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不可約算子直和之研究

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中文摘要

已知任何一個在可分希伯特空間上的有界線性算子均可表示成不可約算子的直積分，但不是每一個算子都可以表示成其直和。在本論文中，我們證明任何算子的簡約子空間的個數或為有限個或為不可數的，而前者成立的充份且必要條件為此算子是有限個兩兩不酉等價的不可約算子的直和。我們也用算子所產生的馮諾曼代數之交換代數的C*-結構來刻劃不可約算子之直和。

不可約算子之直和

房軍生，蔣春瀾，吳培之

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Direct Sums of Irreducible Operators

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Abstract

It is known that every operator on a (separable) Hilbert space is the direct integral of irreducible operators, but not every one is the direct sum of irreducible ones. We show that an operator can have either finitely or uncountably many reducing subspaces, and the former holds if and only if the operator is the direct sum of finitely many irreducible operators no two of which are unitarily equivalent. We also characterize operators T which are direct sums of irreducible operators in terms of the C^*-structure of the commutant of the von Neumann algebra generated by T.

Keywords: *Irreducible operator, reducing subspace, von Neumann algebra.*

AMS Subject Classification: 47A15, 47C15.

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1. INTRODUCTION

A bounded linear operator on a complex separable Hilbert space H is *irreducible* if it has no reducing subspace other than $\{0\}$ and H; otherwise, it is *reducible*. In this paper, we are concerned with the problem of characterizing operators which are expressible as the direct sum of irreducible operators. Examples of such operators include any finite-dimensional operator, compact operator, completely nonnormal essentially normal operator, completely nonnormal hyponormal operator with finite multiplicity (cf. [7, Section 2.1]) and any Cowen-Douglas operator (cf. [3, Prop. 1.18]). On the other hand, not every operator can be expressed as such a direct sum. This is the case even for normal operators since it can be easily seen that a normal operator is irreducible if and only if it acts on a one-dimensional space, and thus it is the direct sum of irreducible operators if and only if it is diagonalizable. In particular, the bilateral shift (the operator of multiplication by the independent variable on the L^2-space of the unit circle) cannot be the direct sum of irreducible operators.

In Section 2 below, we first show in Theorem 2.1 that no operator can have countably infinitely many reducing subspaces, that is, the number of reducing subspaces of any operator is either finite or $\aleph_1$, the cardinal number of real numbers. Moreover, an operator has finitely many reducing subspaces if and only if it is the direct sum of finitely many irreducible operators no two of which are unitarily equivalent. These are proved by making use of the structure theorem of two projections (Lemma 2.2).

An equivalent condition for irreducibility can be formulated in terms of the von Neumann algebra generated by the operator. Indeed, if $W^*(T)$ denotes the von Neumann algebra generated by an operator T on H and $W^*(T)'$ denotes its commutant, then using the von Neumann double commutant theorem we can easily show the equivalence of the following three conditions: (1) T is irreducible, (2) dim $W^*(T)'=1$, and (3) $W^*(T)$ equals $\mathcal{B}(H)$, the algebra of all operators on H. In Section 3, we will generalize this to the situation for direct sums of irreducible operators. We show in Theorem 3.1 that T is such a direct sum if and only if $W^*(T)'$ is $*$-isomorphic to the direct sum of full matrix algebras $M_{n_i}(\mathbf{C})$ with various sizes n_i, $1 \leq n_i \leq \infty$. Here $M_{n_i}(\mathbf{C})$, $1 \leq n_i \leq \infty$, denotes the algebra of all n_i-by-n_i complex matrices, and $M_\infty(\mathbf{C})$ is understood to be $\mathcal{B}(l^2)$. As a corollary (Corollary 3.2), we have the equivalence of T being the direct sum of finitely many irreducible operators and dim $W^*(T)' < \infty$.

If all the n_i's are finite in the above representation for $W^*(T)'$, that is, if $W^*(T)'$ is $*$-isomorphic to the direct sum of full finite matrix algebras, then $W^*(T)'$, as an approximately finite algebra, can be characterized in terms of its (scaled ordered) K_0-group. (For results on the K-theory of C^*-algebras, the reader can consult [13].) However, in our present situation, the full infinite matrix algebra $M_\infty(\mathrm{C})$ may appear, which renders the K_0-group characterization as inappropriate. In our final section, we show that for this case the characterization can be obtained in terms of the semigroup $V(W^*(T)')$.

We conclude this section with two further remarks. Firstly, it is known that on an infinite-dimensional separable Hilbert space H, there are plenty of irreducible operators in the sense that such operators are dense in $\mathcal{B}(H)$ in the norm topology (cf. [4]). In [4], it was asked whether reducible operators are also dense. This is answered positively by Voiculescu [12]. In fact, an even stronger result is true, namely, for any operator T and any $\varepsilon > 0$, there is a compact operator K with $||K|| < \varepsilon$ such that $T + K$ is the direct sum of infinitely many irreducible operators (cf. also [6, Prop. 4.21 (iv) and (v)]).

Secondly, although not every operator is the direct sum of irreducible operators, every one can be decomposed as the direct integral of irreducible ones. This is what the next proposition says.

Proposition 1.1. *Every operator is the direct integral of irreducible operators.*

Proof. This is an easy consequence of [1, Theorem 3.6] on the direct integral decomposition of operator algebras. Indeed, since for any operator T, the weakly closed algebra Alg T generated by T and I can be expressed as $\int_\Lambda^\oplus \mathcal{A}_\lambda d\mu(\lambda)$, where Λ is a separable metric space, μ is (the completion of) a σ-finite regular Borel measure on Λ, and $\mathcal{A}_\lambda$ is a weakly closed irreducible operator algebra for almost all λ in Λ (an operator algebra is *irreducible* if it has no nontrivial reducing subspace), we have $T = \int_\Lambda^\oplus T_\lambda d\mu(\lambda)$, where T_λ is in $\mathcal{A}_\lambda$ for almost all λ. Hence Alg $T \subseteq \int_\Lambda^\oplus$ Alg $T_\lambda d\mu(\lambda) \subseteq \int_\Lambda^\oplus \mathcal{A}_\lambda d\mu(\lambda) =$ Alg T, which implies that Alg $T_\lambda = \mathcal{A}_\lambda$ for almost all λ. The irreducibility of $\mathcal{A}_\lambda$ then implies that of T_λ. Thus $T = \int_\Lambda^\oplus T_\lambda d\mu(\lambda)$ is the asserted decomposition of T. □

For any C^*-algebra $\mathcal{A}$ and natural number n, let $M_n(\mathcal{A})$ denote the C^*-algebra of n-by-n matrices with entries from $\mathcal{A}$.

