

行政院國家科學委員會補助專題研究計畫期中進度報告

計畫編號：NSC 96 - 2628 - M - 009 - 015 - 計畫類別： 個別型計畫

執行期間： 96 年 8 月 1 日至 97 年 7 月 31 日

計畫名稱： 具封閉性質的距離正則圖(1/3)

計畫主持人： 翁志文 (國立交通大學應用數學系)

weng@math.nctu.edu.tw

計畫參與人員： 博士生：潘業忠、黃喻培、黃皓文。 碩士生：卜文強、曾雅榕、汪政緯、陳雅玲、方仁洲、廖彥琳。

97 年 5 月 22 日

一、 中文摘要

我們研究無三角形且有五邊形的具古典參數之距離正則圖，已得到主要的分類上的結果，此結果將成為博士生潘業忠的博士論文主要部分。另一方面在有平行四邊形的距離正則圖上，我們發現也能探討測地封閉距離政則子圖的存在性，這與之前的研究都建立在無平行四邊形這假設完全不同。不過想要很廣泛證明這些子圖的存在性，需要適當的假設及嚴密的推理，希望在暑假期間能完成部份結果。

關鍵詞： 距離正則圖、 測地封閉子圖

二、 英文摘要

We obtain a major classification of a triangle-free distance-regular graph with classical parameters and a pentagon. This result will be included in Pan, Yeh-jong's Ph.

D. thesis. We also find a way to study the geodetically closed subgraphs in a distance-regular graph which contains parallelogram.

Keywords: distance-regular graph, geodetically closed subgraphs

三、 緣由與目的

The D -bounded distance-regular graphs were introduced in 1997[7] by the project investigator. This became an important concept in the classification of classical distance-regular graphs of negative type in 1999[4]. There are many interesting geometric structures constructed from a D -bounded graph that need to be investigated. They also have applications to pooling designs [11], [12], [13]. Recently the project investigator notices that several authors devote themselves to the study of D -bounded distance-regular graphs as results shown in [17], [18], [19], [20], [21]. This lures the project investigator going back to this line of study.

Recall that a sequence x, y, z of vertices of G is *geodetic* whenever

$\partial(x, y) + \partial(y, z) = \partial(x, z)$. Hence a sequence x, y, z of vertices of G is *weak-geodetic* whenever

$$\partial(x, y) + \partial(y, z) \leq \partial(x, z) + 1.$$

A vertex subset O of G is *weak-geodetically closed* if for any weak-geodetic sequence x, y, z of G , $x, z \in O \Rightarrow y \in O$. A weak-geodetically closed subset is also called a *strongly closed* subset in some literatures, e.g. [21]. G is i -bounded if for any vertices x, y of G with distance at most i , x, y are contained in a common regular weak-geodetically closed subgraph of diameter $\partial(x, y)$. It was shown in 1998[5] that if G is distance-regular with diameter $D \geq 3$, intersection numbers $a_1 \neq 0$, $c_2 \neq 1$ and without parallelograms of length up to $i + 1$, then G is i -bounded. For the case $a_1 = 0$, recently we show G is 3-bounded under an additional assumption, Q -polynomial.

Let G denote a D -bounded distance-regular graph, where D is the diameter of G . Putting all the weak-geodetically subgraphs together ordering by reversed inclusion we have a poset structure. It was shown in [7], this poset is a ranked meet semi-lattice with lower semi-modular atomic intervals. In [19], [20], the Mobius functions on intervals of the poset are investigated. The dual interval with a single vertex as the least element is studied in [17] and with more assumption, a strongly regular graph will be found from the set of rank 2 elements of the dual interval. The project

investigator believes there are many more properties in this poset to be found.

四、 結果與討論

Let G be triangle-free distance-regular with classical parameters and a pentagon. We have shown that the graph G is 3-bounded. By using the 3-bounded property of G , we construct a ranked lattice of rank 3. We conclude from the study in this lattice that the intersection number c_2 of G is at most 2. The above line of study is based on the 3-bounded property which implies the graph G does not contain a parallelogram. We also find a way to study the existence of geodetically closed subgraphs in a distance-regular graph which does contain a parallelogram.

