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Decision-making in a single-period inventory environment with fuzzy demand

Rung-Hung Su [a,*], Dong-Yuh Yang [b], W.L. Pearn [a]

[a] Department of Industrial Engineering and Management, National Chiao Tung University, Hsinchu, Taiwan, ROC
[b] Institute of Information Science and Management, National Taipei College of Business, Taipei, Taiwan, ROC

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ABSTRACT

This paper first defines the profitability to be the probability of achieving a target profit under the optimal ordering policy, and introduces a new index (achievable capacity index; I_A) which can briefly analyze the profitability for newsboy-type product with normally distributed demand. Note that since the level of profitability depends on the demand mean μ and the demand standard deviation σ if the related costs, selling price, and target profit are given, the index I_A is a function of μ and σ. Then, we assess level performance which examines if the profitability meets designated requirement. The results can determine whether the product is still desirable to order/manufacture. However, μ and σ are always unknown, and the demand quantity is common to be imprecise, especially for new product. To tackle these problems, a constructive approach combining the vector of fuzzy numbers is introduced to establish the membership function of the fuzzy estimator of I_A. Furthermore, a three-decision testing rule and step-by-step procedure are developed to assess level performance based on fuzzy critical values and fuzzy p-values.

1. Introduction

The classical newsboy problem (single-period problem) deals with the purchasing inventory problem for short shelf-life products with the uncertainty of demand. For such problems, the managers should determine ordering quantity at the beginning of each period. Products cannot be sold in the next period and need additional cost (excess cost) to dispose it if the ordering quantity exceeds actual demand. Therefore, the determination of the ordering quantity is critical in the classical newsboy problem. Several extensions to the newsboy problem have been proposed and solved in the literature. Among those extensions are alternative objective functions such as minimizing the expected cost (Nahmias, 1993), maximizing the expected profit (Khouja, 1995), maximizing the expected utility (Ismail & Louderback, 1979; Lau, 1980), and maximizing the probability of achieving a target profit (Ismail & Louderback, 1979; Khouja, 1996; Lau, 1980; Sankarasubramanian & Kumaraswamy, 1983; Shih, 1979). In fact, these maximum and minimum values can be adopted to measure product's capacity. For example, the maximum expected profit and maximum probability of achieving the target profit can measure product's profitability.

In this paper, we consider the newsboy-type product with normally distributed demand and assume that the profitability is defined to be the probability of achieving a target profit under the optimal ordering policy. Furthermore, in order to simplify the calculation, we develop a new index, which has a simple form and can correspond to the profitability, and so-called "achievable capacity index (ACI)", and be denoted by I_A. Note that since the level of profitability depends on the demand mean μ and the demand standard deviation σ if the related costs, selling price, and target profit are given, the index I_A is a function of μ and σ. Then, we assess level performance which examines if the profitability meets designated requirement. However, μ and σ are always unknown. To tackle this problem, one should collect the historical data of demand, and then implement the following hypothesis testing, $H_0 : I_A \leqslant C$ versus $H_1 : I_A > C$, where C is a designated requirement. Critical value of the test must be calculated to determine the results. The results can determine whether the product is still desirable to order/manufacture. But in practice, especially for new product, the demand quantity is difficult to acquire due to lack of information and historical data. In this case, the demand quantity is approximately specified based on the experience. Some papers have dealt with this case by applying fuzzy theory. Petrovic, Petrovic, and Vujosevic (1996) first proposed a newsboy-type problem with discrete fuzzy demand. Dutta, Chakraborty, and Roy (2007) studied the newsboy problem with reordering opportunities under fuzzy demand. Zhen and Xiaoyu (2006) considered the multi-product newsboy problem with fuzzy demands under budget constraint. Kao and Hsu (2002) compared the area of fuzzy numbers to obtain the optimal order quantity. To the best of our knowledge, no researchers have investigated the fuzzy hypothesis testing for assessing level performance. In this study, we first use a

* Corresponding author. Address: Department of Industrial Engineering and Management, National Chiao Tung University, 1001 University Road, Hsinchu, Taiwan 300, ROC. Tel.: +886 3 5731630; fax: +886 3 5268330.
E-mail address: runghung316@gmail.com (R.-H. Su).

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new approach in fuzzy statistics to estimate the demand mean and variance parameters of normal distribution (Buckley, 2004, 2005a, 2005b). Then, a general method combining the vector of fuzzy numbers of sample mean $\tilde{\bar{x}}$, and sample variance $\tilde{s}^2$ is proposed to derive the membership function of the fuzzy estimator of I_A. Furthermore, a three-decision testing rule for assessing level performance according to two different criteria, critical value and fuzzy p-value are proposed. Based on the test, we develop a step-by-step procedure for managers to use so that decisions made in examining the profitability are more reliable. The rest of the paper is organized as follows. In the next section, we calculate the profitability, and develop a new index I_A to correspond profitability. Section 3 discusses the statistical properties of estimation for I_A based on crisp data. In Section 4, we present some basic definitions, notations of fuzzy sets and the α-cuts of fuzzy estimation for I_A. Section 5 deals with implementing fuzzy hypothesis testing for assessing level performance. Following critical value and fuzzy p-value, decision rules and testing procedures are developed. In Section 6, a numerical example is discussed to illustrate the procedure of solving the problem. Some conclusions are given in the final section.

