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A random graph-based model to analyze packet interference between frequency hopping systems with an application to Bluetooth ☆

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ABSTRACT

In this paper, we present a graph-based model for analyzing interference between two frequency hopping systems for wireless communication. Informally, frequency hopping means that successive packets are transmitted on different frequencies from a pseudo-random sequence. The frequency band occupied by a packet may be narrow or wide. Our model is based on the concept of probabilistic graphs, where a node represents a "channel" and an edge denotes the probability of interference between two packets belonging to two different channels. Two packets on two channels are said to be mutually interfering if the two packets overlap in time and are transmitted on the same frequency. Thus, from the viewpoint of probabilistic graphs, the expected number of nodes with at least one incident edge is a measure of packet interference in a collection of wireless channels. We apply this model of packet interference to a heterogeneous cluster of Bluetooth piconets, where a piconet could be either of 79-hop type or of 23-hop type. Though the 23-hop type has been phased out, we use it as an example in this paper.

1. Introduction

In wireless communication, frequency hopping (FH) is a well established technique for reducing the effect of frequency-selective fading on data rate between two communicating devices. Frequency hopping also allows multiple channels (or, independent networks) to share the same frequency band in a non-collaborative way. In a non-collaborative strategy, a network remains oblivious to the presence of other networks in the same vicinity. Intuitively, in a frequency hopping scheme, successive data packets in a network are transmitted on different frequencies from a pseudo-random sequence of frequencies. The pseudo-random sequence is determined solely by one or more participants in the network. The frequency band occupied by a packet can vary from a small band to a very large one. For example, the Bluetooth system uses frequency hopping at the physical level [5]. Also, most cordless phones use frequency hopping technology that operates within the same spectrum, i.e. 2.4 GHz, as that of Bluetooth system. Bluetooth-enabled devices include laptops, headphones, cell phones, and PDAs, and are becoming part of our life. We, therefore, use Bluetooth system as an example of frequency hopping system in our study.

The presence of multiple frequency hopping channels (or, networks) in the same area will lead to interference between networks. This is because two devices from two different networks may transmit two packets on the same frequency, and the two packets may overlap in time. Depending on the locations of the destinations of those packets, the receivers are likely to receive interfered signals. The consequence of this interference is higher bit error rate and a resulting drop of the two packets. This packet drop will cause a drop in the data rates seen by the two networks. Existence of multiple wireless networks in the same vicinity is not unusual. In hotspots like shopping malls, airports, and university campuses, one may find a high density of devices equipped with Bluetooth or FH interfaces. The following parameters influence interference in a collection of channels:

- the number of distinct hops on each channel,
- the length of a packet on each channel,
- the total number of channels,
- the spectrum width occupied by a packet, and
- synchronization of channels.

Intuitively, larger the number of distinct hops, smaller is the probability of packet interference between two channels. If two

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channels transmit packets of identical lengths, interference is independent of packet length. However, if packets of differing lengths are used on two different channels, their ratio affects the probability of their interference. The larger the spectrum width, the larger is the probability of packet interference. Assuming that packets are of identical lengths, synchronization of packets refers to whether or not two packets on two different channels start at the same time – two channels are said to be synchronized if transmissions of their packets begin at the same time instant. If channels are synchronized, packet interference is expected to be smaller than the interference in case they were not synchronized.

In this paper, we give a model for analyzing packet interference by considering the number of distinct hops on a channel and the total number of channels in both the cases of channel synchronization. In our study we assume that all the packets on all the channels are of identical length. We do not consider how spectrum width affects packet interference.

Our model of interference is based on the concept of probabilistic graphs, where a node represents a "channel" and an edge denotes the probability of interference between two packets belonging to two different channels. Two packets on two channels are said to be mutually interfering if the two packets overlap in time and are transmitted on the same frequency. Thus, the expected number of nodes with at least one incident edge is a measure of packet interference in a collection of wireless channels. We apply this model of packet interference to a heterogeneous cluster of Bluetooth piconets, where a piconet could be either of 79-hop type or of 23-hop type. Here, a piconet is a physical realization of a channel. El-Hoiydi [2] has given a model of packet interference for a homogeneous cluster of piconets. (In a homogeneous cluster, all the piconets are either of 79-hop type or of 23-hop type.) However, extending the model in [2] to a heterogeneous cluster is not a straightforward task.

