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子計畫九：適用於晶片上電力傳輸分析之階層模組簡化技術

(3/3)

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共同主持人： 周景揚

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低功率系統之設計及自動化

子計畫九：適用於晶片上電力傳輸分析之階層模組簡化技術(3/3)

“Power Delivery Network Analysis with Hierarchical Model Order Reduction Techniques”

計畫編號：NSC 94－2220－E009－022

執行期間： 94 年 8 月 1 日 至 95 年 7 月 31 日

主持人：李育民 交通大學電信工程系教授

一、中文摘要

本計劃是整合型計畫『低功率系統之設計及自動化』中的一個三年的子計劃，主要目的為發展晶片上電力傳輸的快速分析工具。

在計劃的前兩年中，我們已利用傳輸線模型（transmission-line-modeling）階層式模組簡化（hierarchical model order reduction）及馬可夫鏈（Markov chain）技術，發展或實現數個準確且高效率的電力傳輸分析工具[R1~R3] 。在計劃的最後一年，我們除了實現以馬可夫鏈技術為基礎的電力傳輸分析工具[R3]及利用模組簡化技術加速晶片上電力傳輸網路的設計[R4]；亦將利用聚成型代數多重網格(aggregation-based algebraic multigrid)的技術發展並實現另一套晶片上電力傳輸網路的快速分析工具[R7]。

這篇報告發展一個以聚成型代數多重網格方法(AbAMG)提供高效率的功率傳輸網路分析，有別於傳統的代數多重網格，我們提出了創新的全域多重映射運算子，不僅增加疏鬆網格運算子的稀疏性，減少運算的複雜度，同時更具有較好的收斂速率。此論文所提出的方法可於 167.6 秒內分析大小為兩百萬以上的電路(包括直流分析和 50 次暫態分析)，且其最大誤差少於 1%。由數據上分析而言，相較於參考文獻[1]，此方法在速度上了超過 26 倍，同時記憶體也只用了 40%。且與先進技術 IEKS[2] 方法相比，速度也快了超過 1.25 倍，記憶體使用量也僅其一半。實驗結果顯示此方法在功率傳輸網路分析上提供了極為準確且非常高的效率。

英文摘要

This report presents an aggregation-based algebraic multigrid (AbAMG) method to efficiently analyze the power delivery network. Different from the conventional algebraic multigrid (AMG) scheme, an innovative constructing method of global inter-grid mapping operator is employed to not only enhance the sparsity of coarse grid operator for reducing the computational complexity but also solve the problem with better convergent rate. The proposed method can solve the circuit with size over two millions in 167.6 CPU seconds (including DC analysis, and transient analysis with 50 time steps), and the maximum error is less than 1% . The significant runtime improvement, over 26X faster than the InductWise[1], over 2X faster than the IEKS method[2] and over 1.25X faster than traditional AMG, and less memory usage, 40％ of the memory usage in [1] and half of the memory usage in [2], are demonstrated. Experimental results show that this AbAMG solver is very efficient in both runtime and memory usage.

二、計畫的緣由與目的

With the ultra deep sub-micron technology, several features of chips (higher operating frequencies, larger number of transistors, smaller feature sizes of transistors and lower supply voltages) have made the integrity issue of power delivery network become a key factor of high performance designs[3]. Generally, the power delivery network contains enormous amount of circuit elements and the efficient analyzers are necessary. Thus, the general circuit solvers, such as SPICE/HSPICE, by using direct methods are not suitable for the power delivery analysis. In the past years, various efficient methods have been developed for the power distribution network analysis. The preconditioned conjugate gradient method is applied for solving power grid analysis in[4]. The hierarchical methods are

developed in [2][5]. The improved extended Krylov subspace (IEKS) method developed in [2] extends the model order reduction technique to deal with time-varying current sources without the moment shifting procedure. Multigrid-like methods are developed in [6][7] to map the original problem to a reduced system by using the geometric properties of circuits. However, the frameworks proposed in [6][7] are hard to handle the coupling effects of mutual inductances. Hence, an adaptive algebraic multigrid (adaptive AMG) method is used in [8] to analyze the power network. It reformulates the system matrix and views the problem as an algebraic problem which doesn't need the geometry information. Hence, it can handle the coupling effects.

