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ABSTRACT

This project proposes a periodic-modulation based blind channel identification scheme for single-carrier block transmission with frequency-domain equalization. The proposed approach relies on the block system model and exploits the circulant channel matrix structure after the cyclic prefix is removed. It is shown that the set of linear equations relating the autocorrelation matrix of the block received signal and the product channel coefficients can be rearranged into one with a distinctive block circulant structure. The identification equations thus obtained lead to a very simple identifiability condition, as well as a natural formulation of the optimal modulating sequence design problem which, based on the block circulant data structure, can be cast as a constrained quadratic problem that allows for a simple closed-form solution. The optimal sequence is shown to result in a consistent channel estimate irrespective of white noise perturbation. Simulation results are used for illustrating the performance of the proposed method.

1. INTRODUCTION

Blind channel identification based on the second-order received signal statistics has been a popular research topic in the past few years, see [4] for a detailed literature review. The transmitter-induced-cyclostationarity (TIC) approach, originally introduced in [11], is known to be one of the major solution branches. In contrast with the multi-channel subspace methods [7], [12], which rely on channel diversity via over-sampling or multi-sensor at the receiver, the TIC approach resorts to signal precoding for facilitating channel identification. The transmitter precoding strategy can lead to identification algorithms free from any restriction on the channel zero location, which is recognized as a fundamental limit of the multi-channel subspace methods [10, p-1930]. Existing TIC precoders include periodic modulation [10], [1], [6], repetition coding [11], and the general filter bank precoder [9], etc. The periodic modulation precoder, in particular, is attractive for it does not introduce redundancy into the symbol streams and thus potentially prevents the loss in the data rate [10].

Single-carrier (SC) block transmission with cyclic prefix (CP) insertion recently attracts much attention for its appealing low-complexity frequency-domain equalization (FDE) [3]. However, most of the existing blind identification schemes for SC modulation devote to the serial transmission case [4]. It is noted that, with the CP based block transmission, the signal model of the SC-FDE system shares the essential features as those of OFDM [3]. As a result, a number of the blind identification algorithms tailored for OFDM, in particular, those exploiting the transmit redundancy due to CP, can also be applied to the SC-FDE systems. Two typical such proposals are the deterministic subspace method [8] and the statistical subspace method [13]. The former, although being attractive for its finite-sample-convergence property, is still subject to the limitation of the channel zero locations and the performance deterioration in the low SNR regime. The latter, on the other hand, is immune to the channel zero pattern but would usually need a large number of data to output a satisfactory identification performance. Instead of CP insertion, another form of guard interval adopted for SC block transmission is through zero-padding [9]. Blind channel identification for such an alternative scheme is addressed in [9]; the proposed approach therein is nonetheless exclusively aimed for the zero-padded transmit redundancy.

Leveraging periodic modulation, this project proposes a correlation matching based blind identification scheme for single-channel SC-FDE systems. The presented study is the block transmission counterpart of the previous work [6] for the serial transmission case. The proposed method exploits the circulant structure of the channel matrix after CP is discarded. The resultant set of identification equations leads to a very simple identifiability condition which depends on the modulating sequence alone. When channel noise is present, the proposed identification framework also provides a natural formulation of the modulating sequence design against the noise effect. The optimal noise-combating solution is in closed-form and also yields a consistent channel estimate irrespective of white noise perturbation. Simulation results are used for illustrating the performance of the proposed method, showing that it is an attractive solution under severe SNR conditions or only a small number of data samples is available.