In Section 2, we explain how the idea of random graphs can be used in modeling interference between two kinds of frequency hopping systems, and present a model of packet interference in a general manner. In Section 3.1, we summarize the operation of a Bluetooth device in the connection state. The notion of packet interference is reviewed in Section 3.2. We present an analytic model of packet interference in the connection state, and show throughput of a piconet cluster under different scenarios in Section 3.3. In this section, we also show that the model developed by El-Hoiydi [2] is in fact a special case of our model. In Section 4, we describe a simulation model of packet interference to validate the analytical model. Some concluding remarks are given in Section 5.

2. Random graph and interference model

We assume that, in a certain geographic area, there is a cluster of N frequency hopping systems (FHSs), or channels. Each channel hops over a number of frequencies. The number of frequencies may be different from channel to channel, giving rise to m different types of hopping systems. In this work, we consider the case of $m = 2$. To develop our model of packet interference in such a system, we first construct a *probabilistic graph* $G = (V, E)$, where V is the set of nodes and E is the set of edges of G. Each node in the set V represents a FHS, and the probability of an edge between two nodes v_i and v_j is denoted by p_{ij} $(\forall i, j)$, $(1 \leqslant i, j \leqslant N)$, where $N = |V|$ and $|V|$ denotes the cardinality of the set V. The quantity p_{ij} denotes the probability of a packet in FHS i being interfered by a packet in FHS j, and vice versa. Let X_N be a random variable representing the number of nodes with no incident edge. We are interested in the expected number of nodes with no incident edge, denoted by $E(X_N)$ (*i.e.* the number of nodes whose packets are not interfered in a given packet transmission slot.) A pair of packets transmitted in two FHSs in their packet transmission slots are said to interfere with each other if the packets are transmitted on the same frequency and the two packets overlap. One must realize that p_{ij} for the case when all clocks of FHSs are synchronized is different from the p_{ij} for the case when all the clocks of FHSs are not synchronized. It is not difficult to see that the synchronous case will lead to less interference than the asynchronous case. To construct an analytical model of $E(X_N)$, we proceed as follows. Notice that there exists an edge between two nodes v_i and v_j if and only if the corresponding FHSs transmit packets on the same frequency at a given instant. We are interested in the expected value of the number of nodes whose packets are not interfered, i.e. the expected number of nodes in G with no incident edges. We then show the model in Theorem 2.1 for a heterogenous cluster of FHSs with two different types of FHS.

Theorem 2.1. *Let G be a probabilistic graph with $N1$ nodes of type a FHS and $N2$ nodes of type b FHS. Let p_{ab} be the probability of an edge between a type a node and a type b node, p_{aa} be the probability of an edge between two type a nodes, and p_{bb} be the probability of an edge between two type b nodes. Also, let $E(X_N)$ be the expected number of nodes of all types having no incident edge. We can represent $E(X_N)$ as follows.*

$$E(X_N) = \sum_{k=1}^{N} k \left(\sum_{r=max(0,k-N2)}^{min(N1,k)} \binom{N1}{r} P_a^r (1-P_a)^{N1-r} \times \binom{N2}{k-r} P_b^{k-r} (1-P_b)^{N2-(k-r)} \right),$$

where P_a is the probability that a node of a type has no incident edge, and P_b is the probability that a node of b type has no incident edge. We express P_a and P_b as follows.

$$P_a = (1-p_{aa})^{N1-1}(1-p_{ab})^{N2} \text{ and}$$
$$P_b = (1-p_{ab})^{N1}(1-p_{bb})^{N2-1}.$$

Proof. A node of the a type has $N1 - 1$ possible links to the nodes of the same type and $N2$ possible links to the nodes of the b type. We, thus, have $P_a = (1-p_{aa})^{N1-1}(1-p_{ab})^{N2}$. Also, for a node of the b type, there are $N2 - 1$ possible links to the nodes of the same type and $N1$ possible links to the nodes of the a type. We, thus, have $P_b = (1-p_{ab})^{N1}(1-p_{bb})^{N2-1}$. Let X_N be the number of nodes of any type with no incident edge in the graph. Also, let X_{N1} be the number of nodes of the a type with no incident edge, and let X_{N2} be the number of nodes of the b type with no incident edge. Since X_{N1} and X_{N2} are independent random variables, we have $X_N = X_{N1} + X_{N2}$. Thus, $X_N = k$ if and only if $X_{N1} = r$ and $X_{N2} = k - r$, for any integer r satisfying $0 \leqslant r \leqslant N1$ and $0 \leqslant k - r \leqslant N2$.

