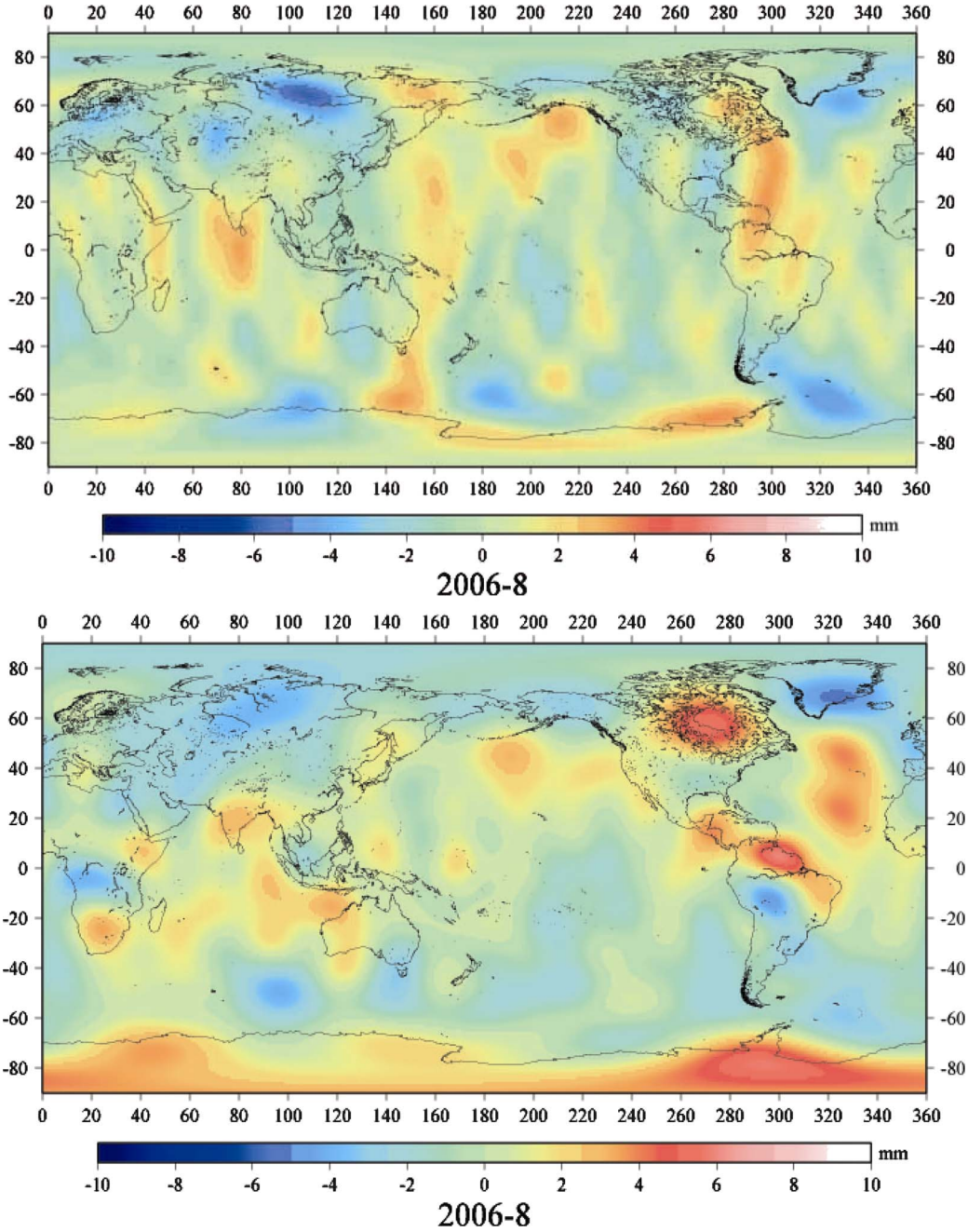


Fig. 7. Geoid variations to spherical harmonic degree 25 from (top) COSMIC and from GRACE.