II. SYSTEM MODEL

We consider the discrete-time baseband model of an SC-FDE system. At the transmitter, the source symbol sequence $s(n)$ is modulated by a periodic sequence $p(n)$ with period N to obtain

$$w(n) = p(n)s(n), \tag{2.1}$$

which is then serial-to-parallel converted to obtain the block signal $\mathbf{w}(n) := \left[w(nN) \quad \cdots \quad w(nN+N-1) \right]^T$, $n \geq 0$. Before transmission, the last L_{cp} symbols in $\mathbf{w}(n)$ are replicated and then attached to the front end of it as the cyclic prefix. The resultant block signal $\mathbf{x}(n) := \begin{bmatrix} \mathbf{L}_{cp}^T & \mathbf{I}_N^T \end{bmatrix}^T \mathbf{w}(n)$, where $\mathbf{L}_{cp} \in \Re^{L_{cp} \times N}$ contains the last L_{cp} rows of $\mathbf{I}_N$, is transmitted in serial through a discrete-time (assuming symbol rate sampling) FIR channel with order L and is contaminated by an additive channel noise $v(n)$. At the receiver, the received samples $y(n)$ are collected into blocks of length $N + L_{cp}$. Assume that the length of CP is greater than or equal to the channel order, that is, $L_{cp} \geq L$. By discarding the first L_{cp} samples in the received data block, the input-output channel characteristics, in terms of block signals, is described as (assuming that the receiver is synchronized with the transmitter)

$$\mathbf{y}(n) = \mathbf{GPs}(n) + \mathbf{v}(n), \tag{2.2}$$

where $\mathbf{y}(n) := \begin{bmatrix} y(Nn) & y(Nn+1) & \cdots & y(nN+N-1) \end{bmatrix}^T \in C^N$, $\mathbf{P} = diag\left\{ p(0), \cdots, p(N-1) \right\}$ and $\mathbf{G} \in C^{N \times N}$ is a circulant matrix with

$$\mathbf{g} := \begin{bmatrix} h(0) & h(1) & \cdots & h(L) & 0 & \cdots & 0 \end{bmatrix}^T \in C^N \tag{2.3}$$

as the first column, and $\mathbf{v}(n)$ is the noise component. The frequency-domain equalization strategy, which exploits the circulant nature of $\mathbf{G}$, can be implemented based on [3]. To this end, the channel information must be known at the receiver. Based on the pre-FFT data model (2.2), this project proposes a blind channel identification algorithm by using the second-order statistics of the received signal and the knowledge of the modulating sequence $p(n)$. We make the following assumptions: *(a)* the source sequence $s(n)$ is i.i.d. with zero mean and $Es(k)s(l)^* = \delta(k-l)$, where $\delta(\cdot)$ is the Kronecker delta function, *(b)* the noise $v(n)$ is white Gaussian with zero mean, variance σ_v^2, and is uncorrelated with $s(n)$, *(c)* the channel order L is known and $L_{cp} \geq L$.

III. BLIND CHANNEL IDENTIFICATION

A. Identification Equations

We will first consider the noiseless case. The proposed approach directly estimates the channel impulse response coefficients $h(n)$, $0 \leq n \leq L$, and lies in exploiting the circulant structure of the channel matrix $\mathbf{G}$. More specifically, let us define the $N \times N$ permutation matrix

$$\mathbf{J} := \begin{bmatrix} 0_{1 \times (N-1)} & 1 \\ \mathbf{I}_{N-1} & 0_{(N-1) \times 1} \end{bmatrix} \in \Re^{N \times N}. \tag{3.1}$$

Since $\mathbf{G}$ is circulant, it can be expressed in terms of its first column as

$$\mathbf{G} = \begin{bmatrix} \mathbf{g} & \mathbf{Jg} & \cdots & \mathbf{J}^{N-2}\mathbf{g} & \mathbf{J}^{N-1}\mathbf{g} \end{bmatrix}. \tag{3.2}$$

Based on (2.2), (3.2), and assumption *a)*, the autocorrelation matrix of the block received signal $\mathbf{y}(n)$ is directly computed as

$$\mathbf{R}_{\mathbf{y}}(0) := E\mathbf{y}(n)\mathbf{y}^H(n) = \sum_{n=0}^{N-1} p(n)^2 \mathbf{J}^n \mathbf{g}\mathbf{g}^H \left(\mathbf{J}^T \right)^n. \tag{3.3}$$