2. Profitability and achievable capacity index I_A

The total profit function, Z, in the newsboy model depends on the demand quantity D and ordering quantity Q, and is formulated as

$$Z = \begin{cases} c_pD - c_e(Q-D) = (c_p+c_e)D - c_eQ, & 0 \leqslant D \leqslant Q, \\ c_pQ - c_s(D-Q) = -c_sD + (c_p+c_s)Q, & Q < D < \infty, \end{cases}$$

where

- c_p the net profit per unit (selling price per unit minus purchasing cost per unit),
- c_e the excess cost per unit (purchasing cost per unit plus disposal cost per unit; $c_p > c_e > 0$),
- c_s the shortage cost per unit ($c_p > c_s > 0$),
- k the target profit which is set according to the product property and the sales experience.

Note that if $c_pQ < k$, then the profit impossibly achieve the target profit, even the demand is large enough. Therefore, the order quantity should be at least k/c_p. For $Q \geqslant k/c_p$, Z increases for $0 \leqslant D \leqslant Q$ and decreases for $D \geqslant Q$, and has a maximum at point $D = Q$. The maximum value of Z is equal and higher than k, i.e., $Z = c_pD = c_pQ \geqslant k$. The target profit will be realized when D is equal to either $LAL(Q)$ or $UAL(Q)$. So the target profit will be achieved in $D \in [LAL(Q), UAL(Q)]$, where

$$LAL(Q) = \frac{c_eQ + k}{c_p + c_e} \quad \text{and} \quad UAL(Q) = \frac{(c_p+c_s)Q - k}{c_s},$$

are the lower and upper achievable limits, respectively, and both are the functions of Q.

Under the assumption that the demand is normally distributed, the probability of achieving the target profit is

$$\begin{aligned} \Pr[Z \geqslant k] &= \Phi\left(\frac{UAL(Q)-\mu}{\sigma}\right) - \Phi\left(\frac{LAL(Q)-\mu}{\sigma}\right) \\ &= \Phi\left(\frac{d(Q)+m(Q)-\mu}{\sigma}\right) - \Phi\left(\frac{-d(Q)+m(Q)-\mu}{\sigma}\right), \end{aligned} \tag{1}$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal distribution, $d(Q) = [UAL(Q) - LAL(Q)]/2$ is the half-length of the achievable interval $[LAL(Q), UAL(Q)]$, and $m(Q) = [UAL(Q) + LAL(Q)]/2$ is the midpoint between the lower and upper achievable limits. Since the necessary condition for maximizing $\Pr[Z \geqslant k]$ is $\mathrm{d}\Pr[Z \geqslant k]/\mathrm{d}Q = 0$, we have

$$\mu = m(Q) - \frac{\omega\sigma^2}{2d(Q)}, \tag{2}$$

where $\omega = \ln[1 + c_pA/c_sc_e] > 0$ and $A = c_p + c_e + c_s$. For $Q \geqslant k/c_p$, the optimal ordering quantity can be obtained by solving Eq. (2), i.e.:

$$Q^* = \frac{k}{c_p} + \frac{c_s(c_p+c_e)(c_p\mu - k)}{c_p(c_pA + 2c_ec_s)} + \sqrt{\left[\frac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)}\right]^2 + \frac{2c_s^2(c_p+c_e)^2\omega\sigma^2}{c_pA(c_pA+2c_ec_s)}} > \frac{k}{c_p}. \tag{3}$$

In addition, the sufficient condition is also calculated as follows:

$$\left.\frac{\mathrm{d}^2\Pr[Z \geqslant k]}{\mathrm{d}Q^2}\right|_{Q=Q^*} = -\frac{(c_p+c_s)e^{-\frac{1}{2}\left(\frac{UAL(Q^*)-\mu}{\sigma}\right)^2}}{\sqrt{2\pi}\sigma^3c_s^2(c_p+c_e)}\left[d(Q^*)(c_pA+2c_ec_s) + \frac{c_pA\omega\sigma^2}{2d(Q^*)}\right] < 0. \tag{4}$$

As a result, it leads to the conclusion that Q^* is the optimal ordering quantity that maximizes the probability of achieving the target profit. By using Eq. (2) and substituting Eq. (3) into Eq. (1), the profitability, $\mathbb{P}$, can be obtained as follows:

$$\mathbb{P} = \Phi\left(\frac{d(Q^*)}{\sigma} + \frac{\omega\sigma}{2d(Q^*)}\right) - \Phi\left(-\frac{d(Q^*)}{\sigma} + \frac{\omega\sigma}{2d(Q^*)}\right). \tag{5}$$

2.1. Achievable capacity index I_A

We develop a new index to express the product's profitability. It is defined as:

$$I_A = \frac{d(Q^*)}{\sigma} = \frac{M(c_p\mu - k)}{\sigma} + \sqrt{\left[\frac{M(c_p\mu-k)}{\sigma}\right]^2 + c_pM\omega}, \tag{6}$$

where $M = A/2(c_pA + 2c_ec_s)$, and call it "achievable capacity index". The numerator of I_A provides the half demand range over which the total profit will achieve the target profit under the optimal order quantity. The denominator gives demand standard deviation. Obviously, it is desirable to have a I_A as large as possible. From the Eq. (5), $\mathbb{P}$ can be rewritten as follows:

$$\mathbb{P} = \Phi\left(I_A + \frac{\omega}{2I_A}\right) - \Phi\left(-I_A + \frac{\omega}{2I_A}\right). \tag{7}$$

It is easy to see that $\mathbb{P}$ is the function of I_A. Taking the first-order derivative of $\mathbb{P}$ with respect to I_A, we obtain

$$\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}I_A} = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(I_A + \frac{\omega}{2I_A}\right)^2}\left[\frac{\omega}{2I_A^2}(e^\omega - 1) + e^\omega + 1\right] > 0. \tag{8}$$

Table 1
The profitability for I_A = 0.5(0.5)4.0 and ω = 0.5(0.5)5.0.