2. NUMBER OF REDUCING SUBSPACES

The main result of this section is the following theorem.

Theorem 2.1. *The number of reducing subspaces of any operator is either finite or uncountably infinite. It is the former case if and only if the operator is the direct sum of finitely many irreducible operators* $\sum_{i=1}^{n} \oplus T_i$ *with* T_i *and* T_j *non-unitarily-equivalent for any* $i \neq j$. *In this case, the number of reducing subspaces is* 2^n.

The preceding result has an analogue in a different context: the number of invariant subspaces of any operator on a finite-dimensional space is either finite or uncountably infinite, and it is the former case if and only if the operator is cyclic (cf. [9]).

To prove Theorem 2.1, we need three lemmas. The first one is a structure theorem for arbitrary two (orthogonal) projections. This result has appeared repeatedly in the literature before; the version we adopt below is from [5].

Lemma 2.2. *Let* P *and* Q *be arbitrary two projections on a Hilbert space. Then there is a unitary operator* U *such that*

$$U^{*}PU = \begin{pmatrix} I_1 & 0 \\ 0 & 0 \end{pmatrix} \oplus I_2 \oplus I_3 \oplus 0 \oplus 0$$

and

$$U^*QU = \begin{pmatrix} A & B \\ B & I_1 - A \end{pmatrix} \oplus I_2 \oplus 0 \oplus I_4 \oplus 0$$

on the space $H_1 \oplus H_1 \oplus H_2 \oplus H_3 \oplus H_4 \oplus H_5$, *where* A *is a positive contraction on* H_1 *and* B *is the positive square root of* $A(I_1 - A)$. *We may require that* $0 < A \leq \frac{1}{2}I_1$, *in which case* A *is unique up to unitary equivalence.*

The preceding lemma is used to prove

Lemma 2.3. *If* T *has countably many reducing subspaces, then* $W^*(T)'$ *is abelian.*

Proof. Let P and Q be two projections in $W^*(T)'$ represented as in Lemma 2.2 with $0 < A \leq \frac{1}{2}I_1$. Since P and Q both commute with T, a simple computation shows that T is of the form $T_1 \oplus T_1 \oplus \sum_{i=2}^{5} \oplus T_i$ on $H_1 \oplus H_1 \oplus \sum_{i=2}^{5} \oplus H_i$ with $T_1 A = AT_1$. For each complex scalar λ, let M_λ be the subspace $\{\lambda Bx \oplus x \oplus 0 \oplus 0 \oplus 0 \oplus 0 : x \in H_1\}$. It is easily seen that the M_λ's are all reducing subspaces of T and are distinct if $H_1 \neq \{0\}$. Since T has only countably many reducing subspaces, this forces $H_1 = \{0\}$. Hence $P = I_2 \oplus I_3 \oplus 0 \oplus 0$ and $A = I_2 \oplus 0 \oplus I_4 \oplus 0$ commute. Since the von Neumann algebra $W^*(T)'$ is generated by the projections it contains, we infer that $W^*(T)'$ is abelian.□

We need one more lemma.

Lemma 2.4. *Let A and B be irreducible operators on H and K, respectively. Then A and B are unitarily equivalent if and only if there is a nonzero operator X such that $XA = BX$ and $XA^* = B^*X$.*

Proof. Assume that $XA = BX$ and $XA^* = B^*X$ for some $X \neq 0$. It is easily seen that ker X and $\overline{\text{ran } X}$ are reducing subspaces of A and B, respectively. If ker $X \neq \{0\}$, then by the irreducibility of A we have ker $X = H$ or $X = 0$, which contradicts our assumption. Hence ker $X=\{0\}$ or X is one-to-one. In a similar fashion, we infer that $\overline{\text{ran } X} = K$ or X has dense range. Therefore, the polar decomposition of X yields $X = UP$, where U is unitary and $P = (X^*X)^{1/2} \geq 0$. Since $X^*XA = X^*BX = AX^*X$, we have $PA = AP$. Hence $UAP = UPA = XA = BX = BUP$. Note that P also has dense range. From above, we conclude that $UA = BU$, which shows the unitary equivalence of A and B as asserted. □

We are now ready for the

Proof of Theorem 2.1. Assume that operator T has a countably infinite number of reducing subspaces. This implies, by Lemma 2.3, that $W^*(T)'$ is abelian. Hence it is generated by some Hermitian operator A (cf. [10, Theorem 7.12]). Note that

$\sigma(A)$, the specturm of A, cannot be a finite set for otherwise A would be of the form $\sum_{i=1}^{n} \oplus \lambda_i I_i$ and $W^*(A)$ would consist of operators of the form $\sum_{i=1}^{n} \oplus \alpha_i I_i$ with scalars α_i , which implies that $W^*(A) = W^*(T)'$ consists of only finitely many projections contradicting our assumption. Thus we can decompose $\sigma(A)$ into countably infinitely many mutually disjoint Borel subsets with each having strictly positive spectral measure. The spectral projections corresponding to various unions of such subsets are all in $W^*(A) = W^*(T)'$. Since there are uncountably many of them, this again contradicts our assumption. Thus the number of reducing subspaces of T cannot be countably infinite.