the same degree is computed by

$$P_{c_{nm}} = P_{s_{nm}} = \frac{1}{\sigma_n^2}. \quad (21)$$

Considering that the orbital inclination of COSMIC is  $72^\circ$ , it is expected that COSMIC GPS data might improve the current gravity models of GRACE at certain regions (the GRACE mission is in polar orbits). In this paper, we also carried out a combined COSMIC and GRACE solution. In this case, the least square solution of  $X$  is

$$X = (A^T P A + \Sigma_g^{-1}) (A^T P L + \Sigma_g^{-1} g) \quad (22)$$

where  $g$  is a vector of time-varying geopotential coefficients from GRACE and  $\Sigma_g$  is the error covariance of  $g$ . Considering that the full error covariance matrices of the GRACE gravity models are not released, only the error variances of the time-varying geopotential coefficients are used for the diagonal elements; hence,  $\Sigma_g$  is in fact a diagonal matrix.

### C. Result

Several experimental gravity solutions were carried out using one month of COSMIC normal point kinematic orbits from the six satellites (Section III-A). Based on numerous tests, we decide to adopt 25 as the maximum degree of expansion (see [1]) for the COSMIC solution. Fig. 6 shows the degree variances



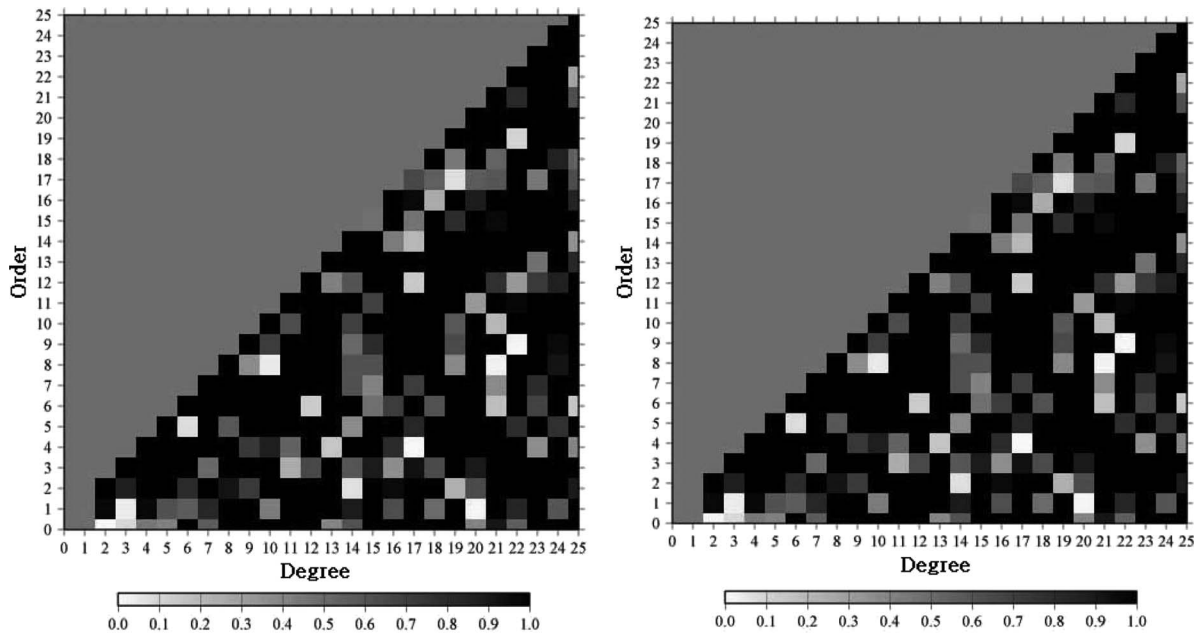


Fig. 8. Relative differences of the COSMIC-derived coefficients with respect to the GRACE-derived coefficients of gravity variation for (left)  $\bar{J}_{nm}$  and  $\bar{K}_{nm}$  up to degree 25.

