Expert Systems with Applications 38 (2011) 2302–2310

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Generalized synchronization of chaotic systems with different orders by fuzzy logic constant controller

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ARTICLE INFO

Keywords:
Generalized synchronization
Different orders
Fuzzy logic constant controller
FLCC

ABSTRACT

The fuzzy logic constant controller (FLCC) is introduced in this paper. Unlike traditional method, a simplest controller is proposed via fuzzy logic design and Lyapunov direct method. Controllers in traditional method by Lyapunov direct method are always complicated or the functions of errors. We propose a new idea to design constant numbers as controllers, while the constant numbers are decided by the upper bound and the lower bound of the error derivatives. Via fuzzy logic rules, the strength of controllers in our new approach can be adjusted according to the error derivatives. Consequently, the slave system becomes exactly and efficiently synchronized to the trajectory of master system through FLCC. Two examples, Lorenz system and four order Chen–Lee system, are presented to illustrate the effectiveness of the new controllers in chaos generalized synchronization.

1. Introduction

Since Pecora and Carroll (1990) proposed the concept of chaotic synchronization, chaos synchronization has become a hot subject in the field of nonlinear science due to its wide-scope potential application in various disciplines. The past two decades has witnessed significant progress on chaotic synchronization in secure communication, life science and information engineering. Typical application of synchronization techniques are in the remote control of nuclear systems and control of distributed power systems. Chaotic synchronization has been investigated extensively. Many kinds of synchronization phenomena and methods have been found in variety of chaotic systems, such as generalized synchronization (Chen, 2009; Chen, Chang, Yan, & Liao, 2008), phase synchronization (Erjaee & Momani, 2008; Li, Chen, & Huang, 2008), lag synchronization (Chen, Chen, & Gu, 2007; Ge & Lin, 2007), inverse synchronization (Chang, Li, & Lin, 2009; Li, 2009), partially synchronization (Chen & Chen, 2009; Wu & Chen, 2009), projective synchronization (Chen, 2005; Hu, Yang, Xu, & Guo, 2008), Q–S synchronization (Hu & Xu, 2008; Wang & Chen, 2006), etc.

In recent years, some chaos synchronizations based on fuzzy systems have been proposed since the fuzzy set theory was initiated by Zadeh (1988), such as fuzzy sliding mode controlling technique (Bagheri & Moghaddam, 2009; Chen, Chen, & Chiang, 2009; Hung & Chung, 2007; Hung, Lin, & Chung, 2007), LMI-based synchronization (Wang, Guan, & Wang, 2003) and extended backstepping sliding mode controlling technique (Li & Khajepour, 2005). The fuzzy logic control (FLC) scheme have been widely developed for almost 40 years and have been successfully applied to many applications (Li, Kuo, & Guo, 2007). Recently, Yau and Shieh (2008) proposed an amazing new idea in designing fuzzy logic controllers – constructing fuzzy rules subject to a common Lyapunov function such that the master–slave chaos systems satisfy stability in the Lyapunov sense. In Yau and Shieh (2008), there are two main controllers in their slave system. One is used in elimination of nonlinear terms and the other is built by fuzzy rules subject to a common Lyapunov function. Therefore, the resulting controllers are nonlinear form. In Yau and Shieh (2008), the regular form is necessary. In order to carry out the new method, the original system must to be transformed into their regular form.

In this paper, we propose a new strategy which is also constructing fuzzy rules subject to a Lyapunov direct method. Error derivatives are used to be upper bound and lower bound. Through this new approach, a simplest controller, i.e. constant controller, can be obtained and the difficulty in realization of complicated controllers in chaos synchronization by Lyapunov direct method can be also coped. Unlike conventional approaches, the resulting control law has less maximum magnitude of the instantaneous control command and it can reduce the actuator saturation phenomenon in real physic system.