For a given $\mathbf{R}_{\mathbf{y}}(0)$, the matrix equation (3.3) defines a set of N^2 scalar nonlinear equations in the unknowns $h(0), \cdots, h(L)$. However, if we consider the product channel coefficients of the form $h(k)h^*(l)$ as unknowns, we have a set of N^2 linear equations. As a result, in lieu of directly solving for $h(0), \cdots, h(L)$, we propose to first compute the product coefficients $h(k)h^*(l)$, $0 \leq k, l \leq L$. We also note from (3.3) that $\mathbf{R}_{\mathbf{y}}(0)$ is a weighted sum of N rank-one matrices, each being an outer product of the circularly shifted zero-padding channel impulse response vector $\mathbf{g}$. In light of this observation, we can further rearrange the N^2 linear equations defined in (3.3) in a more tractable expression, based on which there will be a systematic way of solving for the product coefficients. Toward this end, we need the following lemma.

Lemma 3.1 [5, p-255]: The matrix equation $\sum_{k=1}^{K} \mathbf{A}_k \mathbf{X} \mathbf{B}_k = \mathbf{C}$ can be equivalently written as $\left[\sum_{k=1}^{K} \mathbf{B}_k^T \otimes \mathbf{A}_k \right] vec(\mathbf{X}) = vec(\mathbf{C})$, where $\otimes$ is the Kronecker product and $vec(\cdot)$ is the vectorized operation. □

Based on Lemma 3.1, we can immediately rewrite (3.3) as

$$\left[\sum_{n=0}^{N-1} p(n)^2 \mathbf{J}^n \otimes \mathbf{J}^n \right] vec(\mathbf{g}\mathbf{g}^H) = vec(\mathbf{R}_{\mathbf{y}}(0)). \tag{3.4}$$

By definition of the Kronecker product, equation (3.4) can be further rearranged into

$$vec(\mathbf{R}_{\mathbf{y}}(0)) = \underbrace{\begin{bmatrix} p(0)^2\mathbf{I}_N & p(N-1)^2\mathbf{J}^{N-1} & \cdots & p(2)^2\mathbf{J}^2 & p(1)^2\mathbf{J} \\ p(1)^2\mathbf{J} & p(0)^2\mathbf{I}_N & \cdots & p(3)^2\mathbf{J}^3 & p(2)^2\mathbf{J}^2 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ p(N-2)^2\mathbf{J}^{N-2} & p(N-3)^2\mathbf{J}^{N-3} & \cdots & p(0)^2\mathbf{I}_N & p(N-1)^2\mathbf{J}^{N-1} \\ p(N-1)^2\mathbf{J}^{N-1} & p(N-2)^2\mathbf{J}^{N-2} & \cdots & p(1)^2\mathbf{J} & p(0)^2\mathbf{I}_N \end{bmatrix}}_{:=\mathbf{Q}} vec(\mathbf{g}\mathbf{g}^H) \quad (3.5)$$

The $N^2 \times N^2$ matrix $\mathbf{Q}$ defined in (3.5), which is characterized by the N circulant matrices $\{p(0)^2\mathbf{I}_N,\ p(N-1)^2\mathbf{J}^{N-1},\ \cdots,\ p(1)^2\mathbf{J}\}$ on the top row block, is a block circulant matrix with circulant blocks. Equation (3.5) forms the basis for the proposed channel identification scheme, which is shown next.

B. Computation of the Product Channel Coefficients

The first step of the proposed approach is to determine the product channel coefficients $h(k)h^*(l)$ for $0 \le k,l \le L$, that is, $vec(\mathbf{g}\mathbf{g}^H)$, based on (3.5). To proceed, it is noted that the vector $\mathbf{g}$ contains $L+1$ channel impulse response $h(n)$ for $0 \le n \le L$ followed by $N-L-1$ trailing zeros. As a result, the $N^2 \times N^2$ outer-product matrix $\mathbf{g}\mathbf{g}^H$, and hence the associated vectorized representation $vec(\mathbf{g}\mathbf{g}^H)$, has actually $(L+1)^2$ nonzero product unknowns. By removing the zero elements in $vec(\mathbf{g}\mathbf{g}^H)$, and the corresponding indexed columns of the matrix $\mathbf{Q}$, equation (3.5) can be further simplified as a set of N^2 scalar equations in $(L+1)^2$ unknowns. Specifically, let $\mathbf{h} := \left[h(0) \ \cdots \ h(L)\right]^T$ be the desired channel impulse response vector. Then we immediately have $\mathbf{g} = \left[\mathbf{h}^T \ \ \mathbf{0}^T_{(N-L-1)\times 1}\right]^T$, and hence