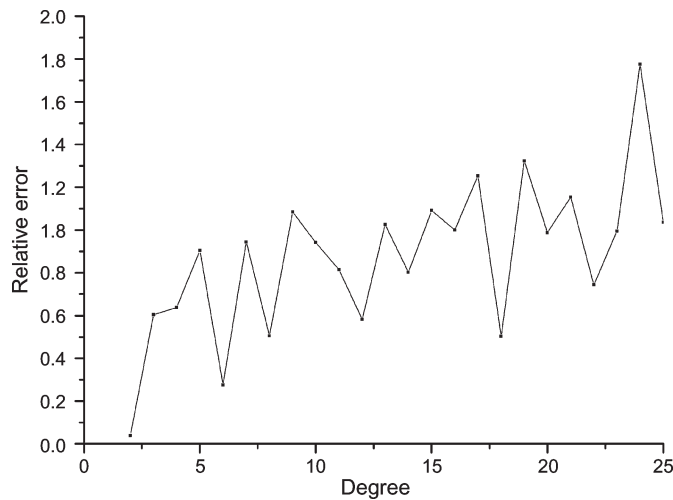


Fig. 9. Same as in Fig. 8 but for the zonal coefficients.

and error degree variances from the COSMIC solution. The error degree variances increase with the degree and are all less than the degree variances below a degree of ten. For spherical harmonic components with degrees lower than ten, the signal-to-noise ratio is larger than one. In Fig. 7, we compare geoid changes from the COSMIC and GRACE solutions to degree 25. Despite the presence of artifacts, the COSMIC solution shows clear geoid highs and lows over Greenland, the Amazon Basin, the India continent, and southern Africa, which resemble those given by the GRACE solution. The geoid variations at latitudes higher than  $72^\circ$  are probably not reliable due to the low inclination angle of COSMIC. Moreover, the GPS data used in the current solutions are from five of the six COSMIC satellites that are at altitudes of about 520 km, where the nongravity forces are difficult to model and the attitude control is not optimal. We expect to see an improved COSMIC gravity solution using data from the operational phase (altitude = 800 km).

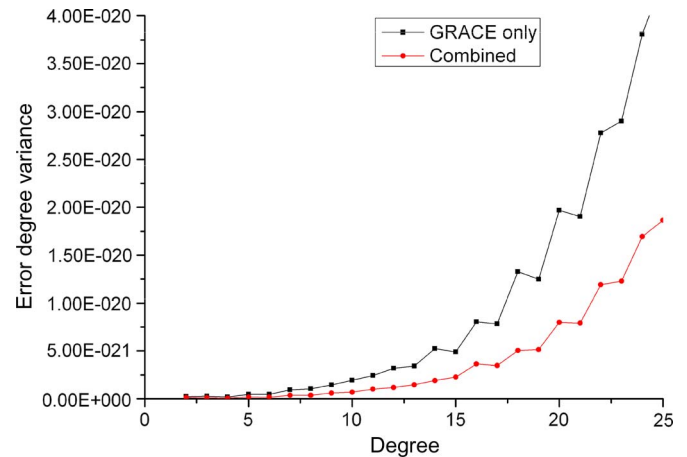


Fig. 10. Formal error degree variances of time-varying geopotential coefficients from the GRACE and combined COSMIC and GRACE solutions.

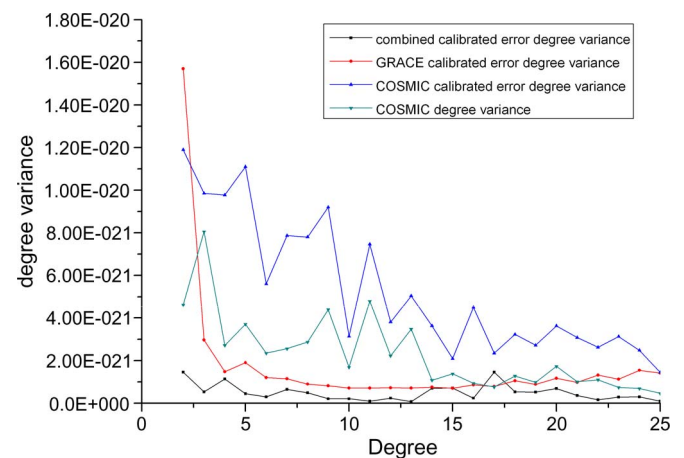


Fig. 11. Degree variances from COSMIC and calibrated error degree variances from COSMIC, GRACE, and combined COSMIC and GRACE solutions.