ω	I_A							
	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
0.5	0.3413	0.6677	0.8610	0.9528	0.9871	0.9972	0.9995	0.9999
1.0	0.2417	0.6247	0.8450	0.9477	0.9858	0.9969	0.9995	0.9999
1.5	0.1359	0.5586	0.8186	0.9391	0.9835	0.9964	0.9994	0.9999
2.0	0.0606	0.4773	0.7825	0.9270	0.9803	0.9957	0.9993	0.9999
2.5	0.0214	0.3891	0.7377	0.9111	0.9759	0.9948	0.9991	0.9999
3.0	0.0060	0.3023	0.6853	0.8914	0.9703	0.9936	0.9989	0.9998
3.5	0.0013	0.2236	0.6267	0.8677	0.9634	0.9920	0.9986	0.9998
4.0	0.0002	0.1573	0.5639	0.8400	0.9550	0.9901	0.9983	0.9998
4.5	0.0000	0.1051	0.4987	0.8083	0.9449	0.9877	0.9978	0.9997
5.0	0.0000	0.0666	0.4330	0.7728	0.9330	0.9848	0.9973	0.9996

As a result, the large value of I_A, the larger value of $\mathbb{P}$. Therefore, in order to simplify the calculation, one also observe the value of I_A to present the profitability. Table 1 displays the $\mathbb{P}$ for I_A = 0.5(0.5)4.0 and ω = 0.5(0.5)5.0.

3. Crisp estimation for I_A

The formula of I_A can be easy to understand and straightforward to apply. But in practice, the demand mean μ and the demand standard deviation σ are usually unknown. Thus, we should collect the historical data of demand to estimate actual I_A. If a demand sample of size n is given as $\{x_1, x_2, \ldots, x_n\}$, the natural estimator $\widehat{I}_A$ is obtained by replacing μ and σ by their estimator $\overline{X} = \sum_{i=1}^{n} x_i/n$ and $s = \left[\sum_{i=1}^{n}(x_i - \overline{X})^2/(n-1)\right]^{1/2}$, respectively. We have the following result

$$\widehat{I}_A = \frac{M(c_p\overline{X} - k)}{s} + \sqrt{\left[\frac{M(c_p\overline{X} - k)}{s}\right]^2 + c_pM\omega}. \tag{9}$$

Under the consideration that D is a normally distributed, $D \sim N(\mu, \sigma^2)$, we have $\overline{X} \sim N(\mu, \sigma^2/n)$. Let us define $Y = T/V$, where $T = M(c_p\overline{X} - k)/\sigma$ and $V = \sqrt{s^2/\sigma^2}$. It is well known that $T \sim N\left(M(c_p\mu - k)/\sigma, c_p^2M^2/n\right)$, so the probability density function (PDF) of T can be expressed in the following:

$$f_T(t) = \frac{1}{c_pM\sqrt{\frac{2\pi}{n}}} exp\left[-\frac{\left(t - \frac{M(c_p\mu - k)}{\sigma}\right)^2}{\frac{2c_p^2M^2}{n}}\right], \quad -\infty < t < \infty.$$

Since the random variable $(n-1)s^2/\sigma^2$ follows the Chi-squared distribution with $n-1$ degree of freedom, $V^2 = s^2/\sigma^2 \sim \Gamma((n-1)/2, 2/(n-1))$. By using the technique of change-of-variable, the PDF of V can be derived as follows:

$$f_V(v) = \frac{2v^{n-2}e^{-\frac{n-1}{2}v^2}}{\Gamma\left(\frac{n-1}{2}\right)\left(\frac{2}{n-1}\right)^{\frac{n-1}{2}}}, \quad v > 0.$$

Because T and V are independent continuous random variables, we easily obtain the PDF of Y by the *convolution formula*

$$\begin{aligned} f_Y(y) &= \int_0^\infty f_T(vy)f_V(v)|v|dv \\ &= \int_0^\infty \frac{2v^{n-1}\left(\frac{2}{n-1}\right)^{-\frac{n-1}{2}}}{c_pM\Gamma\left(\frac{n-1}{2}\right)\sqrt{\frac{2\pi}{n}}} exp\left[-\frac{\left(vy - \frac{I_A^2 - c_pM\omega}{2I_A}\right)^2}{\frac{2c_p^2M^2}{n}} - \frac{n-1}{2}v^2\right]dv, \\ &\quad -\infty < y < \infty. \end{aligned}$$

Subsequently, we define $R = \widehat{I}_A = Y + \sqrt{Y^2 + c_pM\omega}$. The CDF and PDF of R can be derived, respectively, as follows:

$$F_R(r) = \int_0^{\frac{r^2 - c_pM\omega}{2r}} f_Y(y)dy, \quad 0 < r < \infty$$

and

$$f_R(r) = B(r)\int_0^\infty g(v, r)dv, \quad 0 < r < \infty,$$

where

$$B(r) = \frac{r^2 + c_pM\omega}{r^2c_pM\sqrt{\frac{2\pi}{n}}\Gamma\left(\frac{n-1}{2}\right)\left(\frac{2}{n-1}\right)^{\frac{n-1}{2}}}$$

and

$$g(v, r) = v^{n-1}exp\left[-\frac{\left(v\frac{r^2 - c_pM\omega}{2r} - \frac{I_A^2 - c_pM\omega}{2I_A}\right)^2}{\frac{2c_p^2M^2}{n}} - \frac{(n-1)v^2}{2}\right].$$

Fig. 1 shows the CDF and PDF plots of R with c_p = 10, c_e = 5, c_s = 3, I_A = 3.0(0.5)4.0 for n = 30, 50, 100 and 200 (from bottom to top in plots). From Fig. 1, the PDF plots of R reveal the following features:

(1) The larger the value of I_A, the larger the variance of $R = \widehat{I}_A$;
(2) The distribution of R is unimodal and is rather symmetric to I_A even for small sample sizes;
(3) The larger the sample sizes n, the smaller the variance of R, which is certain for all sample estimators.