The above restrictions on r are equivalent to $0 \leqslant r \leqslant N1$ and $k - N2 \leqslant r \leqslant k$. Combining these restrictions, we can write $max(0, k - N2) \leqslant r \leqslant min(N1, k)$. We, thus, have

$$P(X_N = k) = \sum_{r=max(0,k-N2)}^{min(N1,k)} \binom{N1}{r} P_a^r (1-P_a)^{N1-r} \times \binom{N2}{k-r} P_b^{k-r} (1-P_b)^{N2-(k-r)}.$$

Therefore,

$$E(X_N) = \sum_{k=1}^{N} k \left(\sum_{r=max(0,k-N2)}^{min(N1,k)} \binom{N1}{r} P_a^r (1-P_a)^{N1-r} \times \binom{N2}{k-r} P_b^{k-r} (1-P_b)^{N2-(k-r)} \right). \qquad \square$$

The reader may note that our analytical model is a generalized one in that whenever the model is used for a cluster of heterogeneous FHSs, we have $|V| = N = N1 + N2$, where $N1$ is the number of the devices of one type and $N2$ is the number of the devices of the other type, and whenever it is used for a cluster of homogeneous FHSs, we simply have N devices of single type. We thus have the following corollary.

Corollary 2.1. *Given a graph G with N nodes of the same type FHS, let $E(X_N)$ be the expected number of nodes having no incident edge. We have*

$$E(X_N) = N(1-p)^{N-1},$$

where p is the probability that a node has no incident edge.

In a cluster of N FHSs, the expected number of packets interfered in a given slot is $N - E(X_N)$. Thus, we express expected packet loss due to interference in a cluster of N FHSs as follows.

$$\text{Packet Loss} = (N - E(X_N))/N. \tag{1}$$

3. Application of the model to Bluetooth

3.1. Background

Bluetooth is a time division multiplexed system, where the basic unit of time division in the connection state is a slot of 625 μs. In a pre-connection state, such as inquiry, paging, or scanning, packet transmission can occur in half slots. In the connection state, a data packet can consist of 1, 3, or 5 slots.

The transmission (Tx) and reception (Rx) parts of data exchanges between devices are synchronized using a real time clock. The Bluetooth clock is a 28 bit counter which is reset to 0 at power up and runs freely thereafter, incrementing every half slot (312.5 μs). Every device has a free running native clock, referred to as CLKN and in a piconet, the Master's CLKN is called *piconet clock*, and it is referred to as CLK in the Master and in all the Slaves in the piconet. CLK provides reference timing for all Tx and Rx operations in a piconet. CLK is a 28 bit clock, and its lowest two bits delimit the slots and half slots for packet transmit and receive. The lower two bits also determine the choice of Tx and Rx, depending on whether the device is operating as a Master or a Slave. For the Master in the connection state, a transmission slot always starts when CLK[1:0] = 00, whereas for a Slave in the connection state, a transmission starts when CLK[1:0] = 10. Since a slot is 625 μs long and CLK increments every half slot, i.e. every 312.5 μs, neither the Master nor a Slave starts a packet transmission when CLK[1:0] = 01 or CLK[1:0] = 11. Thus, it is said that the Master transmits in even numbered slots, whereas a Slave transmits in odd numbered slots. Originally, two kinds of Bluetooth devices had been proposed: one with 79 hop frequencies (for instance, in the US) and another with 23 hop frequencies (for instance, in France).