The layout of the rest of the paper is as follows. In Section 2, generalized synchronization by fuzzy logic constant controller (FLCC) scheme is presented. In Section 3, simulation results are shown. In Section 4 conclusions are given.

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doi:10.1016/j.eswa.2010.08.018

2. Generalized synchronization by FLCC scheme

2.1. Generalized synchronization scheme

There are two nonlinear dynamical systems, while the master system controls the slave system. The master system is given by

$$\dot{x} = Ax + f(x) \tag{2-1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes a state vector, A is an $n \times n$ constant coefficient matrix and f is a nonlinear vector function.

The slave system is given by

$$\dot{y} = By + g(y) + u \tag{2-2}$$

where $y = [y_1, y_2, \ldots, y_n]^T \in R^n$ denotes a state vector, B is an $n \times n$ constant coefficient matrix, g is a nonlinear vector function, and $u = [u_1, u_2, \ldots, u_n]^T \in R^n$ is a constant control input vector.

Our goal is to design appropriate fuzzy rules and corresponding constant controllers u so that the state vector of the chaotic system (2-1) asymptotically approaches the state vector of the master system (2-2).

The generalized chaos synchronization can be accomplished in the sense that the limit of the error vector $e(t) = [e_1, e_2, \ldots, e_n]^T$ approaches zero:

$$\lim_{t\to\infty} e = 0 \tag{2-3}$$

where

$$e = H(x) - y \tag{2-4}$$

where $H(x)$ is a given vector function of x. From Eq. (2-4) we have

$$\dot{e} = \frac{\partial H(x)}{\partial x}\dot{x} - \dot{y} \tag{2-5}$$

$$\dot{e} = \frac{\partial H(x)}{\partial x}[Ax + f(x)] - Ay - f(y) - u \tag{2-6}$$

A Lyapnuov function $V(e)$ is chosen as a positive definite function

$$V(e) = \frac{1}{2}e^T e \tag{2-7}$$

Its derivative along any solution of the differential equation system consisting of Eq. (2-6) is

$$\dot{V}(e) = e^T \times \left(\frac{\partial H(x)}{\partial x} \times [Ax + f(x)] - Ay - f(y) - u\right) \tag{2-8}$$

If fuzzy constant controllers u can be appropriately chosen so that $\dot{V} = Ce^T e$, C is a diagonal negative definite matrix, and $\dot{V}$ is a negative definite function of e. By Lyapunov theorem of asymptotical stability:

$$\lim_{t\to\infty} e = 0 \tag{2-9}$$

The generalized synchronization is obtained. The design process of FLCC is introduced in the following section.

2.2. Fuzzy logic constant controller design process

The basic configuration of the fuzzy logic system is shown in Fig. 1. It is composed of five function blocks (Shieh, 2003):

1. A rule base contains a number of fuzzy if-then rules.
2. A database defines the membership functions of the fuzzy sets used in fuzzy rules.
3. A decision-making unit performs the inference operations on the rules.

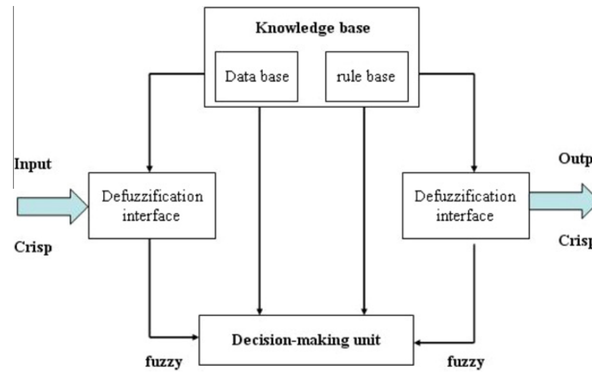

Fig. 1. The configuration of fuzzy logic controller.

4. A fuzzification interface transforms the crisp inputs into degrees of match with linguistic value.
5. A defuzzification interface transforms the fuzzy results of the inference into a crisp output.