Generally, the mapping operator of AMG [9] is determined by each row equation of $Ae \approx 0$, where A is the system matrix and e is the error vector of unknown variables. The quality of mapping operator strongly counts on the selection of coarse grids, and the constructed mapping operator only contains the local information of A. This mapping operator may lose a few of important error terms because of the inadequate choice of coarse grids, and hence, degrade the convergence rate. Therefore, an adaptive choosing method of coarse grids is developed in [8] to remedy this undesirable behavior. However, this choosing strategy needs to reconstruct the mapping operator at each time step and may boost the CPU time. To solve the above problems, we develop a global mapping operator construction procedure. Our mapping operator construction procedure is based on the aggregation AMG methods[10~12]. The idae of aggregation originated in economics[13], where products in a large scale system were aggregated to become a small system. This procedure can achieve significant reduction in the problem size, and still maintains the accurate representation of overall behaviors. In multigrid terminology, the set of coarse grids is a subset of fine grids. An algebraic partition is performed to the fine grids and the system matrix is partitioned into several aggregated sub-matrices. The mapping operator of aggregation AMG method is constructed from the system's global eigen-decomposition property. Theoretically, the error in the direction of an eigenvector associated with a large eigenvalue is rapidly reduced by relaxation, and the error in the direction of an eigenvector associated with a small eigenvalue is reduced by a factor which may approach 1 as the eigenvalue closes to 0 [14]. The eigenvectors associated with small eigenvalues of each sub-matrix are calculated to approximate the smooth error components of the system matrix and the mapping operator P is composed by these eigenvectors. The mapping operator can project the system matrix to a reduced system matrix which is much sparser than the transformed system of conventional AMG[8], and also achieves better convergence rate.

In Section 三.C.Ⅴ, we will present the *resistance dominant property* of the system matrix of power delivery network when determining the aggregation. With this *resistance dominant property*, the dimension of each aggregated sub-matrix is not greater than 4, and, hence, the eigenvectors of those sub-matrices can be easily solved by using the QR algorithm[15]. Furthermore, an innovative matrix compensation algorithm with a global error estimation procedure is developed to further improve the quality of mapping operator. The mapping operator construction procedure of AbAMG only needs to be performed once for all time steps. With these properties, the AbAMG method can construct a much better mapping operator than the traditional AMG method, and achieve better performance for solving the power distribution network problem.

The remainder of this report is organized as follows. The formulation of power delivery network problem is given in Section 三.A. Then, the general AMG method is introduced in Section 三.B. After that, the key part of the AbAMG method, the global mapping operator construction, is described in Section 三.C. Finally, the experimental results, and conclusion are given in Section 三.D, and Section 四, respectively.

三、研究方法及成果

A *Formulation of Power Delivery Network Problem*

The power delivery network can be modeled as a RLKC (resistance, inductance, susceptance, and capacitance) network, and the devices connecting to the power delivery network are viewed as many time-varying current sources with gate capacitances[5]. By using the modified nodal analysis (MNA) method[16], the circuit equation of a linear RLKC circuit can be represented as

$$\tilde{G}\tilde{x}(t) + \tilde{C}\frac{d}{dt}\tilde{x}(t) = \tilde{B}\tilde{u}(t) + \tilde{G}_E v_E(t), \qquad (1)$$

where

$$\tilde{G} = \begin{bmatrix} G_n & -A_{l_n}^T \\ A_{l_n} & 0 \end{bmatrix}, \qquad \tilde{C} = \begin{bmatrix} C_n & 0 \\ 0 & L \end{bmatrix},$$

$$\tilde{x}(t) = \begin{bmatrix} v_n(t) \\ i_l(t) \end{bmatrix}, \qquad \tilde{G}_E = \begin{bmatrix} G_E \\ L_E \end{bmatrix}.$$

The $v_E(t)$, $v_n(t)$, and $i_l(t)$ correspond to the vectors of independent voltage sources, unknown nodal voltages, and branch currents flowing through inductors, respectively. The G_n is the stamping matrix of resistors not connecting to $v_E(t)$. The C_n, and L are the stamping matrices of capacitors, and inductors, respectively, The A_{l_n} is the coefficient matrix related to the inductors not connecting to $v_E(t)$. The $\tilde{u}(t)$ is the vector of independent current sources. The $\tilde{B}$, G_E, and L_E are the coefficient matrices related to $\tilde{u}(t)$, the stamping of resistors between $v_n(t)$ and $v_E(t)$, and the connecting of L and $v_E(t)$, respectively.