$$\mathbf{g}\mathbf{g}^H = \begin{bmatrix} \mathbf{h}\mathbf{h}^H & \mathbf{0}_{(L+1)\times(N-L-1)} \\ \mathbf{0}_{(N-L-1)\times(L+1)} & \mathbf{0}_{N-L-1} \end{bmatrix}. \quad (3.6)$$

Let $\bar{\mathbf{Q}} \in \Re^{N^2 \times N(L+1)}$ be the matrix obtained from $\mathbf{Q}$ by deleting its last $N(N-L-1)$ columns, and define

$$\mathbf{J}_L := \begin{bmatrix} \mathbf{I}_{L+1} \\ \mathbf{0}_{(N-L-1)\times(L+1)} \end{bmatrix} \in \Re^{N\times(L+1)}, \quad (3.7)$$

which is simply the matrix containing the first $L+1$ columns of $\mathbf{I}_N$. From (3.6) and by definition of the $vec(\cdot)$ operation, it can be shown equation (3.5) can thus be reduced to

$$\underbrace{\bar{\mathbf{Q}}\left(\mathbf{I}_{L+1} \otimes \mathbf{J}_L\right)}_{:=\tilde{\mathbf{Q}}} vec(\mathbf{h}\mathbf{h}^H) = vec(\mathbf{R}_{\mathbf{y}}(0)). \quad (3.8)$$

Assume that the matrix $\tilde{\mathbf{Q}} \in \Re^{N^2 \times (L+1)^2}$ defined in (3.8) is of full column rank. Then (3.8) defines a set of overdetermined and consistent linear equations, from which the product channel coefficient vector can be uniquely recovered as

$$vec(\mathbf{h}\mathbf{h}^H) = \left(\tilde{\mathbf{Q}}^T\tilde{\mathbf{Q}}\right)^{-1}\tilde{\mathbf{Q}}^T vec(\mathbf{R}_{\mathbf{y}}(0)) \quad (3.9)$$

It is noted that the solution (3.9) for product unknowns yields the exact solution provided that the autocorrelation matrix $\mathbf{R}_{\mathbf{y}}(0)$ is obtained perfectly and when noise is absent. When only a time average of $\mathbf{R}_{\mathbf{y}}(0)$ is available (as in practical situations), the solution (3.9) then gives the least squares solutions.

C. Identification of the Channel Impulse Response

Assume that the product unknowns $h(k)h^*(l)$, $0 \le k,l \le L$, are available from (3.9). To identify the channel, let us form the $(L+1)\times(L+1)$ Hermitain matrix defined as $\mathbf{H} := [h(k)h^*(l)]$, $0 \le k,l \le L$. Theoretically, the matrix $\mathbf{H}$ is of rank-one and can be factorized as $\mathbf{H} = \mathbf{h}\mathbf{h}^H$. The channel impulse response $h(0),\ \cdots,\ h(L)$, can thus be identified, up to a scalar ambiguity, by computing the unit-norm eigenvector associated with the maximal eigenvalue of the matrix $\mathbf{H}$. We note that similar “bilinear” identification strategy is also used in [2] and [6].

Remark: Based on periodic modulation, similar time-domain correlation matching approach is adopted in [6] for the SC serial transmission case. There is however a substantial difference in the resultant algorithm features between the previous work [6] and the present study for SC-FDE systems. In particular, the work [6] exploits the Toeplitz structure of the channel matrices in the associated block system model; the autocorrelation matrix therein is shown to define $L+1$ decoupled groups of linear equations with product channel taps as unknowns (cf. [6, p-2878]). For the SC-FDE systems considered in this project, the ciuculant property of the channel matrix, on the other hand, leads to a set of coupled equations with a distinctive block circulant structure (cf. (3.5)).