Assume next that T has finitely many reducing subspaces. By Lemma 2.3, the von Neumann algebra $W^*(T)'$ is generated by, say, the mutually commuting projections $P_1, \cdots, P_n$. Thus, in particular, $W^*(T)'$ consists of linear combinations of the products $P_{i_1}, \ldots, P_{i_k}$, where $0 \leq k \leq n$ and $1 \leq i_1 < \ldots < i_k \leq n$. This shows that $m \equiv \dim W^*(T)' \leq 2^n < \infty$ and thus $W^*(T)'$ consists of operators of the form $\sum_{i=1}^{m} \oplus \alpha_i I_i$ on $\sum_{i=1}^{m} \oplus H_i$ with scalars α_i. The von Neumann double commutant theorem then implies that $W^*(T) = W^*(T)'' = \{\sum_{i=1}^{m} \oplus A_i : A_i \in \mathcal{B}(H_i) \quad \text{for all i}\}$. In particular, we have $T = \sum_{i=1}^{m} \oplus T_i$. If P is a projection commuting with T_i, then $0 \oplus \ldots \oplus 0 \oplus \underset{ith}{P} \oplus 0 \oplus \ldots \oplus 0$ is in $W^*(T)'$ and hence is of the form $\sum_{i=1}^{m} \oplus \alpha_i I_i$. It follows that P is either 0 or I_i. This shows that T_i is irreducible. Next we prove that no two of the T_i's are unitarily equivalent. For this, assume otherwise that there is a unitary operator U such that $UT_i = T_jU$, where $1 \leq i < j \leq m$. For any scalar λ, let $M_\lambda = \{0 \oplus \ldots \oplus \underset{ith}{x} \oplus 0 \oplus \ldots \oplus 0 \oplus \underset{jth}{\lambda Ux} \oplus \ldots \oplus 0 : x \in H_i\}$. Then the M_λ's are distinct reducing subspaces of T. Since there are infinitely many of them, this contradicts our assumption on T.

Conversely, assume that $T = \sum_{i=1}^{n} \oplus T_i$ on $H = \sum_{i=1}^{n} \oplus H_i$, where the T_i's are all irreducible and no two of them are unitarily equivalent. Let $P = [P_{ij}]_{i,j=1}^{n}$ be a projection commuting with T. Then $P_{ij}T_j = T_iP_{ij}$ for all i and j. From this we obtain $P_{ij}T_j^* = P_{ji}^*T_j^* = (T_jP_{ji})^* = (P_{ji}T_i)^* = T_i^*P_{ji}^* = T_i^*P_{ij}$. Since T_i and T_j are irreducible and are not unitarily equivalent for $i \neq j$, Lemma 2.4 implies that $P_{ij} = 0$ and hence also $P_{ji} = 0$. Thus P_{ii} is a projection commuting with T_i. The irreducibility of T_i implies that $P_{ii} = 0$ or I_i. This shows that P is one of the 2^n projections obtained by taking the direct sum of some of the I_i's with the 0's. Equivalently, this says that the reducing subspaces of T are the 2^n subspaces obtained by taking the direct sum of some of the H_i's with the $\{0\}$'s , completing the proof. □

3. FULL MATRIX ALGEBRAS

In this section, we will characterize the direct sum of irreducible operators in terms of the C^*-algebra structure of the commutant of its generated von Neumann algebra.

For any operator T on H and any integer n, $1 \leq n \leq \infty$, let $T^{(n)}$ denote the direct sum of n copies of T on $H^{(n)} = \underbrace{H \oplus \ldots \oplus H}_{\text{n}}$.

Theorem 3.1. *An operator T on H is the direct sum of irreducible operators, say, $\sum_{i=1}^{n} \oplus T_i^{(n_i)}$ on $\sum_{i=1}^{n} \oplus H_i^{(n_i)}$, where $1 \leq n \leq \infty, 1 \leq n_i \leq \infty$ for all i and the T_i's are pairwise non-unitarily-equivalent, if and only if $W^*(T)'$ is $*$-isomorphic to $\sum_{i=1}^{n} \oplus M_{n_i}(\mathbf{C})$. Moreover, the T_i's are unique up to permutation and unitary equivalence. More precisely, if $T = \sum_{k=1}^{m} \oplus S_k^{(m_k)}$ is another direct sum representation of irreducible operators for T with pairwise-non-unitarily-equivalent S_k's, then $n = m$ and there is a permutation π of $\{1, \ldots, n\}$ and a unitary operator U in $W^*(T)'$ such that $n_i = m_{\pi(i)}$ and $UT_i = S_{\pi(i)}U$ for all i.*

Since every finite-dimensional (unital) C^*-algebra is $*$-isomorphic to the direct sum of finitely many full (finite) matrix algebras (cf. [11, Theorem 11.2]), an easy consequence of the preceding theorem is

Corollary 3.2. *T is the direct sum of finitely many irreducible operators if and only if* dim $W^*(T)' < \infty$

We need the following lemmas for the proof of Theorem 3.1.

Lemma 3.3. *If T is irreducible on H and X is such that $XT = TX$ and $XT^* = T^*X$, then X is a scalar operator.*

Proof. Since X^*X commutes with T, the same is true for any spectral projection P of X^*X. The irreducibility of T then implies that $P = 0$ or I. Thus the spectrum of X^*X must be a singleton $\{\alpha\}$ and hence $X^*X = \alpha I$. On the other hand, from the assumptions $XT = TX$ and $XT^* = T^*X$ we also have that ker X is a reducing subspace of T. Thus ker $X = \{0\}$ or H. This says that either X is one-to-one or $X = 0$. Similarly, by considering $\overline{\text{ran } X}$, we deduce that either X has dense range or $X = 0$. Thus for our purpose we may assume that X is one-to-one with dense range. Hence $X = U(X^*X)^{1/2} = \sqrt{\alpha}U$, where U is unitary, by the polar decomposition. We may assume that $\alpha \neq 0$. Then $UT = TU$ and $UT^* = T^*U$. Arguing as above, we obtain $U = \beta I$. Thus $X=\sqrt{\alpha}\beta I$ is a scalar operator. □

Lemma 3.4. *Let P be a projection in $W^*(T)'$. Then $T \mid (\text{ran } P)$ is irreducible if and only if P is a minimal projection in $W^*(T)'$.*

Recall that a projection p in a C^*-algebra is *minimal* if there is no projection q, other than 0 and p, such that $pq = q$.