4. Fuzzy estimation for I_A

4.1. Definitions and notations for fuzzy set theory

In the traditional precise set, the degree of an element belongs to a set is either one or zero. In order to deal with the imprecise data, Zadeh (1965) proposed the fuzzy set theory. The definitions and notations are shown as follows:

$\mathbb{R}$ the universal set,
$\widetilde{A}$ the fuzzy set,
$\eta_{\widetilde{A}}$ the membership function of $\widetilde{A}$, $\eta_{\widetilde{A}} : \mathbb{R} \to [0, 1]$,
$\widetilde{A}^{[\alpha]}$ the α-cuts of fuzzy number $\widetilde{A}$ (the set of elements that belong to the fuzzy set $\widetilde{A}$ at least to the degree of membership α, $\widetilde{A}^{[\alpha]} = \{x|\eta_{\widetilde{A}} \geqslant \alpha, x \in \mathbb{R}\}$),
$L_{\widetilde{A}}(\alpha)$ the lower bound of the closed interval of $\widetilde{A}^{[\alpha]}$,
$U_{\widetilde{A}}(\alpha)$ the upper bound of the closed interval of $\widetilde{A}^{[\alpha]}$.

4.2. α-Cuts of a fuzzy estimation for I_A

An important fact that the demand mean and the demand standard deviation are usually unknown. One should estimates these two parameters from observations. However, there is a great deal of uncertainty in the model due to these parameters not being known precisely. According to Buckley and Eslami (2004), we consider additional uncertainty in the unknown parameters, μ and σ^2, by using fuzzy estimators. The fuzzy estimators, $\tilde{\bar{x}}$ and $\tilde{s}^2$, are constructed from a set of confidence intervals. As the triangular fuzzy numbers with α-cuts are constructed, the fuzzy numbers are given as follows:

$$\tilde{\bar{x}}^{[\alpha]} = [L_{\tilde{\bar{x}}}(\alpha), U_{\tilde{\bar{x}}}(\alpha)] = \left[\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right], \quad \text{for } \forall\alpha \in (0, 1], \tag{10}$$

$$\tilde{s}^{2[\alpha]} = [L_{\tilde{s}^2}(\alpha), U_{\tilde{s}^2}(\alpha)] = \left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right], \quad \text{for } \forall\alpha \in (0, 1], \tag{11}$$

where $t_{\alpha/2,n-1}$ is the upper $\alpha/2$ quantile of the t distribution with $n-1$ degrees of freedom, $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the upper $\alpha/2$ and $1-\alpha/2$ quantiles of the chi-square with $n-1$ degrees of freedom, respectively. Figs. 2 and 3 display the membership functions of fuzzy estimation for $\tilde{\bar{x}}$ and $\tilde{s}^2$, respectively, $\bar{x} = 25$, $s^2 = 4$ and k = 200 with n = 30, 50, 100, and 200.

In order to obtain an α-cuts of fuzzy number $\widehat{I}_A$, let $\mu \in \tilde{\bar{x}}^{[\alpha]}$ and $\sigma^2 \in \tilde{s}^{2[\alpha]}$ then