D. Identifiability Condition

From the discussions in the previous section, it is easy to see that the channel can be identified if the product channel coefficients $h(k)h^*(l)$, $0 \le k,l \le L$, are uniquely determined from equation (3.9), which is the case if the matrix $\tilde{\mathbf{Q}}$ is of full column rank. Since $\tilde{\mathbf{Q}}$ is a submatrix of $\mathbf{Q}$ obtained by deleting its columns, a sufficient condition for channel identifiability is that $\mathbf{Q}$ is nonsingular. Based on the block circulant structure of $\mathbf{Q}$, there is an elegant way of specifying this sufficient condition. More precisely, we have the following proposition.

Proposition 3.2: Let $\mathbf{F}$ be the $N \times N$ FFT matrix and define

$$\mathbf{p}^T := [\,p(0)^2 \quad p(1)^2 \quad \cdots \quad p(N-1)^2\,] \in \Re^N . \quad (3.10)$$

Then the matrix $\mathbf{Q}$ defined in (3.5) is nonsingular if and only if the vector $\mathbf{Fp}$ contains no zero entries. □

Prop. 3.2 asserts that channel identifiability is guaranteed if there are no "spectral nulls" associated with the *N*-dimensional vector $\mathbf{p}$ defined in (3.10). This condition holds for almost all choices of the modulation sequence $p(n)$. In particular, we can appropriately choose $p(n)$ to guarantee channel identifiability and, moreover, to obtain a well-conditioned equation (3.8) against noise perturbation and finite-sample effect. This will be shown in next section.

IV. OPTIMAL DESIGN OF MODULATING SEQUENCE

This section considers the noisy case and addresses the problem of modulating sequence design for combating the noise effect. By examining the noise-perturbed equations, we will first introduce the design criterion. By further exploiting the BCCB structure of the matrix $\mathbf{Q}$, the optimization problem is then formulated as a constrained quadratic problem which yields an analytic solution. Some properties regarding the optimal solution are discussed.

A. Optimality Criterion

Assume that the channel noise is present. From (2.2) and (3.3), the autocorrelation matrix of the block received signal is thus

$$\mathbf{R}_{\mathbf{y}}(0) = \left(\sum_{n=0}^{N-1} p(n)^2 \mathbf{J}^n \mathbf{g}\mathbf{g}^H \left(\mathbf{J}^T\right)^n \right) + \sigma_v^2 \mathbf{I}_N . \quad (4.1)$$

By following the same procedures as shown in Section III, the equations for computing the product channel coefficients, viz., the noisy version of (3.8), then becomes

$$vec(\mathbf{R}_{\mathbf{y}}(0)) = \tilde{\mathbf{Q}} vec(\mathbf{h}\mathbf{h}^H) + \sigma_v^2 vec(\mathbf{I}_N) . \quad (4.2)$$

Since the noise variance σ_v^2 is unknown, it is in general impossible to determine the exact solution $vec(\mathbf{h}\mathbf{h}^H)$ from the noisy data. For a given $\mathbf{R}_{\mathbf{y}}(0)$, we can only instead compute the least squares solution, given as

$$vec(\mathbf{h}\mathbf{h}^H)_{LS} := \left(\tilde{\mathbf{Q}}^T \tilde{\mathbf{Q}}\right)^{-1} \tilde{\mathbf{Q}}^T vec(\mathbf{R}_{\mathbf{y}}(0)) =$$

$$vec(\mathbf{h}\mathbf{h}^H) + \sigma_v^2 \left(\tilde{\mathbf{Q}}^T \tilde{\mathbf{Q}}\right)^{-1} \tilde{\mathbf{Q}}^T vec(\mathbf{I}_N) . \quad (4.3)$$