Lemma 3.4 is an easy consequence of the definitions of irreducibility and minimal projection.

Proof of Theorem 3.1. Assume that $T = \sum_{i=1}^{n} \oplus T_i^{(n_i)}$ on $H = \sum_{i=1}^{n} \oplus H_i^{(n_i)}$, where the T_i's are pairwise-non-unitarily-equivalent irreducible operators. If X is an operator in $W^*(T)'$, then $X = \sum_{i=1}^{n} \oplus X_i$ with X_i in $W^*(T_i^{(n_i)})'$ by Lemma 2.4. Letting $X_i = [Y_{jk}^i]_{j,k=1}^{n_i}$, we obtain that Y_{jk}^i belongs to $W^*(T_i)'$. Therefore Y_{jk}^i is a scalar operator by Lemma 3.3. Say, $Y_{jk}^i = \lambda_{jk}^i I_i$, where I_i is the identity operator on H_i. Then $X = \sum_{i=1}^{n} \oplus [\lambda_{jk}^i I_i]_{j,k=1}^{n_i}$. Obviously, the mapping $X \longmapsto \sum_{i=1}^{n} \oplus [\lambda_{jk}^i]_{j,k=1}^{n_i}$ defines a $*$-isomorphism from $W^*(T)'$ onto $\sum_{i=1}^{n} \oplus M_{n_i}(\mathbf{C})$.

Conversely, let Φ be a $*$-isomorphism from $W^*(T)'$ onto $\mathcal{A} \equiv \sum_{i=1}^{n} \oplus M_{n_i}(\mathbf{C})$, and let E_{ij} denote the element $0 \oplus \ldots \oplus e_{ij} \oplus \ldots \oplus 0$ in $\mathcal{A}$, where e_{ij} is the n_i-by-n_i matrix whose (j,j)-entry equals 1 and all others equal 0. Then the $\Phi^{-1}(E_{ij})$'s are mutually orthogonal minimal projections in $W^*(T)'$ with sum equal to I. Obviously, $\Phi^{-1}(E_{ij})H$ is a reducing subspace of T with $T_{ij} \equiv T \mid \Phi^{-1}(E_{ij})H$ irreducible (by Lemma 3.4), and $T= \sum_{ij} \oplus T_{ij}$. Since for any pair j and k the matrices E_{ij} and E_{ik} are unitarily equivalent (via a unitary operator, say, U in $\mathcal{A}$), we infer that T_{ij} and T_{ik} are unitarily equivalent (via the unitary $\Phi^{-1}(U) \mid \Phi^{-1}(E_{ij})H)$. Thus T is the direct sum of irreducible operators $\sum_{i=1}^{n} \oplus T_{i1}^{(n_i)}$ as asserted.

To prove the uniqueness, let $T = \sum_{k=1}^{m} \oplus S_k{}^{(m_k)}$ on $H = \sum_{k=1}^{m} \oplus L_k{}^{(m_k)}$ be another direct sum of irreducible operators for T with pairwise non-unitarily-equivalent S_k's, where $1 \leq m \leq \infty$ and $1 \leq m_k \leq \infty$ for all k. If P_{kl}, $1 \leq k \leq m$ and $1 \leq l \leq m_k$, denotes the projection from H onto the lth component in $L_k{}^{(m_k)}$, then the mutually orthogonal projections $F_{kl} \equiv \Phi(P_{kl})$ in $\mathcal{A}$ are such that $\sum_{k,l} F_{kl} = I$. Moreover, since each F_{kl} is minimal by Lemma 3.4, it can only "live" in some $M_{n_i}(\mathrm{C})$ and can only have rank one. Also note that for any fixed k, the different F_{kl}'s are all in the same $M_{n_i}(\mathrm{C})$ with $\sum_l F_{kl} = I_{n_i}$, the identity matrix of size n_i. This is because for a fixed k, the different P_{kl}'s are unitarily equivalent via a unitary operator in $W^*(T)'$, and thus the different F_{kl}'s are unitarily equivalent via a unitary operator in $\mathcal{A}$. This latter unitary operator, begin a direct sum of operators from the $M_{n_j}(\mathbf{C})$'s, can intertwine only operators in the same $M_{n_i}(\mathbf{C})$. Since $\sum_l F_{kl} = I_{n_i}$ and the mutually orthogonal F_{kl}'s each has rank one, we infer that $m_k = n_i$ and the F_{kl}'s (for different l's) are simultaneously unitarily equivalent to the E_{ij}'s (for different j's). From $\sum_{k,l} F_{kl} = I = \sum_{i,j} E_{ij}$ and the above, we conclude that $m = n$ and, after a permutation of the indices, the F_{kl}'s (for different k's and l's) are simultaneously unitarily equivalent to the E_{ij}'s (for different i's and j's). Our assertion of the uniqueness of the irreducible summands for T then follows from applying Φ^{-1} to the F_{kl}'s and the intertwining unitary operator in $\mathcal{A}$. □

We next consider the problem when two operators have isomorphic reducing subspace lattices. When the operators are normal, this has been solved by Conway and Gillespie [2]. Using their result, we may settle the problem when the two operators are both direct sums of irreducible ones. This covers in particular the cases for operators on finite-dimensional spaces and compact operators.

For any operator T, let Red T denote the lattice of its reducing subspaces.