By applying the trapezoidal approximation to Equation (1) with time step h, it becomes

$$\begin{bmatrix} 2C_n/h + G_n & -A_{l_n}^T \\ A_{l_n} & 2L/h \end{bmatrix} \begin{bmatrix} v_n(t+h) \\ i_l(t+h) \end{bmatrix} = 2\begin{bmatrix} G_E \\ L_E \end{bmatrix} v_E(t) + \begin{bmatrix} 2C_n/h - G_n & A_{l_n}^T \\ -A_{l_n} & 2L/h \end{bmatrix} \begin{bmatrix} v_n(t) \\ i_l(t) \end{bmatrix} + \tilde{B}\begin{bmatrix} \tilde{u}(t+h) + \tilde{u}(t) \\ 0 \end{bmatrix}. \qquad (2)$$

We can observe that the system matrix in Equation (3) is not symmetric positive definite (SPD). Since the multigrid method requires the matrix to be SPD[14], we split the variable vector into a nodal voltage vector and a branch current vector. By using block matrix operations, the system equations can be reformulated as follows.

$$\left(\frac{2C_n}{h} + G_n + \frac{h}{2}A_{l_n}^T L^{-1} A_{l_n}\right) v_n(t+h) = \left(\frac{2C_n}{h} - G_n - \frac{h}{2}A_{l_n}^T L^{-1} A_{l_n}\right) v_n(t) + 2A_{l_n}^T i_l(t) + \tilde{B}(\tilde{u}(t+h) + \tilde{u}(t)) + hA_{l_n}^T L^{-1} L_E v_E(t) + 2G_E v_E(t) \qquad (3)$$

$$i_l(t+h) = i_l(t) - \frac{h}{2}L^{-1}A_{l_n}\left(v_n(t+h) + v_n(t)\right) + hL^{-1}L_E v_E(t) \qquad (4)$$

Since the matrices G_n, C_n, and $K \equiv L^{-1}$ are SPD, we can prove that the system matrix of Equation (3) is still SPD. The K matrix[17] is much sparser than the L matrix, and the above symmetric property can save 50 % of the memory usage. Equation (4) is equivalent to solve $Ax = b$, where A is equal to $(2C_n/h + G_n + hA_{l_n}^T L^{-1} A_{l_n}/2)$, x is the unknown variable vector $v_n(t+h)$ and b is a known vector, and this problem can be solved by the two-level AMG method[9]. After $v_n(t+h)$ being solved, $i_l(t+h)$ can be calculated by Equation (5).

B *Multigrid Method*

A two-level AMG method[9] is stated as follows. Given an $Ax = b$ problem, where A is a $N \times N$ system matrix and x is the $N \times 1$ unknown variable vector, a *relaxation* step is applied to eliminate the oscillatory error components on those fine nodes (unknown variables) with dimension N and the residual vector on fine nodes is calculated by $r = b - Ax$. Then, the residual vector is *restricted* to a few of coarse nodes with a small dimension $M < N$ by $r^c = Rr$, and the reduced system matrix is constructed by the Galerkin operator $A^c = RAP$. Here, R is a $M \times N$ matrix, P is a $N \times M$ matrix, and $R = P^T$. On the coarse nodes, the residual equation, $A^c e^c = r^c$, is solved and the error correction vector e^c is *interpolated* back to the fine nodes by $e = Pe^c$.

The smooth error components which are not eliminated well by the relaxation on fine nodes can be eliminated by the error correction vector e^c. A complementary two-level solution scheme can be constructed to overcome the stalling behavior of smooth error components in general iterative methods[9]. The corrective solution is obtained by $\hat{x} = x + e$, and a post-relaxation step is applied on fine nodes to ensure that the oscillatory error is not introduced through the coarse-node correction step. The two-level solution method can be easily extended to multilevel, and the efficiency of the multigrid method strongly depends on the quality of mapping operator. The constructions of mapping operators of AMG and aggregation AMG are discussed in Section 三.B.I and 三.B.II.

B.I *Mapping Operator Construction of AMG*

Since the purpose of AMG method is to develop a multilevel scheme to efficiently eliminate all error components, the smooth error components which the relaxation cannot eliminate well must be represented by the mapping operator of AMG. The traditional AMG methods construct the mapping operator with the property of algebraic smoothness, $(Ae)_i \approx 0$, which means that the residue is small after several relaxations at each row i. The equation of $(Ae)_i \approx 0$ can be expanded as

$$a_{ii} e_i \approx -\sum_{j \neq i} a_{ij} e_j . \qquad (5)$$

Using the *Color Scheme* algorithm[9], a coarser discretion is performed to the set of fine grid nodes and the set of coarse nodes can be determined according to the connections of each node i in the matrix graph of A. If $|a_{ij}| \geq \theta |a_{ii}|$, we say that node j strongly influences i and vice-versa. Here, θ is a threshold factor between 0 and 1, and is often chosen to be 0.25. The nodes which strongly influence many nodes are defined as coarse nodes since they can approximate other nodes well. However, the above selecting procedure of coarse nodes only considers local connections of each node i, and may result in unsuitable coarse nodes which can degrade the convergence rate. To further discuss the construction of mapping operator, Equation (6) can be rewritten as

$$a_{ii} e_i \approx -\sum_{j \in C_i} a_{ij} e_j - \sum_{k \in F_i^S} a_{ik} e_k - \sum_{m \in F_i^W} a_{im} e_m \qquad (6)$$