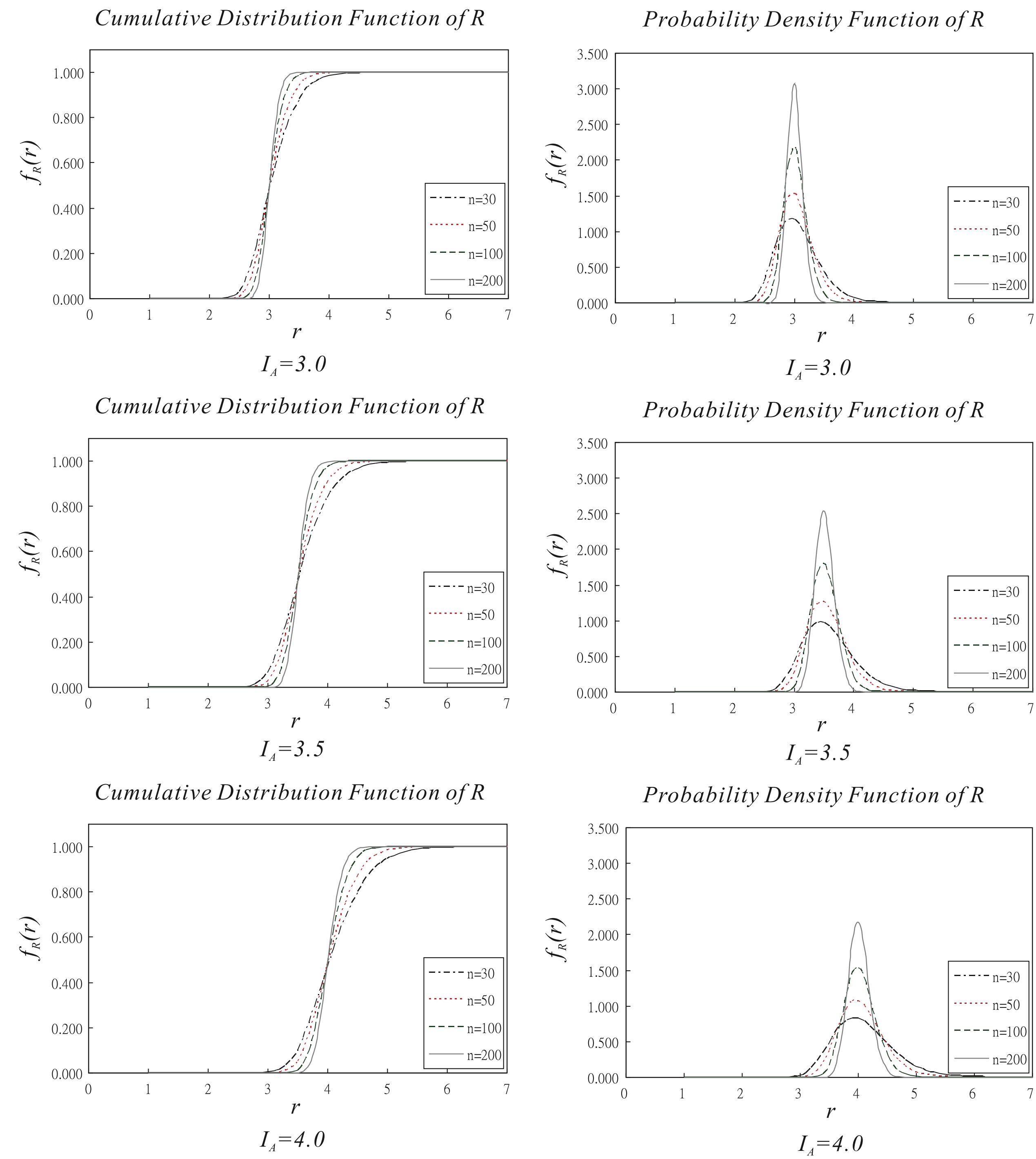

Fig. 1. The CDF and PDF plots of R for sample sizes n = 30, 50, 100, 200 (from bottom to top in plots).

$$\widetilde{\widehat{I}}_A^{[\alpha]} = \left[L_{\widetilde{\widehat{I}}_A}(\alpha), U_{\widetilde{\widehat{I}}_A}(\alpha)\right] = \left[\frac{M(c_p L_{\tilde{\bar{x}}}(\alpha)-k)}{\sqrt{U_{\tilde{s}^2}(\alpha)}} + \sqrt{\left[\frac{M(c_p L_{\tilde{\bar{x}}}(\alpha)-k)}{\sqrt{U_{\tilde{s}^2}(\alpha)}}\right]^2 + c_p M\omega}, \frac{M(c_p U_{\tilde{\bar{x}}}(\alpha)-k)}{\sqrt{L_{\tilde{s}^2}(\alpha)}} + \sqrt{\left[\frac{M(c_p U_{\tilde{\bar{x}}}(\alpha)-k)}{\sqrt{L_{\tilde{s}^2}(\alpha)}}\right]^2 + c_p M\omega}\right]. \tag{12}$$

Fig. 4 plots the membership function of fuzzy estimation $\widetilde{\widehat{I}}_A$ with $\widehat{I}_A = 2.5705$ for n = 30, 50, 100 and 200.

5. Fuzzy hypothesis testing for assessing level performance

To test whether the profitability meets the designated requirement, we consider the following testing hypothesis, procedure with the null hypothesis $H_0 : I_A \leqslant C$, versus the alternative $H_1 : I_A > C$, where C is a designated requirement. Based on CDF of R, given designated requirement C, sample size n, and level of Type I error θ, the critical value c_0 can be calculated by solving the following equation

$$\Pr\{R \geqslant c_0 | I_A = C, n\} = 1 - \int_0^{\frac{c_0^2 - c_p M\omega}{2c_0}} f_Y(y)dy = \theta. \tag{13}$$

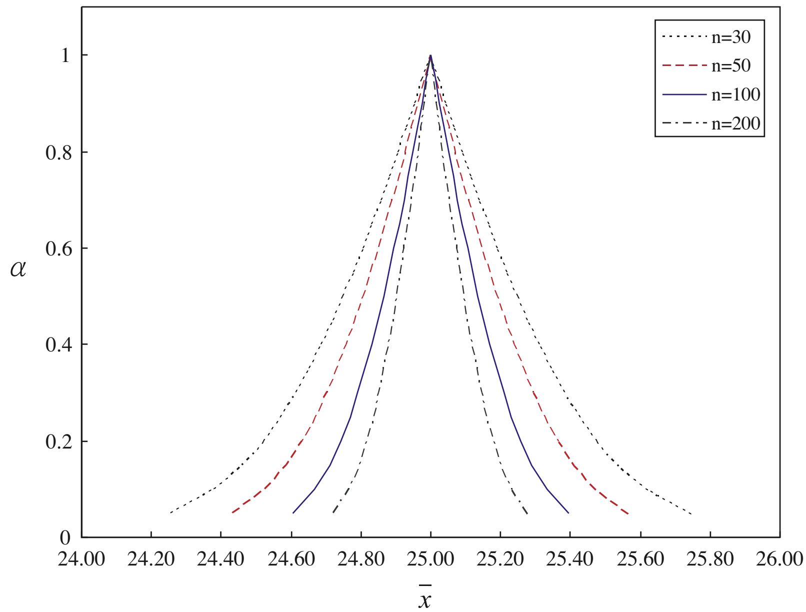

Fig. 2. The membership functions of fuzzy estimation for $\bar{x}$ with n = 30, 50, 100, 200.