Proposition 3.5. *Let* $A = \sum_{j=1}^{n} \oplus A_j^{(n_j)}$ *and* $B = \sum_{k=1}^{m} \oplus B_k^{(m_k)}$ *be direct sums* *of irreducible operators with pairwise non-unitarily-equivalent A_j's and B_k's, where $1 \leq n, m \leq \infty$ and $1 \leq n_j, m_k \leq \infty$ for all j and k, and the n_j's and m_k's are decreasing. Then* Red A *is isomorphic to* Red B *if and only if $n = m$ and $n_j = m_j$ for all j.*

To prove this, we need the following

Lemma 3.6. *If T is irreducible, then, for any $1 \leq n \leq \infty$,* Red $T^{(n)}$ *is isomorphic to* Red I_n*, where I_n denotes the identity operator on an n-dimensional space.*

Proof. If $P = [P_i^j]_{i,j=1}^n$ is any projection commuting with $T^{(n)}$, then for any i and j we deduce using Lemma 3.3 that $P_{ij} = \lambda_{ij} I$, where λ_{ij} is some scalar. The mapping $P \mapsto [\lambda_{ij}]_{i,j=1}^n$ then induces a lattice isomorphism from Red $T^{(n)}$ onto Red I_n. □

Proof of Proposition 3.5. Using Lemma 2.4, we may infer that Red A and $\sum_j \oplus$ Red $A_j^{(n_j)}$ are isomorphic. This latter lattice is isomorphic to $\sum_j \oplus$Red $(1/j)I_{n_j}$ (by Lemma 3.6) or Red $\sum_j \oplus(1/j)I_{n_j}$. Hence Red A is isomorphic to Red $\sum_j \oplus (1/j)I_{n_j}$. A similar assertion holds for B. Hence if Red A and Red B are isomorphic, then the same is true for Red $\sum_j \oplus(1/j)I_{n_j}$ and Red $\sum_k \oplus(1/k)I_{m_k}$. For normal operators, this implies that $n = m$ and $n_j = m_j$ for all j (cf. [2, Theorem 3.2]). A reversal of the above implications yields the converse. This completes the proof. □

The next result will be useful in Section 4.

Proposition 3.7. *If $T^{(k)}$ is a direct sum of irreducible operators, where k is a natural number, then so is T.*

Proof. Assume that $T^{(k)}$ is unitarily equivalent to the direct sum $S \equiv \sum_{i=1}^{n} \oplus T_i^{(n_i)}$, where $1 \leq n \leq \infty, 1 \leq n_i \leq \infty$ for all i and the T_i's are pairwise-non-unitarily-equivalent irreducible operators. Then there are mutually orthogonal projections $P_j, j = 1, \ldots, k$, commuting with S and satisfying $\sum_j P_j = I$ such that $S \mid (\text{ran } P_j), j = 1, \ldots, k$, are mutually unitarily equivalent. Using Lemma 2.4, we deduce that P_j is of the form $\sum_i \oplus Q_{ij}$, where the Q_{ij}'s are mutually orthogonal projections commuting with $T_i^{(n_i)}$ and satisfying $\sum_j Q_{ij} = I_{n_i}$ such that $T_i^{(n_i)} \mid (\text{ran } Q_{ij}), j = 1, \ldots, k$, are mutually unitarily equivalent. Thus we are reduced to proving the following: if $A^{(k)}$ is unitarily equivalent to $B^{(n)}, 1 \leq n \leq \infty$, where B is irreducible, then A is a direct sum of irreducible operators. We may further assume that $n = \infty$ for otherwise $W^*(A^{(k)})' = M_k(W^*(A)')$ is finite-dimensional by Corollary 3.2, which implies the same for $W^*(A)'$ and thus our assertion for A follows by Corollary 3.2 again. Under the assumption $n = \infty$, $A^{(k)}$ is unitarily equivalent to $C^{(k)}$, where $C = B^{(\infty)}$. The unitary equivalence of A and C then follows from an analogous argument in proving the first test problem in [8]. This completes the proof. □

4. K-THEORETIC CHARACTERIZATION

In the preceding section, direct sums of irreducible operators are characterized in terms of the structure of certain C^*-algebras. We now proceed to describe the latter in terms of some ~~gradients~~ ingredients from the K-theory.

If $\mathcal{A}$ is the C^*-algebra $\sum_{i=1}^{n} \oplus M_{n_i}(\mathrm{C})$ with $1 \leq n \leq \infty$ and $1 \leq n_i < \infty$ for all i, then $\mathcal{A}$ is an approximately finite algebra and hence can be characterized by its (scaled ordered) K_0-group (cf. [13, Theorem 12.1.3]). However, if we allow some n_i's to be ∞, then the K_0-group can no longer distinguish one from the other. This is because the K_0-group of $M_\infty(\mathrm{C})$ is the trivial one (cf. [13, Examples 6.2.3]). However, for any C^*-algebra $\mathcal{A}$ its K_0-group is defined through an abelian semigroup $V(\mathcal{A})$, and it turns out that the latter is strong enough to distinguish $M_n(\mathrm{C})$ between the finite and infinite values of n. Indeed, it is known that

$$V(M_n(\mathrm{C})) \cong \begin{cases} \mathrm{N}_+ & \text{if } 1 \leq n \leq \infty, \\ \mathrm{N}_+ \cup \{\infty\} & \text{if } n = \infty, \end{cases}$$

where $\mathrm{N}_+ = \{0, 1, 2, \ldots\}$ (cf. [13, Examples 6.1.4]), and hence $V(\sum_{i=1}^{n} \oplus M_{n_i}(\mathrm{C})) \cong \mathrm{N}_+^{(k_1)} \oplus (\mathrm{N}_+ \cup \{\infty\})^{(k_2)}$, where k_1 (resp. k_2) is the number of finite (resp. infinite) n_i's, and for a semigroup V, $V^{(k)}$ denotes the direct sum of k copies of V. Our purpose in this section is to prove the following