3.2. Review of packet interference

The 2.4 GHz band, commonly known as the ISM (Industrial, Scientific and Medical) band, will soon be populated with various kinds of devices. Most of these devices are likely to be in the area of wireless personal area networking (WPAN) using the Bluetooth technology and wireless local area networking (WLAN) using the Wi-Fi technology (IEEE 802.11b) [7]. Given the proliferation of such devices, study of packet interference will gain importance. Most of the reports published at the web site of IEEE P802.15.2 task group [12] focus on simulation-based study of interference between packets in the Bluetooth and Wi-Fi technologies. Ennis [3] gives an analytical model of the probability of an overlap, in both time and frequency, of a continuous sequence of Bluetooth packets and an IEEE 802.11b direct sequence. Shellhammer [11] has developed an analytical model of the probability of an 802.11 packet error in the presence of a Bluetooth piconet. Mitter et al. [8] present empirical data from actual measurements of packet error rate in a test bed comprising of one pair of 802.11b transmitter and receiver and one pair of Bluetooth transmitter and receiver. They also measure packet error rate in the said test bed as a function of the distance between the WLAN station and the Bluetooth Slave. Lansford et al. [7] also present measured throughput of Wi-Fi in the presence of Bluetooth as a function of distance. Since distance is highly dependent on the actual physical environment, they use the received signal strength to represent distance. Golmie [4] and Lansford et al. [7] present simulation-based study of the impact of Bluetooth data traffic on the throughput performance of 802.11b.

3.3. Specialization of the model to Bluetooth

3.3.1. Preliminaries

We assume that, in a certain geographic area, there is a cluster of N Masters, each of which communicates with one or more Slaves. We also assume that all the slots of a channel are used for packet transmission – a Master sends packets in the even numbered slots, whereas the Slaves send packets in the odd numbered slots.

A pair of packets transmitted in two piconets in their Tx (Rx) slots are said to interfere with each other if the packets are transmitted on the same frequency and the two packets overlap. We need to further clarify what we mean by "overlap" using Fig. 1. Referring to Fig. 1, let CLK1 be the CLK of a Master M1. Also, CLK21, CLK22, CLK23, and CLK24 are four different instances of the CLK of a second Master M2. The transmission (Tx) slots of both the Masters have been shown to start when the lower two bits of their respective CLK's take on value 00. Similarly, the reception (Rx) slots of both the Masters have been shown to start when the lower two bits take on value 10.

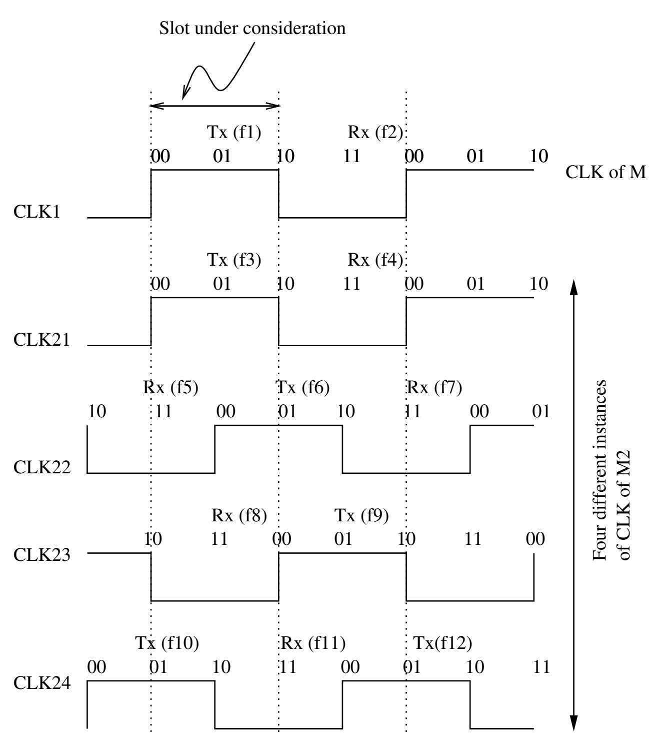

Fig. 1. Packet Interference.

For the purpose of studying interference, we consider just one Tx slot of Master M1 and four different scenarions in Master M2. These four different cases are outlined below.