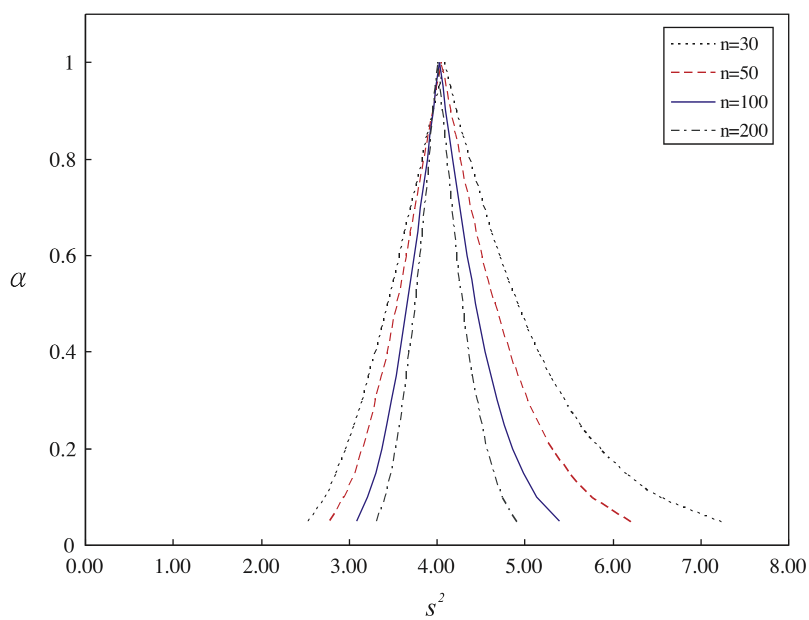

Fig. 3. The membership functions of fuzzy estimation for s^2 with n = 30, 50, 100, 200.

On the other hand, the p-value is generally used for making decisions in hypothesis testing. The p-value corresponding to a specific value of $\widehat{I}_A$ calculated from the sample data, c^*, can be obtained from the following equation

$$p\text{-value} = \Pr\{R \geqslant c^* | I_A = C, n\} = 1 - \int_0^{\frac{c^{*2} - c_p M\omega}{2c^*}} f_Y(y)dy. \tag{14}$$

If $\widehat{I}_A > c_0$ or p-value < θ, we reject the null hypothesis, and conclude that the profitability is better than requirement with significance level θ in non-fuzzy statistics. For the imprecise data, the testing problem must extend the three-decision by Filzmoser & Viertl (2004) & Neyman & Pearson (1933), that is: (a) accept H_0 and reject H_1, (b) reject H_0 and accept H_1, (c) both H_0 and H_1 are neither accepted nor rejected. The final decision will become to depend on the relationship between fuzzy set $\widehat{I}_A$ and c_0.

5.1. Decision-making by critical value

In this subsection, we use the finite interval $\left[L_{\widehat{\tilde{I}}_A}(\alpha), U_{\widehat{\tilde{I}}_A}(\alpha)\right]$ to test with critical value c_0, and make the decision according to the three-decision testing rule:

(a) IF $L_{\widehat{\tilde{I}}_A}(\alpha) > c_0$ THEN reject H_0 and accept H_1;

(b) IF $U_{\widehat{\tilde{I}}_A}(\alpha) < c_0$ THEN accept H_0 and reject H_1;

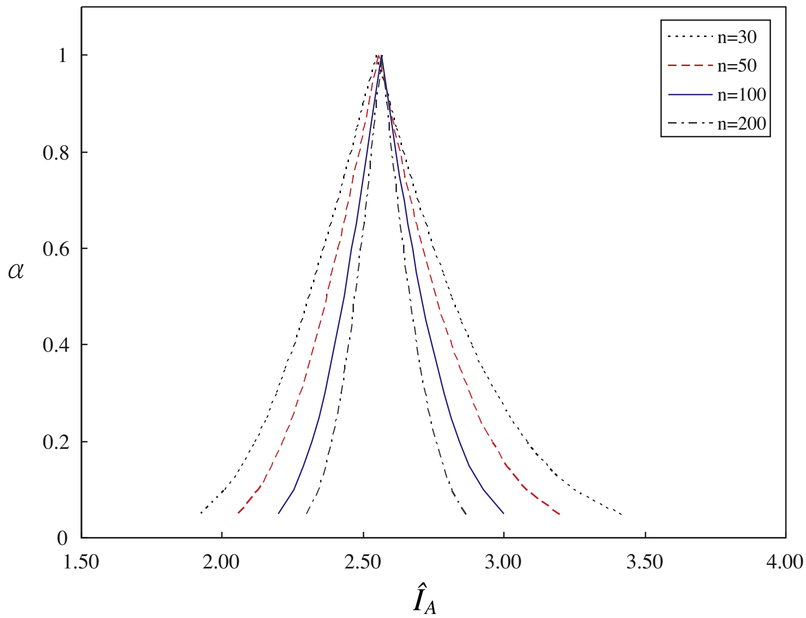

Fig. 4. The membership functions of fuzzy estimation for $\widehat{I}_A$ with n = 30, 50, 100, 200.

(c) IF $L_{\tilde{\hat{I}}_A}(\alpha) \leqslant c_0 \leqslant U_{\tilde{\hat{I}}_A}(\alpha)$ THEN both H_0 and H_1 are neither accepted nor rejected.

(Note: IF $L_{\tilde{\hat{I}}_A}(\alpha) = U_{\tilde{\hat{I}}_A}(\alpha)$ THEN the hypothesis testing is reduced to crisp data with a binary testing rule.)

