# 行政院國家科學委員會專題研究計畫　成果報告

# 喪失銷貨定期盤存制經驗採購法則之研究

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# 喪失銷貨定期盤存制經驗採購法則之研究
(NSF92-2416-H-009-007)

（一）計畫中文摘要

大多數定期盤存制假設需求無法立刻滿足時可延後交貨，本計畫探討需求無法立刻滿足即喪失之定期盤存制採購政策的訂定。與延後交貨定期盤存制相較，喪失銷貨定期盤存制在存貨管理文獻上仍有許多可探討的空間。

由於最佳採購政策僅適用於前置時間為零之喪失銷貨定期盤存制，本計畫探討前置時間不為零之下經驗採購法則的訂定。一般而言，喪失銷貨定期盤存制經驗採購法則有二類:短視政策與基礎存貨政策。與短視政策相較，基礎存貨政策計算上較簡易。本計畫旨在設計較一般基礎存貨更精確的訂購後存貨水準。

本計畫的貢獻有二。首先本計畫針對喪失銷貨定期盤存制設計一更精確的經驗採購法則;其次由於所設計的經驗採購法則導入簡易，可立即提供國內各企業訂定物料採購政策，以適度削減存貨，降低營運成本。
**關鍵詞**:定期盤存制;喪失銷貨;經驗法則

（二）計畫英文摘要

In this research, we study periodic inventory systems in which all demand not filled immediately is lost. Though the lost-sales periodic review systems are more common in the retailing industries, these systems have remained relatively unexplored in the inventory literature compared to periodic systems with backorders. Exact analysis can be done only with the case of zero supply lead-times. In this research, we devise heuristic ordering policies for periodic review systems with positive supply lead-times.

There are in general two types of heuristic ordering policies: myopic policies and base-stock policies (i.e., order-up-to policies). While myopic polices tend to produce near-optimal solutions for periodic review systems with lost sales, base-stock polices often require less computational efforts. In this research, we focus mainly on base-stock polices. We will present a simple rule for deciding the base-stock level.

The contribution of this research is two fold. First, we compute an exact base-stock level for periodic review systems with lost sales. Second, the base-stock level can be obtained through a simple proccedure on a computer. As a result, practitioners should be able to use it immediately to reduce operating costs.
**Keywords**: Periodic Review System, Lost Sales, Heuristic Policy

# I. Introduction

Though the use of computer systems has made continuous review inventory models more attractive, periodic review models are still applied in many situations (see, e.g., Chiang [1] and Chiang and Gutierrez [2]), especially for inventory systems in which many different items are purchased from the same supplier and the coordination of ordering and transportation is important. Also, as Porteus [7] observes, continuous review systems that keep inventory records current, but order periodically are equivalent to periodic review systems. Often, periodic systems have the review periods that are possibly longer than the supply lead-times.

In this research, we study periodic inventory systems in which all demand not filled immediately is lost. Though the lost-sales periodic review systems are more common in the retailing industries (many supermarkets and retail stores have witnessed such a situation), these systems have remained relatively unexplored in the inventory literature compared to periodic systems with backorders. Exact analysis can be done only with the case of zero supply lead-times. With positive supply lead-times (deterministic or stochastic), the lost-sales periodic review problem is difficult to solve, even if the fixed cost of ordering is zero. See, e.g., Hadley and Whitin [3, p. 285], Morton [4], Nahmias [6], and Zipkin [8, pp. 411-413]. Heuristic approaches need to be used. In this research, we devise heuristic ordering policies for periodic review systems with positive supply lead-times.

There are in general two types of heuristic ordering policies: myopic policies and base-stock policies (i.e., order-up-to policies). While myopic polices tend to produce near-optimal solutions for periodic review systems with lost sales (see, e.g., Zipkin [8, p. 413]), base-stock polices often require less computational efforts. In this research, we focus mainly on base-stock polices.

# II. Purpose of Research

The purpose of this research is to present a simple rule for deciding the base-stock level. The contribution of this research is two fold. First, we compute an exact base-stock level for periodic review systems with lost sales. Second, the base-stock level can be obtained through a simple procedure on a computer. As a result, practitioners should be able to use it immediately to reduce operating costs.

# III. Research Methodology

We will not use the dynamic programming approach in this research. The dynamic program formulated indeed contains the number of states that grow exponentially in the lead-time. As Zipkin [8, pp. 411-412] notes, such a dynamic program are very hard and virtually impossible to solve (the curse of dimensionality!). Instead, we will estimate the average cost per period, which include the procurement cost, holding cost, and shortage cost. Then we minimize the average period cost to obtain the base-stock level.