Case 1: The pair CLK1 and CLK21 represent the fact that Masters M1 and M2 start transmitting packets precisely at the same time.
Case 2: The pair CLK1 an CLK22 represent the fact that the Tx slot of Master M1 overlaps with a part of a Tx and a part of an Rx slot of M2. That is, while M1 is transmitting in its Tx slot, a Slave of M2 transmits in the Rx slot of M2 followed by a transmission from M2.
Case 3: The pair CLK1 and CLK23 represent the fact that M1 and one of the Slaves of M2 transmit at the same time.
Case 4: The pair CLK1 and CLK24 represent the fact that while M1 is transmitting in its Tx slot, Master M2 transmits in its Tx slot followed by a transmission from one of M2's Slaves.

It may be noted that **case 3** is similar to **case 1**, whereas **case 4** is similar to **case 2** in so far as the probability of a packet transmitted in the Tx slot of M1 being interfered is concerned.

Lemma 3.1. *Referring to* **case 1***, if Masters M1 and M2 transmit at frequencies f_1 and f_3, respectively, where f_1 and f_3 are arbitrary members of a set of frequencies of size S, then the probability of f_1 being the same as f_3 is 1/S.*

Lemma 3.2. *Referring to* **case 2***, let Master M1 transmit at frequency f_1, and Master M2 and one of its Slaves transmit at frequencies f_5 and f_6, respectively. Let f_1, f_5, and f_6 be arbitrary members of a set of frequencies of size S, such that $f_5 \neq f_6$. Then, the probability of f_1 being the same as f_5 or f_6 is $w \times 2/S$, where w is the ratio of the duration occupied by the data packet in a slot to the total length of the slot. For single slot data packets, $w = 366/625$.*

Lemma 3.3. *Referring to* **case 1***, if Masters M1 and M2 transmit at frequencies f_1 and f_3, respectively, where f_1 is an arbitrary member of a set of frequencies of size S_1 and f_3 is an arbitrary member of a set of frequencies of size S_2, then the probability of f_1 being the same as f_3 is $1/S$, where S is maximum of S_1 and S_2.*

Lemma 3.4. *Referring to* **case 2***, let Master M1 transmit at frequency f_1, and Master M2 and one of its Slaves transmit at frequencies f_5 and f_6, respectively. Let f_1 be an arbitrary member of a set of frequencies of size S_1, and f_5 and f_6 be arbitrary members of a set of frequencies of size S_2, such that $f_5 \neq f_6$. Then, the probability of f_1 being the same as f_5 or f_6 is $w \times 2/S$, where S is the maximum of S_1 and S_2, and $w = 366/625$.*

3.3.2. Calculating the packet interference and aggregated throughput for a cluster of Bluetooth piconets using the model

Since a node in the probabilistic graph can be of the 23-hop or the 79-hop type, and for discussion convenience, we summarize Lemmas 3.1–3.3, and 3.4 in Table 1. We then show the model for the Bluetooth system in Theorem 3.1.

Theorem 3.1. *Let G be a probabilistic graph with N1 nodes of the 23-hop type and N2 nodes of the 79-hop type, where the probabilities of edges between nodes are specified as in the above three cases. Let $E(X_N)$ be the expected number of nodes of all types having no incident edge. We can represent $E(X_N)$ as follows.*

$$E(X_N) = \sum_{k=1}^{N} k \left(\sum_{r=max(0,k-N2)}^{min(N1,k)} \binom{N1}{r} P_a^r (1-P_a)^{N1-r} \times \binom{N2}{k-r} P_b^{k-r} (1-P_b)^{N2-(k-r)} \right),$$

where P_a is the probability that a node of the 23-hop type has no incident edge, and P_b is the probability that a node of the 79-hop type has no incident edge. We express P_a and P_b as follows.

$$P_a = (1-P_1)^{N1-1}(1-P_3)^{N2} \text{ and}$$
$$P_b = (1-P_3)^{N1}(1-P_2)^{N2-1} = (1-P_2)^{N-1},$$

where the values of P_1, P_2, and P_3 can be obtained from Table 1.