Theorem 4.1. *An operator T on H is the direct sum of irreducible operators if and only if $V(W^*(T)')$ is isomorphic to $\mathbf{N}_+^{(k_1)} \oplus (\mathbf{N}_+ \cup \{\infty\})^{(k_2)}$ for some integers k_1 and k_2, $0 \leq k_1, k_2 \leq \infty$.*

Here we briefly recall the definition of $V(\mathcal{A})$. Two projections p and q in $M^\infty(\mathcal{A})$, the collection of all finite matrices with entries from $\mathcal{A}$, are said to be *equivalent* if there is a v in $M^\infty(\mathcal{A})$ such that $v^*v = p$ and $vv^* = q$. The equivalence class containing p is denoted by $[p]$ and the set of all these classes is $V(\mathcal{A})$. $V(\mathcal{A})$ is an abelian semigroup with the addition defined by

$$[p] + [q] = [\text{diag }(p, q)],$$

where diag (p, q) is the matrix $\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$ (cf. [13, Section 6.1]).

Theorem 4.1 will be proved after the following series of lemmas.

Lemma 4.2. *Let P and Q be two projections in $W^*(T)'$ which are orthogonal to each other. If P is unitarily equivalent to Q via a unitary operator in $W^*(T)'$, then*

$T \mid (\text{ran } P)$*is unitarily equivalent to* $T \mid (\text{ran } Q)$.

Proof. Let U be a unitary operator in $W^*(T)'$ such that $UP = QU$, and let $W = U \mid (\text{ran } P)$. Then W is a unitary operator from ran P onto ran Q and satisfies $W(T \mid (\text{ran } P)) = (T \mid (\text{ran } Q))W$. □

Lemma 4.3. *let* T *be an operator on* H *with* $V(W^*(T)') \cong (\mathrm{N}_+)^{(k_1)} \oplus (\mathrm{N}_+ \cup \{\infty\})^{(k_2)}$*, where* $0 \leq k_1, k_2 \leq \infty$*. Let* $l = k_1 + k_2$*,* $\{e_i\}_{i=1}^{l}$ *be the* l *free generators of* $V(W^*(T)')$*, and* $P \neq 0$ *be a projection in* $W^*(T)'$*. Then* $T \mid (\text{ran } P)$ *is irreducible if and only if* $[P] = e_i$ *for some* i.

Proof. Assume that $T \mid (\text{ran } P)$ is irreducible and let $[P] = \sum_{i=1}^{l} \oplus \alpha_i e_i$, where the α_i's are integers, $0 \leq \alpha_i \leq \infty$. Assume that more than one of the α_i's is nonzero, say, $\alpha_1, \alpha_2 \neq 0$. Then $f \equiv \alpha_1 e_1$ and $g \equiv \sum_{i=2}^{\infty} \oplus \alpha_i e_i$ are nonzero elements in $V(W^*(T)')$. Hence a natural number m exists for which there are mutually orthogonal projections Q and R in $M_m(W^*(T)') = W^*(T^{(m)})'$ such that $[Q] = f$ and $[R] = g$. If $S = Q + R$, then $[S] = [Q] + [R] = f + g = \sum_{i=1}^{l} \oplus \alpha_i e_i = [P]$. Hence S and $P \oplus 0^{(m-1)}$ are unitarily equivalent via a unitary operator in $W^*(T^{(m)})'$, where 0 denotes the zero operator on H. Lemma 4.2 then implies that $T^{(m)} \mid (\text{ran } S)$ is unitarily equivalent to

$T^{(m)} \mid (\text{ran } (P \oplus 0^{(m-1)}))$. But the former equals $(T^{(m)} \mid (\text{ran } Q)) \oplus (T^{(m)} \mid (\text{ran } R))$ while the latter coincides with the irreducible $T \mid (\text{ran } P)$. This is a contradiction. Hence we can have only one of the $e_i's$ to be nonzero, which proves that $[P] = e_i$ for some i.

Conversely, assume that $[P] = e_1$ and $T \mid (\text{ran } P)$ is reducible. Then there are nonzero projections Q and R in $W^*(T)'$ such that $QR = 0$ and $P = Q + R$. Let $[Q] = \sum_{i=1}^{l} \oplus \alpha_i e_i$ and $[R] = \sum_{i=1}^{l} \oplus \beta_i e_i$, where $0 \leq \alpha_i, \beta_i \leq \infty$ for all i. From $e_1 = [P] = [Q] + [R] = \sum_{i=1}^{l} \oplus (\alpha_i + \beta_i) e_i$, we deduce that $\alpha_1 + \beta_1 = 1$ and $\alpha_i + \beta_i = 0$ for all $i \geq 2$. Hence $\alpha_1 = 0$ or $\beta_1 = 0$ and $\alpha_i = \beta_i = 0$ for all $i \geq 2$. This shows that $[Q] = 0$ or $[R] = 0$, which is a contradiction. Thus $T \mid (\text{ran } P)$ is irreducible. □

Lemma 4.4. *Assume that A on H is a direct sum of irreducible operators and B on K has no reducing subspace on which it is irreducible. If X is such that $XA = BX$ and $XA^* = B^*X$, then $X = 0$.*