A simple step-by-step procedure for judging whether the profitability meets the designated requirement based on critical value, is summarized as follows:

Step 1: Determine the designated requirement C, the target profit k, the sample size n, the significance level θ and the user-approved degree of imprecision α on sample data.
Step 2: Calculate the α-cuts of fuzzy numbers $\tilde{\bar{x}}$ and $\tilde{s}^2$ by Eqs. (10) and (11).
Step 3: Calculate the α-cuts of fuzzy number $\widetilde{\widehat{I}}_A$, $\widetilde{\widehat{I}}_A^{[\alpha]} = \left[L_{\tilde{\hat{I}}_A}(\alpha), U_{\tilde{\hat{I}}_A}(\alpha)\right]$ by Eq. (12).
Step 4: Find the critical value c_0 from Eq. (13).
Step 5: Conclude that the profitability is better than requirement $I_A > C$ if $L_{\tilde{\hat{I}}_A}(\alpha) > c_0$. Conclude that the profitability is lower than requirement $I_A \leqslant C$ if $U_{\tilde{\hat{I}}_A}(\alpha) < c_0$. Conclude that the profitability is not proven to be better or lower than requirement, and that further study is needed, if $L_{\tilde{\hat{I}}_A}(\alpha) \leqslant c_0 \leqslant U_{\tilde{\hat{I}}_A}(\alpha)$.

For example, if the $\widehat{I}_A = 2.5705$ ($\bar{x} = 25$, $s^2 = 4$, $k = 200$ and $n = 100$), C = 2.0, θ = 0.05, α = 0.7, we obtain $L_{\tilde{\hat{I}}_A}(0.7) = 2.4871 > c_0 = 2.1966$. One can conclude that the profitability is better than designated requirement, $I_A > 2.0$. If the designated requirement C = 2.5, we obtain $U_{\tilde{\hat{I}}_A}(0.7) = 2.6442 < c_0 = 2.7713$. One can conclude that the profitability is lower than designated requirement, $I_A < 2.5$. Fig. 5 displays the membership function $\eta_{\tilde{\hat{I}}_A}(\widehat{I}_A)$. The α-cuts $\widetilde{\widehat{I}}_A^{[\alpha=0.7]}$ is also exhibited in the Fig. 5. Fig. 6 depicts the PDF plots of $\widehat{I}_A$ for $I_A = C$ = 2.0, 2.5 and n = 100 associated with the critical values c_0 with the significance level θ = 0.05.

5.2. Decision-making by fuzzy p-value

The p-value is also widely used for making decisions in hypothesis testing. Therefore, we employ the finite interval

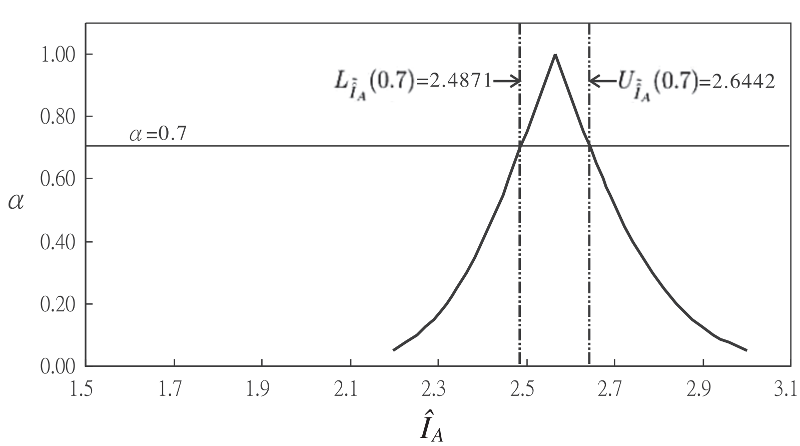

Fig. 5. The membership function $\eta_{\tilde{\hat{I}}_A}\left(\hat{I}_A\right)$ and the α-cuts $\tilde{\hat{I}}_A^{[\alpha=0.7]}$.

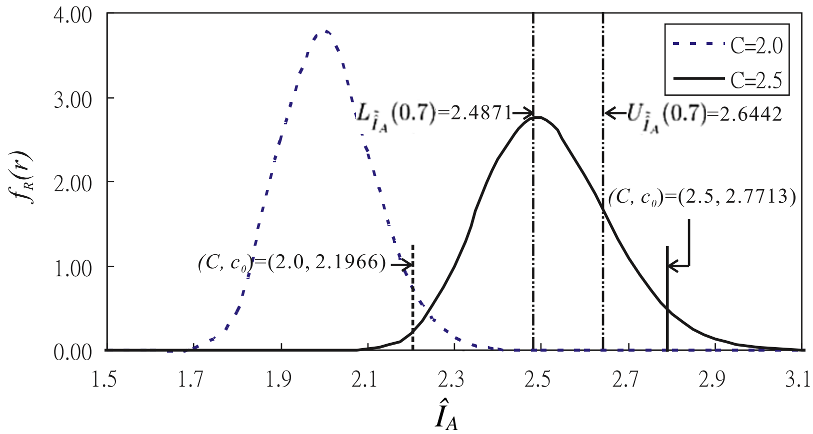

Fig. 6. The PDF plots of $\widehat{I}_A$ for $I_A = C = 2.0, 2.5$, $n = 100$ associated with the critical values c_0 with $\theta = 0.05$.