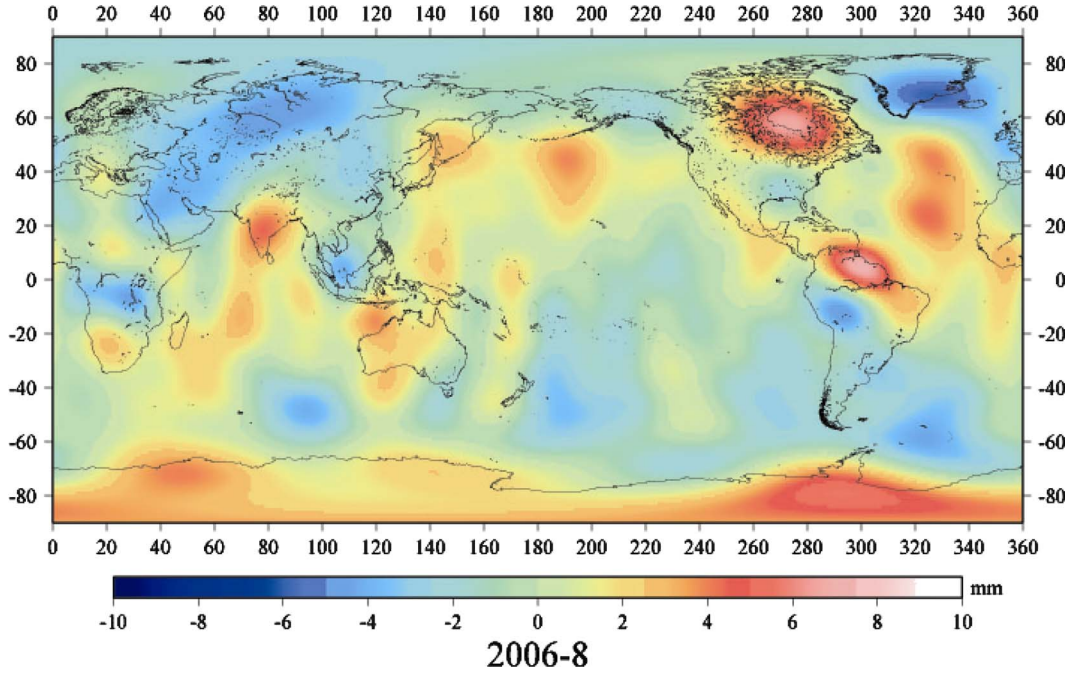


Fig. 12. Geoid variation to spherical harmonic degree 25 from the combined COSMIC and GRACE solution.

In order to compare the geopotential coefficients from COSMIC and from GRACE, we compute the relative differences of coefficients as follows (regarding GRACE-derived geopotential coefficients as the reference values):

$$\varepsilon_{nm}^C = \frac{\bar{J}_{nm}^C - \bar{J}_{nm}^G}{\bar{J}_{nm}^G} \quad \varepsilon_{nm}^S = \frac{\bar{K}_{nm}^C - \bar{K}_{nm}^G}{\bar{K}_{nm}^G} \quad (23)$$

where  $(\bar{J}_{nm}^C, \bar{K}_{nm}^C)$  and  $(\bar{J}_{nm}^G, \bar{K}_{nm}^G)$  are the estimated geopotential coefficients from COSMIC and GRACE, respectively, and  $(\varepsilon_{nm}^C, \varepsilon_{nm}^S)$  represents the relative differences. The relative differences of all coefficients and those of the zonal coefficients are shown in Figs. 8 and 9. The recovered coefficients, with relative differences smaller than one, are regarded as significantly close to the GRACE coefficients. From Figs. 8 and 9, a large portion of the recovered geopotential coefficients from COSMIC is consistent with those of GRACE. However, a significant portion of the COSMIC-derived coefficients disagrees with the GRACE-derived coefficients, which is expected, given the different data qualities from the two missions.