Proof. Let $A = \sum_{n=1}^{\infty} \oplus A_n$ on $H = \sum_{n=1}^{\infty} \oplus H_n$, where A_n is irreducible for all n. (A similar argument applies if A is the direct sum of finitely many irreducible operators.) Let X^* be represented as $[X_1\ X_2\ \cdots]^t$ from K to $\sum_n \oplus H_n$. We now show that $X_1 = 0$. Indeed, from $XA = BX$ and $XA^* = B^*X$ a simple computation yields $X_1B = A_1X_1$ and $X_1B^* = A_1^*X_1$. Hence $(X_1X_1^*)A_1 = A_1(X_1X_1^*)$ and $(X_1X_1^*)A_1^* = A_1^*(X_1X_1^*)$. Since A_1 is irreducible, Lemma 3.3 implies that $X_1X_1^*$ is a scalar operator, say, $X_1X_1^* = \lambda I_{H_1}$. Assuming that $X_1 \neq 0$, we want to derive a contradiciton. Indeed, in this case, we have $\lambda \neq 0$. If $U = \lambda^{-1/2}X_1$, then $UU^* = I_{H_1}$ and $Q \equiv U^*U$ is a projection on K satisfying $QB = BQ$. Let $p = I_{H_1} \oplus 0$ and $q = 0 \oplus Q$ be operators on $H_1 \oplus K$ and let $p' = p \oplus 0$ and $q' = q \oplus 0$ on $(H_1 \oplus K) \oplus (H_1 \oplus K)$. Letting $C = A_1 \oplus B$, we claim that p' and q' are unitarily equivalent via a unitary operator in $W^*(C^{(2)})'$. To prove this, let $v = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$ on $H_1 \oplus K$. Then v is a partial isometry in $W^*(C)'$ with $vv^* = p$ and $v^*v = q$. Our assertion then follows from [13, Prop. 5.2.12]. By Lemma 4.2, we infer that $C^{(2)} \mid (\text{ran } p')$ is unitarily equivalent to $C^{(2)} \mid (\text{ran } q')$. But the former coincides with the irreducible A_1 and the latter $B \mid (\text{ran } Q)$. Thus $B \mid (\text{ran } Q)$ is irreducible, which contradicts our assumption. This proves that $X_1 = 0$. Similarly, we have $X_n = 0$ for all $n \geq 2$ and hence $X = 0$ as asserted. □

We are now ready for

Proof of Theorem 4.1. The necessity follows from the paragraph before the statement of the theorem. For the sufficiency, we assume that $V(W^*(T)')$ is isomorphic to $\mathbf{N}_+^{(k_1)} \oplus (\mathbf{N}_+ \cup \{\infty\})^{(k_2)}$, where $0 \leq k_1, k_2 \leq \infty$. Let P be a projection in some $M_k(W^*(T)') = W^*(T^{(k)})'$ (k is a natural number) such that $[P]$ is one of the free generators of $V(W^*(T)')$. By Lemma 4.3, $T^{(k)} \mid (\text{ran } P)$ is irreducible (here we embed $W^*(T)'$ into $M_k(W^*(T)')$ under the canonical embedding $A \mapsto \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix}$, which results in the identification of $V(W^*(T)')$ and $V(M_k(W^*(T)'))$; cf. [13, Lemma 6.2.10]). Using Zorn's lemma, we can find a maximal family of mutually orthogonal projections $\{P_j\}_{j=1}^n$, $1 \leq n \leq \infty$, in $W^*(T^{(k)})'$ such that $T^{(k)} \mid (\text{ran } P_j)$ is irreducible for all j. Letting $Q = \sum_j P_j$, we will show that $Q = I^{(k)}$, the identity operator on $H^{(k)}$. Assume this is not the case. Since Q is a projection in $W^*(T^{(k)})'$, the operators $T_1 \equiv T^{(k)} \mid (\text{ran } Q)$ and $T_2 \equiv T^{(k)} \mid (\text{ran } (I^{(k)} - Q))$ are acting on nontrivial spaces. Moreover, T_1 is the direct sum of irreducible operators and T_2 has no reducing subspace on which it is irreducible. Hence we may apply Lemma 4.4 to infer that $W^*(T^{(k)})' = W^*(T_1)' \oplus W^*(T_2)'$. Therefore, $V(W^*(T^{(k)})') \cong V(W^*(T_1)') \oplus V(W^*(T_2)')$ (cf. [13, Prop. 6.2.1]). Since both $V(W^*(T^{(k)})') = V(W^*(T)')$ and $V(W^*(T_1)')$ are torsion-free semigroups, the same is true for $V(W^*(T_2)')$. Let R be a projection in $W^*(T_2^{(m)})$ (m is a natural number) for which $[R]$ is one of the free generators of $V(W^*(T_2)')$. From Lemma 4.3, we know that $T_2^{(m)} \mid (\text{ran } R)$ is irreducible. Arguing as above, we can find a nonzero projection Q_1 in $W^*(T_2^{(m)})'$ such that $T_3 \equiv T_2^{(m)} \mid (\text{ran } Q_1)$ is the direct sum of irreducible operators and $T_4 \equiv T_2^{(m)} \mid (\text{ran } (I - Q_1))$

has no reducing subspace on which it is irreducible. Applying Lemma 4.4, we obtain that $W^*(T_2^{(m)})' = W^*(T_3)' \oplus W^*(T_4)'$. Thus Q_1 commutes with every operator in $W^*(T_2^{(m)})'$, that is, Q_1 is in $W^*(T_2^{(m)})''$ or $W^*(T_2^{(m)})$ by the von Neumann double commutant theorem. Therefore, Q_1 is of the form $S^{(m)}$, where S is a nonzero projection in $W^*(T_2)$, and hence $T_3 = T_2^{(m)} \mid (\text{ran } Q_1) = (T_2 \mid (\text{ran } S))^{(m)}$. Since T_3 is the direct sum of irreducible operators, the same is true for $T_2 \mid (\text{ran } S)$ by Proposition 3.7. This contradicts the fact that T_2 has no reducing subspace on which it is irreducible. Hence we must have $Q = I^{(k)}$. Thus $T^{(k)}$ is a direct sum of irreducible operators. By Proposition 3.7, the same is true for T. This completes the proof. □

We end this paper by noting that Theorem 4.1 cannot be generalized to arbitrary C^*-algebras, that is , a (unital) C^*-algebra $\mathcal{A}$ with $V(\mathcal{A})$ isomorphic to $\mathbf{N}_+^{(k_1)} \oplus (\mathbf{N}_+ \cup \{\infty\})^{(k_2)}$, $0 \leq k_1, k_2 \leq \infty$, may not be $*$-isomorphic to $\sum_i \oplus M_{n_i}(\mathbf{C})$, where $1 \leq n_i \leq \infty$. One example of such C^*-algebra is $\mathcal{A} = \{\lambda I + K : \lambda \in \mathbf{C},\ K \text{ compact operator on } H\}$, where H is an infinite-dimensional separable Hilbert space. It can be verified that $V(\mathcal{A})$ is isomorphic to $\mathbf{N}_+ \cup \{\infty\}$ (cf. [13, Examples 6.1.4]), but $\mathcal{A}$ is not $*$-isomorphic to $\mathcal{B}(H)$ since their K_0-groups are different (cf. [13, Examples 6.2.3]). Whether there is an example of such von Neumann algebras seems to be unknown.