The fuzzy rules base consists of collection of fuzzy if-then rules expressed as the form if a is A then b is B, where a and b denote linguistic variables, A and B represent linguistic values which are characterized by membership functions. All of the fuzzy rules can be used to construct the fuzzy associated memory.

We use two signals, $e(t) = [e_1, e_2, \ldots, e_m, \ldots, e_n]^T$ in Eq. (2-4) and $e(t) = [\dot{e}_1, \dot{e}_2, \ldots, \dot{e}_m, \ldots, \dot{e}_n]^T$ Eq. (2-5), as the antecedent part of the proposed FLCC to design the control input u in Eq. (2-8) that will be used in the consequent part of the proposed FLCC as follows:

$$u = [u_1, u_2, \ldots u_m, \ldots u_n]^T \tag{2-10}$$

where u is a constant column vector and the FLCC accomplishes the objective to stabilize the error dynamics (2-6). In this paper, we are not going to use the original fuzzy rule base, but using it in each error dynamics separately. In order to obtain the simplest controllers, the ith if-then rule of the fuzzy rule base of the FLCC is of the following form:

$$Rule\ i: if\ e_m\ is\ X_i\ then\ \dot{e}_m\ is\ Y_i\ and\ u_{mi} = constant \tag{2-11}$$

where X_i is the input fuzzy sets of e_m, $m = 1 \sim n$, Y_i is the output fuzzy sets of $\dot{e}_m$ and u_{mi} is the i-rd output of $\dot{e}_m$ which is a constant controller. For given input sign of the process variables e_m, then the output sign of $\dot{e}_m$ would be decided and its degree of membership μ_{X_i}, $i = 1 \sim 3$ called rule-antecedent weights are calculated. The centriod defuzzifier evaluates the output of all rules as follows:

$$u_m = \frac{\sum_{i=1}^{3}\mu_{x_i} \times u_{mi}}{\sum_{i=1}^{3}\mu_{x_i}} \tag{2-12}$$

The fuzzy rule base is listed in Table 1, in which the input variables in the antecedent part of the rules are e_m and the output variable, in the consequent part are $\dot{e}_m$ andu_{mi}.

The membership function is obtained via the method shown in Fig. 2. After designing appropriate fuzzy logic constant controllers,

Table 1
Rule-table of FLCC.

Rule	Antecedent	Consequent part 1	Consequent part 2
	e_m	$\dot{e}_m$	u_{mi}
1	Positive (P)	Negative (N)	u_{m1}
2	Negative (N)	Positive (P)	u_{m2}
3	Zero (Z)	Zero (Z)	u_{m3}

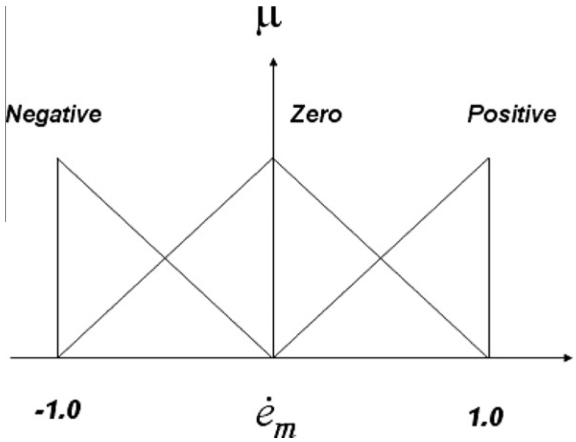

Fig. 2. Membership function.

a negative definite of $\dot{V}$ in Eq. (2-9) can be obtained and the asymptotically stability of Lyapunov theorem can be achieved.