# IV. The Model

We assume that all demand not satisfied immediately is lost. Let $\tau$ be the supply lead-time (which is positive) and $c$ the purchase cost per unit. Demand is stochastic with mean rate $\mu$ per day, and is assumed to be non-negative and independently distributed in disjoint time intervals. Let $T$ denote the period length. It is assumed that $T$ is not small such that an order is always placed at a review epoch. Also, let $g(\cdot|T+\tau)$ be the conditional probability density function of demand during $T$ plus $\tau$. Demand is assumed to be continuous for convenience of notation.

Let $R$ be the inventory position (i.e., inventory on hand plus inventory on order) after an order is placed at a review epoch. Assume that $\tau$ (if random) is iid and independent of $T$. We suggest the base-stock policy of Hadley and Whitin [3, pp. 240-242], but with the correction described below. Alternatively, assuming further that the maximum $\tau$ is less than $T$, we could also employ the modified base-stock policy proposed by Moses and Seshadri [5]. Let $h$ be the holding cost per unit per day and $p$ the shortage cost per unit. The expected undiscounted cost (excluding the constant procurement cost) of the upcoming time interval between the arrival of two consecutive orders is expressed by

$$J(R) = hT(R - \mu E[\tau] - 0.5T\mu) + (0.5hT + p - c)\int_R^{\infty}(Z - R)E_{\tau}[g(Z|T+\tau)]dZ \tag{1}$$

There is an error in expression (5-11) of Hadley and Whitin. The integral should be multiplied by a factor of 0.5. Since the base-stock policy orders filled demands just as it does in a backlogged periodic review model (except that now lost sales are not counted), the average order quantity (i.e., cycle stock) is given by

$$0.5\{T\mu - \int_R^{\infty}(Z - R)E_{\tau}[g(Z|T+\tau)]dZ\} \tag{2}$$

instead of $0.5T\mu$. Expression (7) of Moses and Seshadri does not correct this either. Adding (2) to the safety stock, given by (5-10) of Hadley and Whitin, would yield the holding cost component of (1). Since $J(R)$ is convex, the optimal base-stock level $R^*$ is obtained by solving the first-order condition of (1), i.e., $R^*$ is the solution to

$$(0.5hT + p - c)\int_R^{\infty}E_{\tau}[g(Z|T+\tau)]dZ = hT \tag{3}$$

Let $Q$ be the quantity ordered at a review epoch that is a function of the inventory on hand at that review epoch denoted by $I$. According to Hadley and Whitin [3], we order $Q(I) = R^* - I$ at a review epoch. Alternatively, according to Moses and Seshadri [5], we would order the amount $Q(I) = R^* - \mu E[\tau] - (I - \mu E[\tau])^+$ at a review epoch.

## V. Numerical Results

In this section, we illustrate the solution method with some examples. Consider the base case: $T$ = 10 days, $\mu$ = 2/day (with Poisson demand), $c$ = \$10, $h$ = \$0.01, $p$ = \$12, and $\tau$ = 4 days. Using (3), we find that $R^*$ = 41. If $p$ is changed to \$16, \$20, \$24, and \$28, respectively in the base case (other things being equal), then $R^*$ is found to be 42, 42, 42, and 43, respectively. This result indicates that $R^*$ is non-decreasing in $p$, which agrees with our intuition that a larger shortage penalty may dictate a higher order-up-to level.

Moreover, if $\tau$ in the base case is changed to 5, 6, 7, and 8 days, respectively, then $R^*$ is found to be 44, 46, 48, and 51, respectively. This result, again, agrees with our common knowledge that longer lead-times make higher order-up-to levels necessary.

## VI. Conclusion and Suggestion

In this research, we propose a (modified) base-stock policy, which is a corrected version of Hadley and Whitin [3] and Moses and Seshadri [5]. As the proposed policy is easily implemented on a digital computer, we hope that inventory practitioners or material purchasers will adopt it in the near future to reduce their inventory-related costs.

# References

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# 計畫成果自評

The major finding of this research is that a more accurate base-stock level should be used for the lost-sales periodic review systems. This result is expected to make a good contribution to the inventory literature. However, the modified base-stock policy proposed has the following restriction (according to Moses and Seshadri): at most one order is outstanding at any time. This may limit the use of the proposed modified base-stock policy in practice.