五、 計畫成果自評

The result about the classification is submitted to JCTB in title [“A note on triangle-free distance-regular graphs with a pentagon”](#). There are two cases remaining open in the above classification, one in $c_2=2$ and the other in $c_2=1$. Further study is necessary. There is another paper in title [“A Combinatorial Representation of Coxeter Graphs over a Field of Two Elements, submitted to JA”](#) submitted to Journal of Algebra.

References

- [1] Yeh-jong Pan and Chih-wen Weng, Triangle-free Distance-regular Graphs, submitted to Journal of Algebraic Combinatorics
- [2] Terwilliger, P. and Weng, C., An inequality for regular near polygons, European Journal of Combinatorics 26, 227-235, 2005
- [3] Terwilliger, P. and Weng, C., Distance-regular graphs, pseudo primitive idempotents, and the Terwilliger algebra, European J. Combin. 25, 287--298, 2004
- [4] Weng, C., Classical Distance-Regular Graphs of Negative Type, Journal of Combinatorial Theory Series B, 76: 93-116, 1999
- [5] Weng, C., Weak-geodetically Closed Subgroups in Distance-Regular Graphs, Graphs and Combinatorics, 14: 275-304, 1998
- [6] Liang, Y. and Weng, C., Parallelogram-Free Distance-Regular Graphs, Journal of Combinatorial Theory Series B, 231-243, 1997
- [7] Weng, C., D-bounded Distance-Regular Graphs, European Journal of Comb. 18, 211-229, 1997
- [8] Weng, C., Kite-Free P- and Q-Polynomial Schemes, Graphs and Combinatorics, 201-207, 1995
- [9] Hau-wen Huang, Yu-pei Huang and Chih-wen Weng, More on Pooling Spaces, submitted to Discrete Mathematics
- [10] Ito T., Terwilliger P. and Weng C., The quantum algebra $U_q(\mathfrak{sl}_2)$ and its equitable presentation, Journal of Algebra 298 (2006) 284-301 .
- [11] A. G. D'yachkov, Frank K. Hwang, A. J. Macula, P. A. Vilenkin and Chih-wen Weng, A Construction of Pooling Designs with Some Happy Surprises, Journal of Computational Biology, 12(8), 1127-1134, 2005
- [12] Tayuan Huang and Chih-wen Weng, Pooling Spaces and Non-Adaptive Pooling Designs, Discrete Mathematics, 282(1-3), 163-169, 2004
- [13] Tayuan Huang and Chih-wen Weng, A note on decoding of superimposed codes, Journal of Combinatorial Optimization, 7, 381-384, 2003
- [14] Shih, C. and Weng. C., On the Templates Corresponding to Cycle-Symmetric Connectivity in Cellular Neural Networks, International Journal of Bifurcation and Chaos, 12:2957-2966, 2002
- [15] Wei, P., Weng, C., A Typical Vertex of a Tree , Discrete Math, 226:337-345, 2001
- [16] Shih, C., Weng, C. Cycle-Symmetric Matrices and Convergent Neural Networks, Physical D, 146:213-220,2000
- [17] Suogang Gao and Jun Guo, A construction of a Distance-regular Graph with Diameter 2, submitted to G & C
- [18] Jun Guo and Suogang Gao, Lattices Generated by Join of Strongly Closed Subgraphs in d-bounded Distance-regular Graphs, submitted to DM

[19] Jun Guo, Wen Liu, Suogang Gao,
Some counting formulae of strongly closed
subgraphs in distance-regular graphs,
preprint

[20] Suogang Gao, Jun Guo, Wen Lin,
Lattices Generated by Strongly Closed
Subgraphs in d -bounded Distance-regular
Graphs, preprint

[21] Hiroshi Suzuki, On Strongly Closed

Subgraphs with Diameter Two and

Q -Polynomial Property, preprint, 2004