From (4.3), it is easy to see that the least squares solution coincides with the exact solution if, and only if, the noise signature $vec(\mathbf{I}_N)$ is orthogonal to the range space of $\tilde{\mathbf{Q}}$, that is,

$$\tilde{\mathbf{Q}}^T vec(\mathbf{I}_N) = \mathbf{0}_{(L+1)^2 \times 1} . \quad (4.4)$$

In fact, if we think of the product channel coefficients $vec(\mathbf{h}\mathbf{h}^H)$ as the signal of interest in (4.2), the range space of $\tilde{\mathbf{Q}}$ then defines the signal subspace, whereas the noise perturbation signature $vec(\mathbf{I}_N)$ spans the noise subspace. Accordingly, the condition (4.4) amounts to the requirement that the signal and noise subspaces are mutually orthogonal. Since the matrix $\tilde{\mathbf{Q}}$ is completely determined by the modulating sequence $p(n)$, one natural design criterion, therefore, is to choose $p(n)$ to meet the orthogonality constraint (4.4), or to match it as exactly as we can if perfect fulfillment is impossible. This suggests the following performance measure

$$\gamma := \max_i \quad \frac{|\, \tilde{\mathbf{q}}_i^T vec(\mathbf{I}_N)\,|}{\|\, \tilde{\mathbf{q}}_i \,\| \cdot \|\, vec(\mathbf{I}_N)\,\|} , \quad (4.5)$$

where $\tilde{\mathbf{q}}_i \in \Re^{N^2}$ is the *i*th column of the matrix $\tilde{\mathbf{Q}}$. We note that γ thus defined is the maximal correlation index among the pairs of vectors $\{\tilde{\mathbf{q}}_i, vec(\mathbf{I}_N)\}$ for all $1 \le i \le (L+1)^2$; it gauges the worst-case tendency of noise contamination upon all the signal components, and thus serves as a good measure of "closeness", and hence orthogonality, between the signal and noise subspaces. Small values of γ, in particular, imply small noise contribution on the desired signals and are expected to yield better channel estimation accuracy. To achieve an utmost noise reduction, we then propose to minimize the quantity γ in (4.5), subject to the following two constraints

$$\frac{1}{N}\sum_{n=0}^{N-1} p(n)^2 = 1, \quad (4.6)$$

and

$$p(n)^2 \geq \delta > 0,\ 0 \leq n \leq N-1. \quad (4.7)$$

The constraint (4.6) normalizes the average transmit power within one block to unity. The constraint (4.7), on the other hand, imposes a threshold on the minimal modulated power for equalization feasibility; this is because zero, or too small, transmit power will prevent symbol recovery since the source sequence is uncorrelated. For the serial transmission counterpart, these two constraints are used in [1] and [6] for modulating sequence selection.

B. Optimal Solution

The proposed optimization problem, which aims for minimizing the worst-case noise corruption, appears to be one of the min-max type. Toward a solution, one has to first determine the maximal correlation index among all the vector pairs $\{\tilde{\mathbf{q}}_i,\ vec(\mathbf{I}_N)\}$, $1 \leq i \leq (L+1)^2$. Since each $\tilde{\mathbf{q}}_i$ is a column of the matrix $\mathbf{Q}$ in (3.5), by exploiting the BCCB structure of $\mathbf{Q}$ there is an elegant expression of γ in terms of $p(n)$. Specifically, it can be shown that

$$\gamma = \sqrt{\frac{N}{\sum_{n=0}^{N-1} p(n)^4}}\ . \quad (4.\ 8)$$

In contrast with equation (4.9) in [6, p-2880], it is somewhat surprising to see from (4.8) that the cost function γ regarding the SC-FDE scenario turns out to be identical with the one reported for the serial transmission case, irrespective of the fundamental difference in the individual signal models. Given the same design constraints (4.6) and (4.7), this coincidence could result from certain "inherent circularity" in the respective identification equations: the noise-corrupted equation group in [6] is described by a circulant matrix (cf. [6, p-2879]), whereas the identification equations (4.2) inherit the BCCB structure from the matrix $\mathbf{Q}$ defined in (3.5). To find the solution, we can adopt the approach used in [6, p-2880], which lies in a natural formulation of the original optimization problem as a constrained quadratic one. More precisely, based on (4.8) the minimization of the worst-case orthogonality measure γ is seen to equivalent to maximizing the quantity $\sum_{n=0}^{N-1} p(n)^4$, which is nonlinear in $p(n)$ but is quadratic if we think of $p(n)^2$'s as unknowns. In terms of the vector $\mathbf{p}$ defined in (3.10), the objective function becomes