Proof. A node of the 23-hop type has $N1-1$ possible links to the nodes of the same type and N2 possible links to the nodes of the 79-hop type. We, thus, have $P_a = (1-P_1)^{N1-1}(1-P_3)^{N2}$. Also, for a node of the 79-hop type, when $S_2 > S_1$ (which, in fact, is the case), the probability of an edge between the node and any other node is always either $1/S_2$ or $w \times 2/S_2$ depending on if the masters are synchronized or not, which results in $P_2 = P_3$. There are $N-1$ possible links to all other nodes. We, thus, have $P_b = (1-P_2)^{N-1}$. Then, the rest of the proof is the same as that of Theorem 2.1. □

Then, the aggregated throughput is given by the following expression:

$$\text{Aggregated throughput} = E(X_N) \times 10^6 \times (366/625) \text{ bits/s.} \quad (2)$$

It is not hard to see that our developed model can be used for both the synchronous case and the asynchronous one by substituting the appropriate value of p_{ij}, i.e. let $P_1 = 1/23$, $P_2 = 1/79$, and $P_3 = 1/79$ for the synchronous case, and let $P_1 = w \times 2/23$, $P_2 = w \times 2/79$, and $P_3 = w \times 2/79$ for the asynchronous case.

Corollary 3.1. *The model of packet interference developed by El-Hoiydi in [2] is a special case of our model.*

Proof. Using Eq. (1) and Corollary 2.1, we have

$$\begin{aligned} \text{Packet Loss} &= (N - E(X_N))/N, \\ &= 1 - E(X_N)/N, \\ &= 1 - (1-p)^{N-1}, \\ &= 1 - \alpha^{N-1}, \quad \text{where } \alpha = 1 - p, \end{aligned}$$

which is identical to the model developed in [2].

3.3.3. Comparisons

To validate our analytical model for the synchronous case, we compare and show that the aggregated throughput calculated from our model for the 79-hop system is identical to that calculated from El-Hoiydi's model. For the asynchronous case, we notice a small difference between our model and El-Hoiydi's. In El-Hoiydi's model, the probability of packet interference between two piconets with S hop frequencies is $2(1-w) \times 1/S + (2 \times w - 1)(2/S - 1/S^2)$, which is derived by ignoring that two consecutive hop frequencies cannot be identical. However, according to the Bluetooth specification, hop frequencies in two consecutive Tx and Rx slots cannot be identical. Thus, the exact probability quantity is $2(1-w) \times 1/S + (2 \times w - 1)(2/S) = w \times 2/S$, which is instantiated as $w \times 2/23$ in the 23-hop system and $w \times 2/79$ in the 79-hop system in our model stated above. Though a difference of $(2 \times w - 1)/S^2$ for $S = 23$ or $S = 79$ does not lead to a significant difference in the packet error rate, we use the accurate quantity

Table 1
Three possible cases of P_{ij}

Cases	Synchronized p_{ij}	Not synchronized p_{ij}
v_i and v_j are of 23-hop type (P_1)	1/23	$w \times 2/23$
v_i and v_j are of 79-hop type (P_2)	1/79	$w \times 2/79$
v_i and v_j are of different hop types (P_3)	1/79	$w \times 2/79$

in our model. In a nutshell, our model is a general one that considers a heterogeneous cluster of piconets, and models both the synchronous and asynchronous clock cases.

In Fig. 2, we show packet loss as a function of the number of piconets in a cluster. Packet losses for asynchronous and synchronous Masters for the 23-hop, 79-hop, and a heterogenous cluster with 50% of 23-hop and 50% of 79-hop types have been shown in Fig. 2. The aggregated throughputs, when all Masters are asynchronous, for the 23-hop, 79-hop, and heterogenous clusters are shown in Fig. 3. We would like to point out that, according to the Bluetooth specification, the more accurate way of calculating the aggregated throughput for a cluster of Bluetooth piconets should be given by Eq. (3), rather than by Eq. (2). For the purpose of comparing the throughput calculated from our model with that from [2], we first adopt Eq. (2) to draw Fig. 3. However, the accurate aggregated throughputs for a cluster of Bluetooth piconets should be the one given in Fig. 4.

$$\text{Aggregated throughput} = E(X_N) \times 10^6 \times (240/625)\ \text{bits/s}. \quad (3)$$