$\left[L_{\widetilde{\widehat{I}_A}}(\alpha), U_{\widetilde{\widehat{I}_A}}(\alpha)\right]$ for defining the corresponding interval of fuzziness of $\tilde{p}$. The α-cuts of $\tilde{p}$ can be represented as follows:

$$\tilde{p}^{[\alpha]} = [L_{\tilde{p}}(\alpha), U_{\tilde{p}}(\alpha)] = \left[\Pr\left\{R \geqslant U_{\widetilde{\widehat{I}_A}}(\alpha)|I_A = C, n\right\}, \Pr\left\{R \geqslant L_{\widetilde{\widehat{I}_A}}(\alpha)|I_A = C, n\right\}\right] \quad \text{for } \forall\alpha \in (0,1]. \tag{15}$$

Then the decision is made according to the three-decision testing rule:

(a) IF $U_{\tilde{p}}(\alpha) < \theta$ THEN reject H_0 and accept H_1;
(b) IF $L_{\tilde{p}}(\alpha) > \theta$ THEN accept H_0 and reject H_1;
(c) IF $L_{\tilde{p}}(\alpha) \leqslant \theta \leqslant U_{\tilde{p}}(\alpha)$ THEN both H_0 and H_1 are neither accepted nor rejected.

(Note: IF $L_{\tilde{p}}(\alpha) = U_{\tilde{p}}(\alpha)$ THEN the hypothesis testing is reduced to crisp data with a binary testing rule.)

A simple step-by-step procedure for judging whether the profitability meets the designated requirement based on p-value, is summarized as follows:

Step 1: Determine the designated requirement C, the target profit k, the sample size n, the significance level θ and the user-approved degree of imprecision α on sample data.
Step 2: Calculate the α-cuts of fuzzy numbers $\tilde{\bar{x}}$ and $\tilde{s}^2$ by Eqs. (10) and (11).
Step 3: Calculate the α-cuts of fuzzy number $\widetilde{\widehat{I}}_A$, $\widetilde{\widehat{I}}_A^{[\alpha]} = \left[L_{\widetilde{\widehat{I}_A}}(\alpha), U_{\widetilde{\widehat{I}_A}}(\alpha)\right]$ by Eq. (12).
Step 4: Calculate the α-cuts of fuzzy number $\tilde{p}$, $\tilde{p}^{[\alpha]} = [L_{\tilde{p}}(\alpha), U_{\tilde{p}}(\alpha)]$ by Eq. (15).
Step 5: Conclude that the profitability is better than requirement $I_A > C$ if $U_{\tilde{p}}(\alpha) < \theta$. Conclude that the profitability is lower than requirement $I_A \leqslant C$ if $L_{\tilde{p}}(\alpha) > \theta$. Conclude that the profitability is not proven to be better or lower than requirement, and that further study is needed, if $L_{\tilde{p}}(\alpha) \leqslant \theta \leqslant U_{\tilde{p}}(\alpha)$.

For the above numerical example, if the designated requirement $C = 2.0$, we obtain $U_{\tilde{p}}(0.7) = 1.6387 \times 10^{-4} < \theta = 0.05$. One can conclude that the profitability is better than designated requirement, $I_A > 2.0$. If $C = 2.5$, we obtain $L_{\tilde{p}}(0.7) = 0.1836 > \theta = 0.05$. One can conclude that the profitability is lower than designated requirement, $I_A < 2.5$.

6. Numerical example

This section considers the following case taken from a publisher selling certain weekly magazine with stocks and investment. The kind of this magazine has the following features:

(1) The magazine cannot be sold in the next week;
(2) The net profit of this magazine is \$10 per unit;
(3) The unsold quantity need the transportation cost to dispose it, c_e = \$5 per unit;
(4) The unsatisfied demand will lost the opportunity cost, c_s = \$3 per unit.

Suppose that the target profit and the designated requirement for this magazine are set to $k = 200$ and $C = 2.5$ by the managers, respectively. To test the profitability meets the designated requirement, one must determine whether the profitability meets $I_A > 2.5$, which is equivalent to having the maximum value of the probability of achieving the target profit larger than 0.9752. The historical date of magazine demand volume per week with sample size $n = 100$ has collected. One first uses the Kolmogorov–Smirnov test for the historical data to confirm if the data is normally distributed. The test result in p-value > 0.05, which means that data is normally distributed.

Since the data given by the retailer has some degrees of imprecision, managers suggest fuzzy inference to assess the profitability with imprecise data. The sample mean, sample standard deviation and sample estimator are calculated as $\overline{X} = 26.0316$, $s = 2.0311$ and $\widehat{I}_A = 2.9216$, respectively. If the user-approved degree of imprecise on the sample data and significance level are set as $\alpha = 0.8$ and $\theta = 0.05$, respectively. We execute the computer software (Mathematica 4.0) with $n = 100$ and $I_A = C = 2.5$ to calculate $\widetilde{\widehat{I}}_A^{[\alpha=0.8]} = \left[L_{\widetilde{\widehat{I}_A}}(0.8), U_{\widetilde{\widehat{I}_A}}(0.8)\right] = [2.8549, 2.9744]$ and $c_0 = 2.7713$. Based on the three-decision testing rule, H_0 is rejected at the significance level $\theta = 0.05$ since $L_{\widetilde{\widehat{I}_A}}(0.8) = 2.8549 > c_0 = 2.7713$. One can conclude that the profitability is higher than designated requirement $I_A > 2.5$. On the other hand, the α-cuts of p-value also leads to the same conclusion as above since $U_{\tilde{p}}(0.8) = 0.0180 < \theta = 0.05$.

7. Conclusion

In this paper, we proposed a method to calculate the I_A index when the precise demand quantity cannot be identified. The fuzzy set theory was applied to tackle this problem. It is important for practical decision-making based on statistical hypothesis testing.

In this case, we described the three-decision testing rule and provided a step-by-step procedure to assess the profitability by two fuzzy inference criteria, the critical value and the fuzzy *p*-value. Using fuzzy inference to assess the profitability with imprecise demand quantity under non-normality would be worthy of further investigation.

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