The set of total original nodes is defined as $C \cup F$, where C is the set of coarse nodes and F is the set of remaining fine nodes. The set C_i is equal to $C \cap N_i$, and the set F_i is equal to $F \cap N_i$. Here, N_i is the set of neighboring nodes of node i. The set F_i can be divided into F_i^s and F_i^w, where F_i^s is the set of nodes which strongly influence i in F_i, and F_i^w is the set of nodes which weakly influence i in F_i.

From Equation (7), an interpolation formula for each fine node error variable, e_i, by its neighboring coarse nodes error variables is defined as $e_i = \sum_{j \in C_i} w_{ij} e_j$ [9], where

$$w_{ij} = -\frac{a_{ij} + \sum_{k \in F_i^s} \left(\frac{a_{ik} a_{kj}}{\sum_{l \in C_i} a_{kl}} \right)}{a_{ii} + \sum_{m \in F_i^w} a_{im}}. \quad (7)$$

Here, the coefficient of each node error variable corresponding to the fine node in F_i^s is approximated by a sum of the coefficients of node error variables corresponding to the coarse nodes in C_i, and the coefficients of each node error variable corresponding to the fine node in F_i^w are simply added to the coefficinet a_{ii}. The selection of F_i^s and F_i^w is only determined by the coefficients in Equation (7) and this might lead to the inappropriate choice of F_i^s and F_i^w since those nodes of F_i^w with large errors should be labeled in the set of F_i^s. This unsuitable selection can decrease the convergent rate of AMG. To remedy the disadvantages of traditional AMG methods, a global mapping operator construction for the aggregation AMG methods is proposed in Section 三.C.

B.II *Mapping Operator Construction of Aggregation AMG*

The idea of the mapping operator construction of aggregation AMG is based on the concept that the smooth error components are in the directions of system's eigenvectors associated with small eigenvalues[12][14]. Firstly, an algebraic partition is performed to the connection graph of system matrix A, and the nodes which represent the unknown variables with strong influence between them are clustered together. After the aggregating procedure, each aggregated sub-matrix is eigen-decomposed and the eigenvectors related to the small eigenvalues are used to compose the mapping operator P. With accurately calculating the directions of system's smooth error components, the aggregation AMG can achieve better convergence rate than the traditional AMG methods. However, the small connected coefficients between aggregations are usually neglected or added to the diagonal elements of the aggregated sub-matrices. Hence, the convergent rate of aggregation AMG might be declined[11]. To improve the convergent rate, an innovative matrix compensation algorithm with a global error estimation procedure is developed in our AbAMG method and is described in the next section.

C *Global Mapping Operator Construction*

In this section, a global mapping operator construction for the AbAMG method is presented. At first, a node-by-node aggregation algorithm is shown in Section 三.C.I. Then, the global error estimation procedure and matrix compensation algorithm are stated in Section 三.C.II and Section 三.C.III, respectively. After that, the mapping operator construction procedure is summarized in Section 三.C.IV. Finally, the practicability of aggregation AMG method for the power delivery network analysis problem is addressed in Section 三.C.V.

C.I *Aggregation Algorithm*

One goal of aggregation method is to reformulate the original system matrix such that the smooth error components of system can be easily calculated from the modified system. Different from the difficulty of performing a geometry partition on the circuit topology because of the mutual inductance coupling effects, the aggregation method provides an easy approach in the algebraic manner and simplifies the problem.

The node i and node j are defined to have a strong connection if any strong influence

relation exists between them. Please refer to Section 三.B.I for the meaning of strong influence. Otherwise, we say that they have a weak connection. The above definition of strong connection provides a reasonable measurement when determining aggregations. A node having the maximum number of strong connections is a good candidate to be the starting node of aggregation algorithm, and those nodes with strong connections between them must be labeled in the same aggregation since their values might be highly correlated. On the other hand, the nodes with weak connections between them should be classed to different aggregations. Each node can only be included in an aggregation. The aggregation algorithm is shown in Fig. 1 and 2.