Fig. 10 shows the error degree variances (formal errors) of time-varying geopotential coefficients from the combined COSMIC and GRACE and the GRACE solutions. The combined solution yields error degree variances that are smaller than those from the GRACE solution. We also compute calibrated standard errors of COSMIC-derived geopotential coefficients using the method given in [22], which was used to calibrate the error estimates of GRACE-derived geopotential coefficients. A scaling factor at degree  $n$  and order  $m$  is computed as

$$f_{nm} = \frac{C_{nm}^C - C_{nm}^G}{E_{nm}^C} \quad (24)$$

where  $C_{nm}^C$  and  $C_{nm}^G$  are the estimated geopotential coefficients from COSMIC and GRACE, respectively, and  $E_{nm}^C$  is the uncalibrated error degree variance from COSMIC. A calibrated standard error is obtained by multiplying the uncalibrated standard error by the factor in (24). Fig. 11 shows a comparison of the calibrated error degree variances from GRACE (available at the Center for Space Research, University of Texas at Austin, <http://www.csr.utexas.edu/grace>), COSMIC, and combined solutions. Again, the combined solution yields smaller error degree variances than those from the GRACE solutions.

Fig. 12 shows the geoid variations from the combined COSMIC and GRACE solution. The combined solution closely resembles the GRACE solution, and this is due to the large weights of GRACE coefficients in the combination. Comparing Figs. 7 and 12, we note that the combined solution enhances the geoid signatures of the GRACE solution over central Africa, Russia, and Greenland, as well as the geoid highs in North America, India, and northern Amazon. More quantitative assessments of accuracy and spatial resolution of COSMIC gravity solutions are yet to be carried out, for example, using terrestrial-based gravity measurements at locations with large gravity changes. However, from the gravity solutions derived in this paper, the COSMIC mission shows a potential for gravity recovery. The COSMIC mission coincides with the GRACE mission since April 2006, and its lifetime is expected to be five years. Following the procedure of gravity recovery in this paper, we plan to produce improved monthly temporal gravity fields from the combined COSMIC and GRACE data for a long-term time series analysis of global gravity variation.

#### IV. CONCLUSION

This paper presents the detail of the force modeling for COSMIC satellites. Surface forces, such as atmospheric drag

and solar radiation, are determined using COSMIC's effective area-to-mass ratios, and the relevant coefficients are estimated using COSMIC kinematic orbits over short arcs (1.5 h). The major tool is GEODYN II that is used in many satellite missions requiring precise force modeling and centimeter orbit. Considering the simple geometry of COSMIC spacecrafts, it is believed that the surface forces are adequately modeled in this paper. The 5-s kinematic orbits from six COSMIC LEOs in August 2006 are resampled into 1-min normal points for gravity recovery. A method based on the analytical orbital perturbation of satellite orbit due to the geopotential is used to estimate time-varying geopotential coefficients. The geoid variation from the COSMIC solution shows some major temporal gravity signatures but contains aliasing effects of unknown origins. The combined COSMIC and GRACE solution enhances some local temporal gravity signatures in the GRACE solution. In the near future, we will follow the practice of the GRACE mission to produce monthly gravity fields using combined COSMIC and GRACE data.