Acknowledgements

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赴匈牙利開會附加說明

因在匈牙利 Szeged 舉行的「泛函分析及應用國際會議」開會日期是八月二日至六日，由台灣前往該地行程需要有兩天的時間，故本人於七月三十一日自台灣出境，八月一日晚上始到達該地，才能趕上八月二日的會議開幕儀式，特說明如上。

國立交通大學應用數學系教授
吳培元
1999. 9. 1

匈牙利開會及訪問報告

國立交通大學應用數學系

吳培元

匈牙利大數學家 Béla Szökefalvi-Nagy 於1998年12月21日因病去世，享年85歲。他在富氏分析、逼近理論、尤其是算子理論等領域貢獻卓著，可以說是一代宗師的典型。因其一生大部份時間都是在 Szeged, Hungary 渡過，故由當地的幾個學術單位，包括 Bolyai Institute of University of Szeged 及匈牙利科學院，聯合發起一個紀念性的學術會議 Functional Analysis and Applica Memorial Conference for Béla Szökefalvi-Nagy，在1999年8月2日至6日在 Szeged 的匈牙利科學院舉行。我也正好利用這一次訪問匈牙利的機會參與該項會議。

我是在7月31日自台北出發，經過十六個半小時後到達阿姆斯特丹，再轉機到布達佩斯，改搭火車到 Szeged，到達旅館時都已是8月1日晚上了。會議在第二天上午開始，各主辦單位代表及實際籌辦人 L. Kérchy 都上台推崇 Sz.-N 的學術貢獻，當地的報紙和電視台也都報導了此項活動。會議一共進行了五天，每天上午安排了三至四位四十分鐘的演講，下午則分 A、B 兩組同時進行三十或二十分鐘的演講。大部份的演講內容是和 Sz.-Nagy 和 Foias 所發展的那一套收縮算子理論的再延伸，故都相當艱澀深入，即使連本行的專家都不太容易確實掌握。其中主要的發展重點有：(1) commutant lifting theorem

的推廣及應用（如 C. Foiaş, C. Sadosky, A. Biswas等人的工作）；(2) dual algebra 技巧用於多個算子不變子空間的研究（如 J. Eschmeier, B. Chevreau, M. Ptak, M. Kosiek 等的工作）；(3) 用 Hilbert module 等代數方法推廣單個壓縮算子的模式理論（如 R. G. Douglas 的工作）。

此次與會者將近一百人，俄國人、法國人、美國人及東歐各國人佔其大半，由東方來的，除了我之外，另有三位日本人，他們大多是隨 T. Ando 的領導從事算子不等式的研究。俄國人則承襲了他們深厚的分析學研究的傳統，在理論和應用方面都有紮實而深厚的功力。相對而言，美國人則較偏於抽象、形式上的思考方向，研究成果流於淺薄。

我的演講被安排在8月5日下午，共三十分鐘，講題是 Polygons and numerical ranges。這是我和我的博士班學生高華隆合作的三篇論文的綜合報告，內容因涉及數百年前投影幾何的一些結果，淺顯易懂，引起與會者高度的興趣，要求寄論文者也很多，應是一次很成功的演講。

開會期間的五天，每天晚上都有酒會等社交活動，其中8月[illegible]日晚上主辦單位特別安排了與會者搭車到當地的墓園，在 Sz.-Nagy 的墳前獻花、默哀、致敬。

8月6日下午，我也被安排主持會議的一個議程，介紹五位演講者演講。

在六年前（1993年）同樣的場址也曾舉辦過一項大型國際泛函分析研討會，慶祝 Sz-Nagy 的八十歲生日，我也曾經與會過。當時 Sz-Nagy 全程參與會議的演講及酒會，而今在同一場地再次開會懷念他的貢獻，令人憑生感慨。哲人已遠，但他對算子理論的貢獻，將隨著後起者的研究繼續留傳下去。

會議結束後，我於9月7日搭火車前往布達佩斯，至當地的匈牙利科學院數學研究所訪問，與 Z. Sebestyén 交換研究心得。數日後於9月10日離開，搭機於9月11日回到新竹，結束了這一趟前後12天的匈牙利訪問。

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8	4	Szeged	參加會議					美金162元×32.27×0.8=4182元			4,182元
8	5	Szeged	參加會議					美金162元×32.27×0.8=4182元			4,182元
合小計				36,500元				25,092元			61,592元

備註：☐派車 ☐供膳 ☐供宿
☐未派車 ☐未供膳 ☐未供宿 ＊國外出差應註明匯率並附證明。

存帳郵局 局號 帳號

上列出差旅費計新台幣　拾　萬　千　百　拾　元整，具領人 吳（蓋章）

校長或授權代簽人	會計室 主任	會計室 審核	人事室（事務組）	處、院、館中心主管	系所主管或組長	計畫主持人	出差人	郵局入帳章
教授 林松	88.10 (甲)			林松	應用數學系 系主任 黃大	吳培元	吳培元	新竹郵局006000-9 薪資轉帳存款專用章 88.10.22

注意：本表應逐欄填寫清楚，如有塗改需加蓋私章。