An example of aggregation algorithm is shown in Fig. 3. Given a system equation $Ax = b$, the system matrix A and its matrix connection graph are shown at the left hand side of Fig. 3. The modified system matrix A after aggregating and rearranging is shown at the top of the right hand side of Fig. 3. After that, this modified system matrix can be further simplified into three isolated aggregated sub-matrices a, b, and c, and the smooth error components of original system can be derived by the eigen-decomposition analysis of these sub-matrices. Because the nodes with weakly connected coefficients might have large errors in the global view, the way of discarding the coefficients or adding them into to the diagonal elements of aggregations[11] is not a suitable simplification strategy. Therefore, a global error estimation procedure is developed in Section 三.C.II to estimate these troublesome error components, and the related matrix compensation algorithm is shown in Section 三.C.III to remedy this defect.

C.II *Global error Estimation*

An intuitive way to get the information of errors which the relaxation cannot efficiently eliminate is applying the relaxation scheme to an auxiliary problem with a known solution. The system homogeneous problem, $Ax = 0$, provides us the proper information for this purpose. The error of the approximated solution of $Ax = 0$ can be known since its solution is a zero vector. By applying the relaxation several times to this problem with a random initial guess, we can get an error vector, e^g, which consists of the error components that the relaxation can not eliminate well. This candidate error vector can be employed in the following matrix compensation algorithm.

Algorithm of Aggregation

Input: The Graph of System Matrix A of Nodes 1, 2,…, n and the Related Weights $w_1, w_2, \ldots, w_n$ of these Nodes

Output: Aggregations 1,2,…,m

```
1   Begin
2     NodeCounter=0, AggCounter=0
3     While NodeCounter!=n
4       MaxWeight=0, StartNode=1
5       For each node i
6         If node i is not in any aggregation
7           If wi>MaxWeight
8             MaxWeight=wi, StartNode=i
9       EndFor
10      AggCounter++
11      j=AggCounter, StartNode is labeled in
        aggregation j
12      NodeCounter++
13      AggreConstruct(j, NodeCounter,
        StartNode and its strongly connect
        nodes)
14  End
```

Fig.1 Algorithm of Aggregation

Algorithm of AggreConstruct

Input: AggCounter j, NodeCounter, The StartNode and it's Strongly Connected Nodes $n_1, n_2, \ldots, n_s$

```
1  Begin
2    For each strongly connected node k of node i
3      If the node k is not in any aggregation
4        the node k is labeled in aggregation j
5        NodeCounter++
6        AggreConstruct(j, NodeCounter, node k
                        and its strongly connected
                        1nodes)
7    EndFor
8  End
```

Fig.2 Algorithm of AggreConstruct

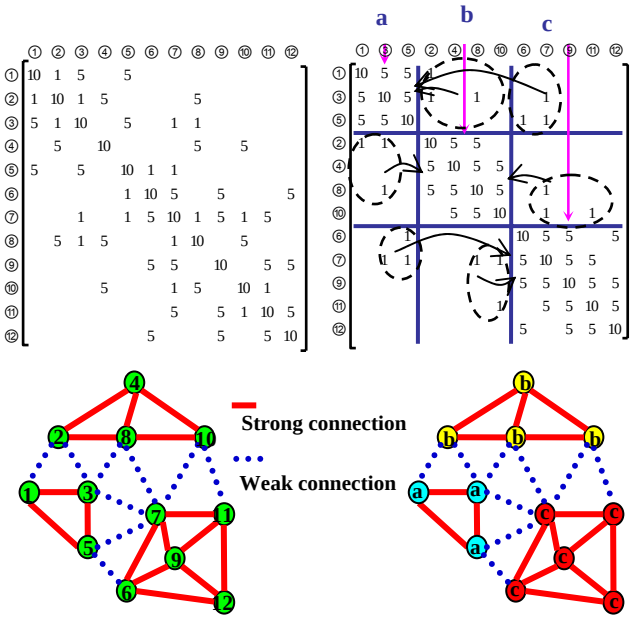

Fig.3 An example of aggregation

Algorithm of Matrix Compensation

Input: *System Matrix A and Aggregation 1, 2, …,n*

Output: *Aggregated Sub-matrices $A^1, A^2, \ldots, A^n$*

```
1   Begin
2     For each aggregation m
3       For each node i in aggregation m,
            sweep the i-th row of A
4         If node j is in aggregation m
5           A_ij^m = A_ij
6         Else If node j is a strong node,
7           sweep the j-th row of A
8           total = 0
9           For each column k in row j
10            If node k is in aggregation m
11              total += A_jk
12          EndFor
13          For each column k in row j
14            If node k is in aggregation m
15              A_ik^m += A_ij × A_jk / total
16          EndFor
17      EndFor
18    EndFor
19  EndFor
```