## REFERENCES

- [1] B. F. Chao, E. Pavlis, C. Hwang, C. C. Liu, C. K. Shum, C. L. Tseng, and M. Yang, "COSMIC: Geodetic applications in improving Earth's gravity model," *Terr. Atmos. Ocean. Sci. (TAO)*, vol. 11, no. 1, pp. 365–378, Mar. 2000.
- [2] C. Hwang, "Gravity recovery using COSMIC GPS data: Application of orbital perturbation theory," *J. Geod.*, vol. 75, no. 2, pp. 117–136, May 2001.
- [3] D. Švehla and M. Rothacher, "Impact of a LEO formation and a LEO/GPS dual constellation on the IGS products," presented at the Int. GNSS Service (IGS) Workshop, Darmstadt, Germany, May 8–12, 2006.
- [4] D. Švehla and M. Rothacher, "Kinematic and reduced-dynamic precise orbit determination of low Earth orbiters," *Advances Geosci.*, vol. 1, pp. 47–56, 2003.
- [5] D. Švehla and M. Rothacher, "Kinematic precise orbit determination for gravity field determination," in *Proc. Int. Assoc. Geod.: A Window on the Future of Geodesy*, F. Sanso, Ed, Sapporo, Japan, Jun. 30–Jul. 11, 2003, vol. 128, pp. 181–188.
- [6] C. Hwang, T. P. Tseng, T. J. Lin, C. L. Fu, and D. Švehla, "Precise orbit determination for FORMOSAT-3/COSMIC and gravity application," presented at the American Geophysical Union (AGU) Fall Meeting, San Francisco, CA, Dec. 11–15, 2006.
- [7] C. Hwang, T. P. Tseng, T. J. Lin, D. Švehla, and B. Schreiner, "Precise orbit determination for the FORMOSAT-3/COSMIC satellite mission using GPS," *J. Geod.*, 2008, to be published.
- [8] G. Seeber, *Satellite Geodesy*, 2nd ed. Berlin, Germany: Walter de Gruyter, 2003.
- [9] A. Long, Jr., J. Cappellari, C. Velez, and A. Fuchs, Eds., *Goddard Trajectory Determination System, Mathematical Theory*. Washington DC: Flight Dyn. Division, Nat. Aeronautics Space Admin., 1989, Revision 1.
- [10] O. Montenbruck and E. Gill, *Satellite Orbit—Models, Methods, and Application*. Berlin, Germany: Springer-Verlag, 2001.
- [11] W. Heiskanen and H. Moritz, *Physical Geodesy*. Graz, Austria: Inst. Phys. Geod., TU Graz, 1985, Reprint.
- [12] D. G. King-Hele and D. M. C. Walker, "Upper-atmosphere zonal winds from satellite orbit analysis," *Planet. Space Sci.*, vol. 31, no. 5, pp. 509–535, May 1983.
- [13] D. Pavlis *et al.*, *GEODYN Operational Manual*. Greenbelt, MD: Hughes/STX Corp., 1996, 5 volumes.
- [14] D. McCarthy, (1995) *IERS Standards, IERS Technical Note 21* [Online] Available: <http://www.iers.org/MainDisp.csi?pid=47-25786>
- [15] C. Cox and B. F. Chao, "Detection of a large-scale mass redistribution in the terrestrial system since 1998," *Science*, vol. 297, no. 5582, pp. 831–833, Aug. 2002.
- [16] S. Luthcke, D. Rowlands, F. G. Lemoine, S. M. Klosko, D. Chinn, and J. J. McCarthy, "Monthly spherical harmonic gravity field solutions determined from GRACE inter-satellite range-rate data alone," *Geophys. Res. Lett.*, vol. 33, no. 2, p. L02 402, Jan. 2006.
- [17] B. D. Tapley, J. Ries, S. Bettadpur, D. Chambers, M. Cheng, F. Condi, B. Gunter, Z. Kang, P. Nagel, R. Pastor, T. Pekker, S. Poole, and F. G. Wang, "GGM02—An improved Earth gravity field model from GRACE," *J. Geod.*, vol. 79, no. 8, pp. 467–478, Nov. 2005.
- [18] A. Hedin, "Extension of the MSIS thermosphere model into the middle and lower atmosphere," *J. Geophys. Res.*, vol. 96, no. A2, pp. 1159–1172, Feb. 1991.
- [19] P. Knocke, J. C. Ries, and B. D. Tapley, "Earth radiation effects on satellites," in *Proc. AIAA/AAS Astrodynamics Conf. 88-4292-CP*, 1988, pp. 577–587.
- [20] R. D. Ray, *A Global Ocean Tide Model From TOPEX/POSEIDON Altimetry: GOT99.2*. Greenbelt, MD: Goddard Space Flight Center, 1999.
- [21] W. M. Kaula, *Theory of Satellite Geodesy*. London, U.K.: Blaisdell, 1966.
- [22] R. Schmidt, F. Flechtner, R. König, U. Meyer, K.-H. Neumayer, C. Reigber, M. Rothacher, S. Petrovic, S.-Y. Zhu, and A. Güntner, "GRACE time-variable gravity accuracy assessment," in *Dynamic Planet*, vol. 130, P. Tregoning and C. Rizos, Eds. Berlin, Germany: Springer-Verlag, 2007, pp. 230–243. DOI: 10.1007/978-3-540-49350-1\_36.



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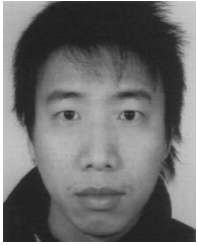
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