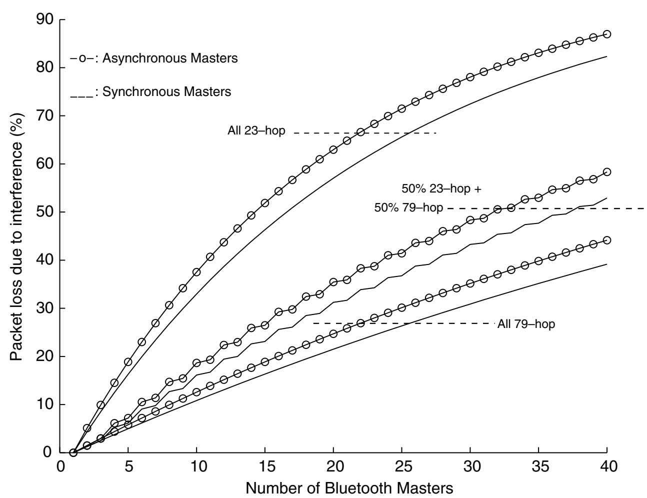

Fig. 2. Packet error rate suffered by a piconet.

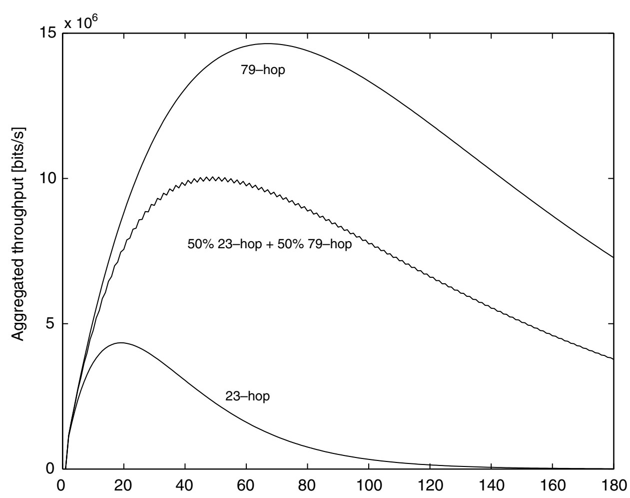

Fig. 3. Aggregated throughput, using $w = 366/625$, in an asynchronous cluster of piconets.

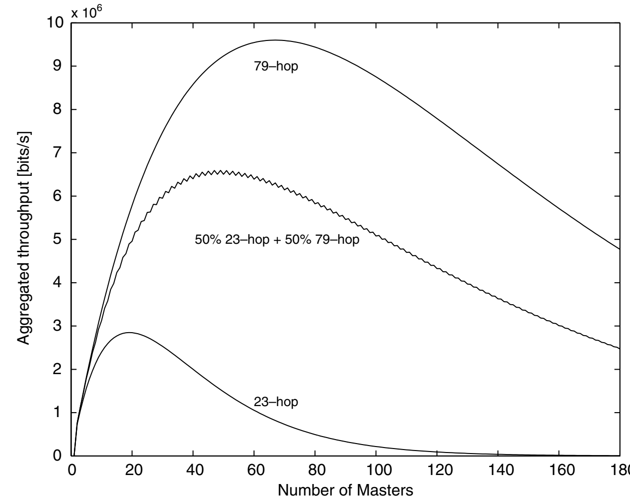

Fig. 4. Aggregated throughput, using $w = 240/625$, in an asynchronous cluster of piconets.

4. Simulations

To validate the analytical model, we have developed a discrete-event simulator in Java. We model each Bluetooth device as an object instance of a `BTDevice` class. The air interface between Bluetooth devices is modeled as an `Air` class. A `Packet` class is used to model different kinds of baseband-layer packets. In the `Air` class, we count the total number of packets transmitted and the number of packets interfered over the entire duration of simulation.

A virtual clock drives the simulation process. For the synchronous case, the interval between two ticks of the virtual clock is assumed to be half-a-slot, that is 312.5 μs. For the asynchronous case, we must make this tick interval very small, say 1 μs, in order to accurately simulate packet interference, and the execution time of the simulator would have been about 312.5 times longer than that of the synchronous case. We thus focus our simulations on the synchronous case. Based on the way we develop the analytical model, it will not hard to understand that once the synchronous case of the model is verified by simulation results, the asynchronous case should also be verified by the simulations.