3. Simulation results

3.1. Example 1: synchronization of master and slave Lorenz system

The master Lorenz system (Lorenz, 1963) is:

$$\begin{cases} \frac{dx_1(t)}{dt} = a(x_2(t) - x_1(t)) \\ \frac{dx_2(t)}{dt} = cx_1(t) - x_1(t)x_3(t) - x_2(t) \\ \frac{dx_3(t)}{dt} = x_1(t)x_2(t) - bx_3(t) \end{cases} \tag{3-1-1}$$

When initial condition $(x_{10},x_{20},x_{30}) = (-0.1, 0.2, 0.3)$ and parameters $a = 10$, $b = 8/3$ and $c = 28$, chaos of the Lorenz system appears. The chaotic behavior of Eq. (3-1-1) is shown in Fig. 3.

The slave Lorenz system is:

$$\begin{cases} \frac{dy_1(t)}{dt} = a(y_2(t) - y_1(t)) + u_1 \\ \frac{dy_2(t)}{dt} = cy_1(t) - y_1(t)y_3(t) - y_2(t) + u_2 \\ \frac{dy_3(t)}{dt} = y_1(t)y_2(t) - by_3(t) + u_3 \end{cases} \tag{3-1-2}$$

When initial condition $(y_{10},y_{20},y_{30}) = (0.5, 0.7, 1.5)$ and parameters are the same as that of Eq. (3-1-1), chaos of the slave Lorenz system appears as well. u_1, u_2 and u_3 are FLCC to synchronize the slave Lorenz system to master one, i.e.,

$$\lim_{t\to\infty} \mathbf{e} = 0 \tag{3-1-3}$$

where the error vector

$$[e] = \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} = \begin{bmatrix} x_1(t) - y_1(t) \\ x_2(t) - y_2(t) \\ x_3(t) - y_3(t) \end{bmatrix} \tag{3-1-4}$$

From Eq. (3-1-4), we have the following error dynamics:

$$\begin{cases} \dot{e}_1 = a(x_2 - x_1) - (a(y_2 - y_1) + u_1) \\ \dot{e}_2 = cx_1 - x_1x_3 - x_2 - ((cy_1 - y_1y_3 - y_2) + u_2) \\ \dot{e}_3 = x_1x_2 - bx_3 - ((y_1y_2 - by_3) + u_3) \end{cases} \tag{3-1-5}$$

Choosing Lyapunov function as:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{3-1-6}$$

Its time derivative is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(a(x_2 - x_1) - (a(y_2 - y_1) + u_1)) \\ &\quad + e_2(cx_1 - x_1x_3 - x_2 - ((cy_1 - y_1y_3 - y_2) + u_2)) \\ &\quad + e_3(x_1x_2 - bx_3 - ((y_1y_2 - by_3) + u_3)) \end{aligned} \tag{3-1-7}$$

In order to design FLCC, we divide Eq. (3-1-7) into three parts as follows: Assume $V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) = V_1 + V_2 + V_3$, then $\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$, where $V_1 = \frac{1}{2}e_1^2$, $V_2 = \frac{1}{2}e_2^2$ and $V_3 = \frac{1}{2}e_3^2$.

$$\begin{aligned} &Part\ 1 : \dot{V}_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - (a(y_2 - y_1) + u_1)) \\ &Part\ 2 : \dot{V}_2 = e_2\dot{e}_2 = e_2(cx_1 - x_1x_3 - x_2 - ((cy_1 - y_1y_3 - y_2) + u_2) \\ &Part\ 3 : \dot{V}_3 = e_3\dot{e}_3 = e_3(x_1x_2 - bx_3 - ((y_1y_2 - by_3) + u_3)) \end{aligned}$$

Part 1: FLCC in *Part 1* can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_1$ (without any controller) can be observed in time history of error