Fig. 4. Algorithm of Matrix Compensation

C.III *Matrix Compensation Algorithm*

Given a linear algebraic system equation $Ae^g = 0$ with dimension N and the global error estimation e^g from Section 三.C.II, a node i is defined as a strong node if $e_i^g \geq \lambda \max(e_1^g, \cdots, e_N^g)$, and is defined as a weak node if $e_i^g < \lambda \max(e_1^g, \cdots, e_N^g)$. Here, $e_1^g, \cdots, e_N^g$ are the elements of vector e^g, and λ is a positive constant which is less than 1 and is chosen to be 0.25 in our algorithm.

The weakly connected coefficients related to the strong nodes are properly distributed to aggregated nodes with suitable weights as shown in Fig. 4, and the effects related to the weak nodes are simply neglected. With this compensation step, each modified aggregated

sub-matrix is isolated from other sub-matrices and a better reduced system can be constructed to achieve a smaller error which is demonstrated in Section 三.D. The matrix compensation algorithm is shown in Fig. 4. The compensation procedure exactly matches with the weight calculation step in the traditional AMG methods. The modified sub-matrices are used for the coarse grid construction introduced in the next subsection.

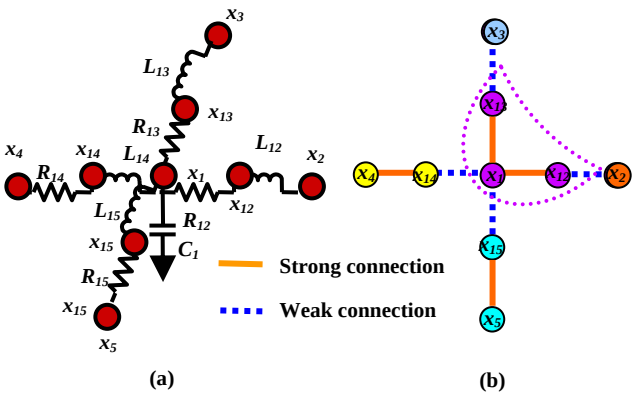

Fig.5 Structure of aggregation with size 3

C.IV *Mapping Operator Construction*

In this section, we summarize the coarse grid construction of the aggregation-based AMG method. After executing the matrix compensation algorithm, those modified aggregated sub-matrices are isolated from each other, and the eigenvalue decomposition procedure is performed to each modified aggregated sub-matrix. Finally, the eigenvector corresponding to the smallest eigenvalue of each sub-matrix is used to compose the inter-grid transfer operator P, and the coarse grid operator can be constructed by the Galerkin operator $A^c = RAP$, where $R = P^T$. This mapping operator construction of AbAMG is determined from the global information of system matrix, and only needs to be performed once. The experimental results are shown in Section 三.D.

C.V *Practicability of Aggregation AMG Method for Power Network Analysis*

Before demonstrating the experimental results of the proposed AbAMG method, we discuss the practicability of aggregation AMG method for the power network analysis. The aggregation AMG method is applied to the $Ax = b$ problem in Equation (4), where A is equal to $(2C_n/h + G_n + hA_{l_n}^T L^{-1} A_{l_n}/2)$, and the unknown variable vector x consists of all nodal voltages of power delivery circuit at each specific time.

From the predictive technology model (PTM) developed by the Berkeley university (http://www.eas.asu.edu/ ptm/), the unit value of R, L, and C in the 0.13 μm technology are $0.046\Omega/\mu m$, $1.69\,pH/\mu m$, and $0.13011\,fF/\mu m$, respectively. For the length of wire segment being around 100 μm, the values of $2C_n/h$, G_n, and $hA_{l_n}^T L^{-1} A_{l_n}/2$ vary from 5E-3 to 8E-3, 2E-1 to 4E-1, and 2E-2 to 5E-2, respectively. The value of G_n term is over 8 times larger than other terms of A. From the aggregation algorithm discussed in Section 三.C.I, we can know that the determination of the aggregation of A is dominated by the effects of the resistances of G_n. This R-dominant phenomenon still exists if we choose different reasonable wire lengths or more advanced technologies such as 0.9 μm or 0.65 μm technologies. For the on-chip power delivery network with mesh structure, most aggregations have the size with 3 as shown in Fig. 5, and the maximum size of aggregation is equal to 4 if the via is connected

to the structure of Fig. 5. Hence, the eigenvectors of each sub-matrix can be easily solved by using the QR algorithm[15].