$$\sum_{n=0}^{N-1} p(n)^4 = \|\mathbf{p}\|^2. \quad (4.9)$$

The constraints (4.6) and (4.7), respectively, can be equivalently expressed in terms of $\mathbf{p}$ as

$$\|\mathbf{p}\|_1 = \sum_{n=0}^{N-1} p(n)^2 = N, \quad (4.10)$$

and

$$\mathbf{p}_n \geq \delta > 0,\ 0 \leq n \leq N-1, \quad (4.11)$$

where $\mathbf{p}_n (= p(n)^2)$ denotes the *n*th component of $\mathbf{p}$. With (4.10)~(4.11), the optimization problem is thus essentially a constrained quadratic optimization problem: Maximize $\|\mathbf{p}\|^2$, subject to the constraints (4.10) and (4.11). The solution to this problem is detailed in [6, p-2880], and resulting optimal sequence is given as, for any fixed $0 \leq m \leq N-1$,

$$|p(m)| = \sqrt{N(1-\delta)+\delta}, \text{ and } |p(n)| = \sqrt{\delta} \text{ for } n \neq m. \quad (4.12)$$

It is noted that, in the serial transmission scenario, the same solution is also reported in [1], through maximizing a certain measure of the frequency-domain SNR. With the optimal sequence (4.12), the resultant optimal orthogonality measure can be found as

$$\gamma_{opt} := \left(\sqrt{N-(N-1)\delta(2-\delta)}\right)^{-1}. \quad (4.13)$$

It can be directly shown that, for $0 < \delta < 1$, $\delta(2-\delta)$ is an increasing function in δ. As a result, for a fixed block length *N*, the optimal measure will decrease as δ is decreased. Hence, a small power threshold δ implies small noise contribution on the signals, and hence attains better channel estimation accuracy. Also, it is shown that the solution (4.12) results in a consistent channel estimate when noise is white.

V. SIMULATION RESULTS

In this section, numerical simulations are used for illustrating the performance of the proposed method. We consider the following test channel

$$\mathbf{h} = [2.3823 + 1.858j, -1.1217 - 1.0801j, -0.725 - 0.8724j,$$
$$0.5284 - 0.6103j, 0.3561 - 0.6360j, -0.2927 - 0.2570j,$$
$$0.2843 - 0.330j, -0.1825 + 0.2194j, 0.1082 - 0.0667j].$$