One of the most complex tasks in the design of the simulator is implementing the *hop selection* box specified in Section 11 of the Baseband Specification of Bluetooth [5]. In order to validate the implementation of the hop selection box, we obtained test data from actual measurement of frequencies used in the communication between two Bluetooth modules as shown in Fig. 5. The two Bluetooth modules shown in Fig. 5 are from Zucotto Wireless Inc. [6], and the Bluetooth protocol analyzer is Arca Technology's *Wavecatcher*™ Bluetooth Protocol Analyzer [1]. We established a

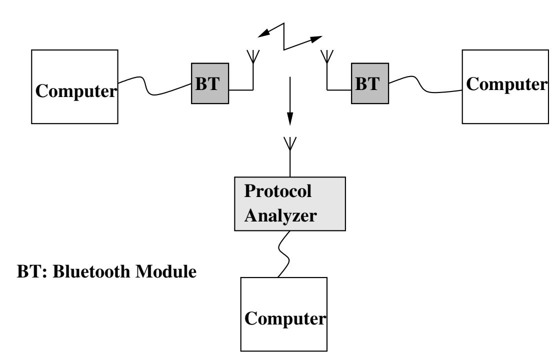

Fig. 5. Using a Bluetooth Protocol Analyzer.

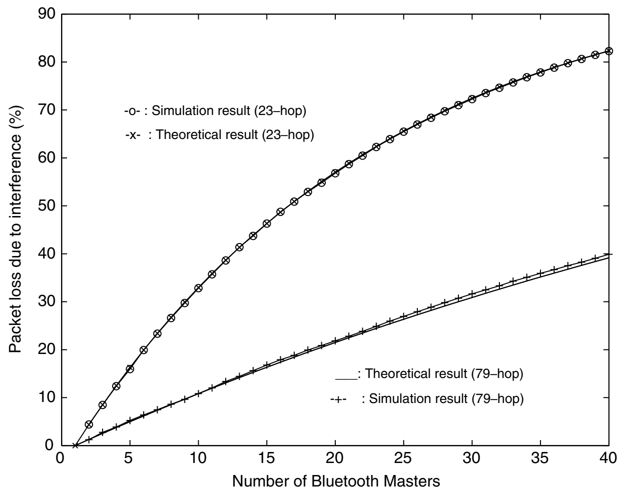

Fig. 6. Packet loss due to interference (sync).

link between the two Bluetooth modules and transmitted data between the Master and the Slave. Using the protocol analyzer we captured packets at the Baseband layer. The protocol analyzer displays packets in a formatted manner with additional information, such as the CLK value of the channel, the Bluetooth device address of the Master, and the frequency at which the packet was transmitted. We collected these data from the analyzer and fed them to our simulator to verify the correctness of implementation of the hop selection scheme.

In Fig. 6, we compare the analytical model of packet loss due to interference, as derived in Eq. (1), with the simulated packet loss. The simulated packet loss shown in Fig. 6 corresponds to **Case 1** in Fig. 1, which means CLK's of all the Masters have their Tx slots synchronized. In Fig. 6, we have shown two different kinds of Bluetooth clusters. First, all Bluetooth devices in a cluster are of the 23-hop type. Second, all Bluetooth devices in a cluster are of the 79-hop type.

We make two important observations from Fig. 6. First, in the 79-hop system, packet loss due to interference increases by almost 1% for each additional Master in a cluster of Bluetooth devices. Second, in a cluster of Masters of the 23-hop type, packet interference rapidly increases with increase in the number of Masters.

5. Conclusion

In this paper, we presented a model for analyzing interference between wireless channels where frequency hopping is used in packet transmission. We identified the control parameters of frequency hopping that affect interference. Our analysis model is based on the concept of probabilistic graphs, where a node represents a channel and an edge between two nodes represents the probability of interference between the two channels. Now, computation of interference becomes a matter of finding the expected number of nodes with at least one incident edge. Such a generalized model can take into account several interference contributing factors, such as the total number of channels in a certain geographic area, the distinct number of frequency hops, and synchrony or asynchrony between channels. We applied this idea to the study of packet interference in a heterogeneous cluster of Bluetooth piconets. We can also apply the proposed model in a study of packet interference in a cluster of piconets that use multiple-slot packets (i.e. a cluster of piconets transmitting 1-slot, 3-slot, or 5-slot packets) [10].

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