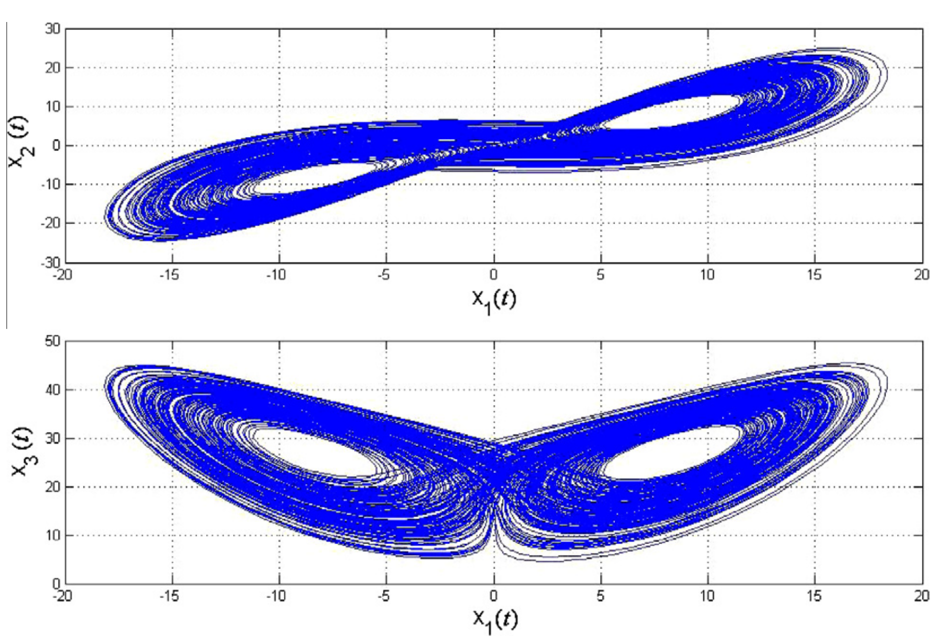

Fig. 3. Projections of phase portrait of chaotic Lorenz system with $a = 10$, $b = 8/3$ and $c = 28$.

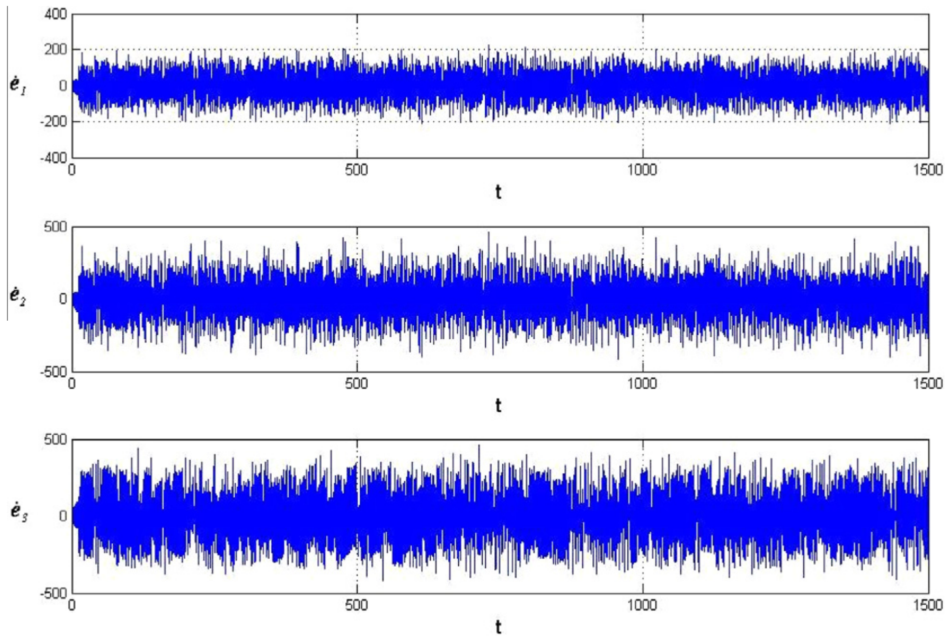

Fig. 4. Time histories of error derivatives for master and slave Lorenz chaotic systems without controllers.

derivatives drawn in Fig. 4. We choose f_1 to be the upper bound value and g_1 to be the lower bound value of $\dot{e}_1$(without any controller), they are satisfied with $f_1 < \dot{e}_1$ (without any controller) $< g_1$ and f_1, g_1 are all constants.