The lengths of the symbol block and CP, respectively, are $N = 32$ and $L_{cp} = 8$; the source symbols are drawn from the QPSK constellation. Throughout the simulations, the index of peak power in the optimal sequence (4.12) is set to be $m = 0$. We compare the performances of the proposed method with the two subspace methods [8], [13] (denoted respectively by the d-subspace and s-subspace methods). For fixed SNR = 10 dB, Figures 1 shows the respective NMSE for different numbers of symbol blocks. From the figure, we can see that the performance of the proposed method (solid line) is improved as δ decreases; this is because a small δ leads to small orthogonality measure (4.13) and hence good estimation accuracy. Also, compared with the subspace methods, the proposed method performs better as long as $\delta \le 0.8$; when $\delta = 0.8$ and the number of symbol blocks is as small as 100, there is about a 10 dB improvement in the NMSE. Figure 2 shows the NMSE of the three methods at various SNR levels (with 300 symbol blocks). The result shows that, in the medium-to-low SNR region (< 10 dB), the proposed method attains the best performance even with $\delta = 0.9$ for small power variation. For SNR larger than 10 dB, the output NMSE of the method [8] exhibits a fast decay. This is not unexpected since, owing to the finite-sample-convergence property, the deterministic based approach can usually yield impressive estimation accuracy when SNR is high [11, p-1789]. We note that, in the series transmission scenario, similar tendency is also observed in [10, p-1942] when modulation-based identification is compared with the (deterministic) multi-channel subspace methods [7] and [12]. We note that, although both the proposed approach and the subspace method [13] are statistical based in nature, the former can better track the channels. This is because the method [13] directly relies on the "raw" estimated autocorrelation matrix for channel identification, and could be more sensitive to the finite-sample estimation errors. For the proposed method, the identification equations (3.8) would usually remain well-conditioned whenever the optimal sequence (4.12) with a moderate δ (or small) is used (for $\delta = 0.7$, the condition number of the matrix $\tilde{\mathbf{Q}}^T\tilde{\mathbf{Q}}$ is about 3.84). This could act as an inbuilt mechanism for combating the effect of imperfect data estimation.

VI. CONCLUSION

In this project we propose a blind identification scheme for the SC-FDE systems based on periodic power modulation. The proposed method exploits the circulant channel matrix property unique to the SC-FDE systems as well as the linear relations between the autocorrelation matrix and the product channel coefficients. The resultant identifiability condition inherits the appealing feature common to most TIC-based approaches: it depends entirely on the modulating sequence but not on the channel characteristics. In fact, almost all sequences yield the channels identifiable. The problem of modulating sequence design against noise amounts to a constrained quadratic problem which yields a closed-form solution. Simulation results show that, in the low-to-medium SNR region or when the number of available data samples is small, the proposed method compares favorably with existing subspace algorithms applicable to SC-FDE systems.

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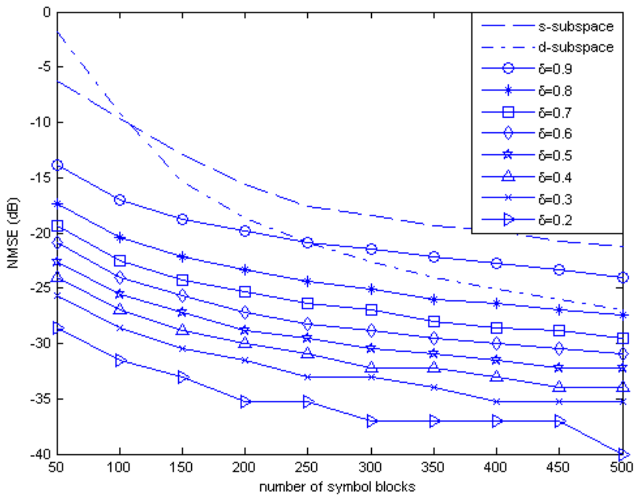

Figure 1. NMSE v.s. number of symbol blocks.

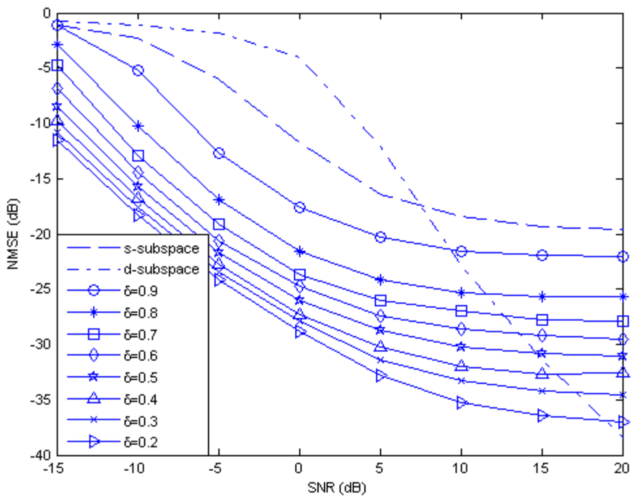

Figure 2. NMSE v.s. SNR.