D *Experimental Results*

This section demonstrates the speed and accuracy of the proposed AbAMG solver, and compares our results with other methods. The power delivery networks are randomly generated as mesh networks which consist of lumped RLKC segments with many independent time varying current sources, and the supply voltage is 1 volt. This solver is implemented in C++ language and tested on a Pentium IV 3.4-GHz machine with 3 GB memory.

First, an efficient and accurate time domain solver InductWise[1] is used to demonstrate the accuracy of our method. The results are shown in Table I. In Table I, ``MaxVD" is the maximum voltage drop of each circuit, and the maximum voltage drop of each circuit is around 15% of the supply voltage. Even without executing the matrix compensation algorithm, our AbAMG solver can still achieve a smaller maximum error than the traditional AMG method[9] which is applied to the power delivery network analysis, and its average error is also less than or equal to the AMG method. With the help of matrix compensation algorithm, the AbAMG can further reduce the maximum error to be less than 0.973% (AMG is 1.173%) for each test circuit, and the average error to be less than 0.067% (AMG is 0.077%). The above demonstrates that the aggregation-based AMG method can accurately capture the error directions which can not be eliminated in the relaxation procedure of fine grid nodes, and our matrix compensation algorithm does improve the accuracy.

To show the efficiency of our AbAMG solver, the analysis of DC and 50 transient time steps are executed and the results are compared with two efficient methods, IEKS[2] and InductWise[1], and the traditional AMG method [9]. The comparison of results is shown in Table II for different RLKC circuits. In Table II, ``RT" is the CPU run time, ``Mem" represents the memory usage, ``RT*" is the run time of AbAMG without compensation, and ``RT**" is the run time of AbAMG with compensation. The speedup of our method for each test circuit is shown in Table III. In Table III, S_{In}, S_{IEKS}, S_{AMG} and S_{*} are the speedup of AbAMG with compensation respect to InductWise, IEKS, AMG, and AbAMG without compensation. The significant speed improvement, over 26 times faster than the InductWise[1], over 2 times faster than IEKS[2], over 1.25 times faster than AMG, and over 1.12 times faster than AbAMG without compensation, and less memory usage, 40% of the memory usage in [1] and half of the memory usage in [2], are observed. The proposed AbAMG solver can solve the DC and transient nodal voltages with 50 time steps of a circuit with size being 2.4472M in 167.6 CPU seconds, and this indicates that the proposed simulator is very efficient in solving power delivery networks and capable of handling the circuit with size over millions.

The number of the non-zero terms of system matrix, and the iterative cycles needed for 100 time step transient analysis of AMG or AbAMG are shown in Table IV. Here, ``Fine Grid NZ", and ``Coarse Grid Nz" are the number of the non-zero terms of original system matrix, and non-zero terms of reduced system matrix, respectively. The ``Cycle", ``Cycle*", and ``Cycle**" are the total number of multilevel cycle of AMG method, the total number of multilevel cycle of AbAMG without compensation, and the total number of multilevel cycle of AbAMG with compensation, respectively. From the results, we can see that the number of non-zero terms in the reduced system matrix of AbAMG is only one third of the reduced system matrix of AMG method. In the other words, the reduced system matrix by using AbAMG is sparser than the matrix by using the AMG method. Hence, its

Circuit Size	Max VD (v)	AMG		AbAMG without compensation		AbAMG with compensation	
		Max Error (%)	Avg Error (%)	Max Error (%)	Avg Error (%)	Max Error (%)	Avg Error (%)
49.6K	0.150	1.173	0.0.77	1.165	0.077	0.973	0.067
199.2K	0.168	1.107	0..066	1.059	0.065	0.885	0.058
448.8K	0.165	1.170	0..065	1.072	0.064	0.955	0.056
798.4K	0.158	1.130	0..067	1.081	0.067	0.967	0.059

TABLE I

ERROR PERCENTAGE OF RLKC CIRCUITS

Circuit Size	InductWise[1]		IEKS[2]		AMG		AbAMG		
	RT(s)	Mem (MB)	RT(s)	Mem (MB)	RT(s)	Mem (MB)	RT(s)	RT**(s)	Mem (MB)
49.6k	78.34	111	6.25	68	3.953	46	3.593	2.972	40
199.2k	391.70	424	29.76	308	15.875	182	14.235	12.719	156
448.8k	1576.00	994	82.56	747	38.187	407	32.594	28.312	351
798.4k	2903.00	157	131.31	1230	68.219	721	59.328	51.812	624
1.248k	X	X	X	X	105.590	1130	93.750	83.156	974
1.7976M	X	X	X	X	152.360	1627	137.312	119.360	1401
2.4472M	X	X	X	X	X	X	196.531	167.600	1907