Rule 1: *if* e_1 *is P, then* $\dot{e}_1$ *is N and we take* $u_{11} = f_1$

Rule 2: *if* e_1 *is N, then* $\dot{e}_1$ *is P and we take* $u_{12} = g_1$

Rule 3: *if* e_1 *is Z, then* $\dot{e}_1$ *is Z and we take* $u_{13} = 0 = e_1$

where $f_1 = -g_1 =$ *constant* = 400 and we choose $u_{13} = 0 = e_1$ when e_1 approaches to zero. We take *Rule* $1 \sim 3$ in Part 1, $\dot{V}_1 = e_1\dot{e}_1$, for explaining:

$$Rule\ 1: if\ e_1\ is\ P,\ then\ \dot{e}_1\ is\ N\ and\ we\ take\ u_{11} = f_{1:}$$

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - a(y_2 - y_1) - f_1)$$

where $e_1 > 0$ and $(a(x_2 - x_1) - a(y_2 - y_1) - f_1) = (\dot{e}_1($*without controller*$) - f_1) < 0$. Therefore, $\dot{V}_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - a(y_2 - y_1) - f_1) < 0$ and is going to approach asymptotically stable.

$$Rule\ 2: if\ e_1\ is\ N,\ then\ \dot{e}_1\ is\ P\ and\ we\ take\ u_{12} = g_1$$

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - a(y_2 - y_1) - g_1)$$

where $e_1 < 0$ and $(a(x_2 - x_1) - a(y_2 - y_1) - g_1) = (\dot{e}_1($*without controller*$) - g_1) > 0$. Therefore, $\dot{V}_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - a(y_2 - y_1) - g_1) < 0$ and is going to approach asymptotically stable.

$$Rule\ 3: if\ e_1\ is\ Z,\ then\ \dot{e}_1\ is\ Z\ and\ we\ take\ u_{13} = 0 = e_1$$

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(a(x_2 - x_1) - a(y_2 - y_1) - e_1)$$

where $e_1 = 0$ and we donot need any controller now. Therefore, $\dot{V}_1 = e_1\dot{e}_1 = 0$ and achieve asymptotically stable. As a results, FLCC in *Part 1* can be obtained from *Rule 1, 2 and 3*:

$$u_1 = \frac{\mu_P \times u_{11} + \mu_N \times u_{12} + \mu_Z \times u_{13}}{\mu_P + \mu_N + \mu_Z} \tag{3-1-8}$$

Part 2: FLCC in *Part 2* can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_2$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 4. We choose f_2 to be the upper bound value and g_2 to be the lower bound value of $\dot{e}_2$ (without any controller), they are satisfied with $f_2 < \dot{e}_2$ (without any controller) $< g_2$ and f_2, g_2 are all constants.

Rule 1: *if* e_1 *is P, then* $\dot{e}_1$ *is N and we take* $u_{11} = f$

Rule 1: *if* e_2 *is P, then* $\dot{e}_2$ *is N and* $u_{21} = f_2$

Rule 2: *if* e_2 *is N, then* $\dot{e}_2$ *is P and* $u_{22} = g_2$

Rule 3: *if* e_2 *is Z, then* $\dot{e}_2$ *is Z and* $u_{23} = 0 = e$

where $f_2 = -g_2 =$ *constant* = 500 and we choose $u_{23} = 0 = e_2$ when e_2 approaches to zero. The process of FLCC designing is the same as *Part 1*, as a results, FLCC in *Part 2* can be obtained from *Rule 1, 2 and 3* and are going to take $\dot{V}_2 = e_2\dot{e}_2 < 0$:

$$u_2 = \frac{\mu_P \times u_{21} + \mu_N \times u_{22} + \mu_Z \times u_{23}}{\mu_P + \mu_N + \mu_Z} \tag{3-1-9}$$

Part 3: FLCC in *Part 3* can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_3$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 4. We choose f_3 to be the upper bound value and g_3 to be the lower bound value of $\dot{e}_3$ (without any controller), they are satisfied with $f_3 < \dot{e}_3$ (without any controller) $< g_3$ and f_3, g_3 are all constants.