TABLE II

RUNTIME OF PLKC CIRCUITS. “X” DENOTES THIS METHODOLOGY FAILED

Circuit Size	Speed up			
	SIN	SIEKS	SAMG	S*
49.6k	26.36	2.10	1.33	1.21
199.2k	30.80	2.34	1.25	1.12
448.8k	55.70	2.92	1.35	1.15
798.4k	56.03	2.53	1.32	1.15
1.248k	-	-	1.27	1.13
1.7976M	-	-	1.28	1.15
2.4472M	-	-	-	1.17

TABLE III

SPEED UP OF AbAMG COMPARED TO OTHER METHODS

Circuit Size	Fine Gird NZ	AMG		AbAMG		
		Coarse Grid NZ	Cycle	Coarse Grid NZ	Cycle*	Cycle**
49.6k	119.5K	92K	118	33.8K	127	101
199.2k	480.0K	370K	113	135.6K	131	100
448.8k	1082.0K	834K	111	305.6K	131	101
798.4k	1925.0K	1484K	114	543.0K	135	103
1.248k	3009.0K	2320K	109	849.0K	131	102
1.7976M	4335.0K	3342K	108	1223.0K	132	100

TABLE IV

COMPARISON BETWEEN AbAMG AND AMG

computational load is lighter than the AMG's. Furthermore, because the matrix compensation algorithm can compensate those big error terms from the information of system homogenous problem, $Ae^g = 0$, the number of multilevel cycle used by AbAMG with compensation approaches the minimum number needed (Ideally, the minimum number of cycle for 100 time steps analysis is 100.) and is much smaller than the AMG method. Therefore, the speed of

analysis can be improved further.

四、結論與討論

In this report, an AbAMG solver for the power distribution network has been developed. The proposed methodology can handle the circuit size with more than two million in 167.6 CPU seconds, and the maximum error of each RLKC test circuit is less than 1% . The significant speed improvement and the less memory usage show that this AbAMG method is very suitable for analyzing the power delivery network. The global construction of mapping operator with the matrix compensation algorithm does improve the performance of AbAMG, and a sparser reduced system matrix is performed with the better convergence rate than the traditional AMG method.

In this research, we have published two journal papers [R1][R2] in IEEE TCAD and four international papers[R3~R6], and submitted one journal paper[R7] to TCAS-I and two papers[R7][R8] to international conferences.

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[R2] Yu-Min Lee, Yahong Cao, Tsung-Hao Chen, Janet Wang, and Charlie Chung-Ping Chen "HiPRIME: Hierarchical and Passivity Peserved Interconnect Macromodeling Engine for RLKC Power Delivery", IEEE Trans. Computer-Aided Design of Integrated Circuits And Systems (TCAD), Vol. 24, No. 6, pp. 797-806, June, 2005.

[R3] Pei-Yu Huang, Chih-Hong Hwang, Po-Han Lai, and Yu-Min Lee, "Hierarchical Power Delivry Network Analysis via Bipartite Markov Chain", *The 13th Workshop on Synthesis and System Integration of Mixed Information Technologies* (*SASIMI*), April 2006.

[R4] Pei-Yu Huang, Yu-Min Lee, Jeng-Liang Tsai, and Charlie Chung-Ping Chen, ‘‘Simultaneous Area Minimization and Decaps Insertion for Power Delivery Network Using Adjoint Sensitivity Analysis with IEKS Method", *IEEE International Symposium on Circuits and Systems* (*ISCAS*), May 2006.

[R5] Cheng-Hsuan Chiu, Pei-Yu Huang, Zhe-Yu Lin, and Yu-Min Lee, "Crosstalk-Driven Placement with Considering On-Chip Mutual Inductance and RLC Noise", *The 13th Workshop on Synthesis and System Integration of Mixed Information Technologies* (*SASIMI*), April 2006.

[R6] Yih-Lang Lin, Pei-Yu Huang, and Yu-Min Lee, ‘‘Performance- and Congestion-Driven Multilevel Router", *The 13th Workshop on Synthesis and System Integration of Mixed Information Technologies* (*SASIMI*), April 2006.

[R7] Yu-Min Lee, Huang-Yu Chou, and Pei-Yu Huang, "An Aggregation-based Algebraic Multigrid Method for Power Grid Analysis", submitted to TCAS-I and ISQED2007.

[R8] Zhe-Yu Lin, Pei-Yu Huang, and Yu-Min Lee, "A Multilevel Placement for Low

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