Rule 1: *if* e_3 *is P, then* $\dot{e}_3$ *is N and* $u_{31} = f_3$

Rule 2: *if* e_3 *is N, then* $\dot{e}_3$ *is P and* $u_{32} = g_3$

Rule 3: *if* e_3 *is Z, then* $\dot{e}_3$ *is Z and* $u_{33} = 0 = e_3$

where $f_3 = -g_3 =$ *constant* = 500 and we choose $u_{33} = 0 = e_3$ when e_3 approaches to zero. The process of FLCC designing is the same as *Part 1*, as a results, FLCC in *Part 3* can be obtained from *Rule 1, 2 and 3* and are going to take $\dot{V}_3 = e_3\dot{e}_3 < 0$:

$$u_3 = \frac{\mu_P \times u_{31} + \mu_N \times u_{32} + \mu_Z \times u_{33}}{\mu_P + \mu_N + \mu_Z} \tag{3-1-10}$$

FLCC are proposed in *Part 1, 2* and *3* and are going to take $\dot{V}_1 = e_1\dot{e}_1 < 0$, $\dot{V}_2 = e_2\dot{e}_2 < 0$ and $\dot{V}_3 = e_3\dot{e}_3 < 0$. Hence, we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 < 0$. It is clear that all of the rules in our FLC can lead the Lyapunov function to approach asymptotically stable and the simulation results are shown in Figs. 5 and 6.

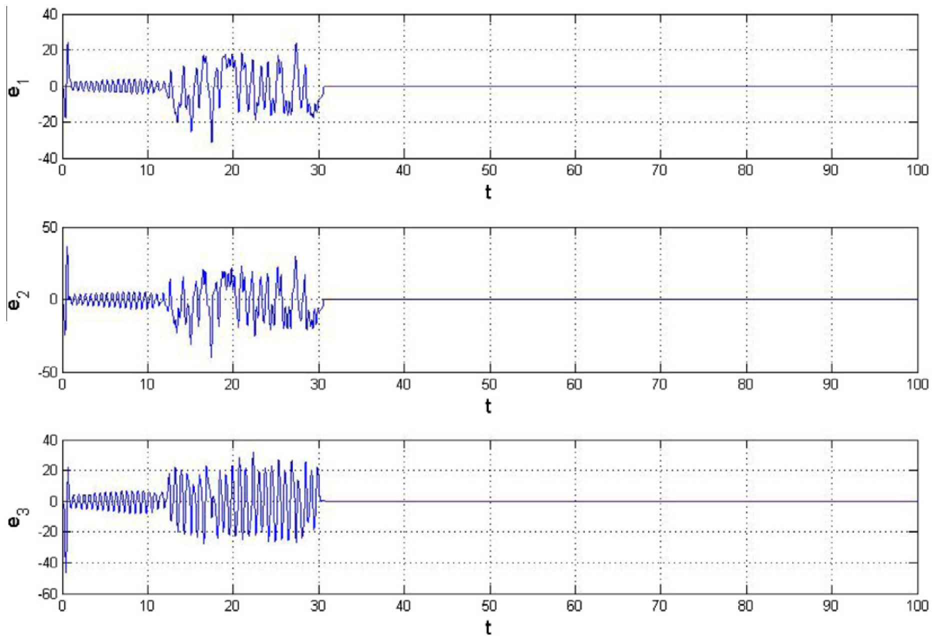

Fig. 5. Time histories of errors for Example 1- the FLCC is coming into after 30s.

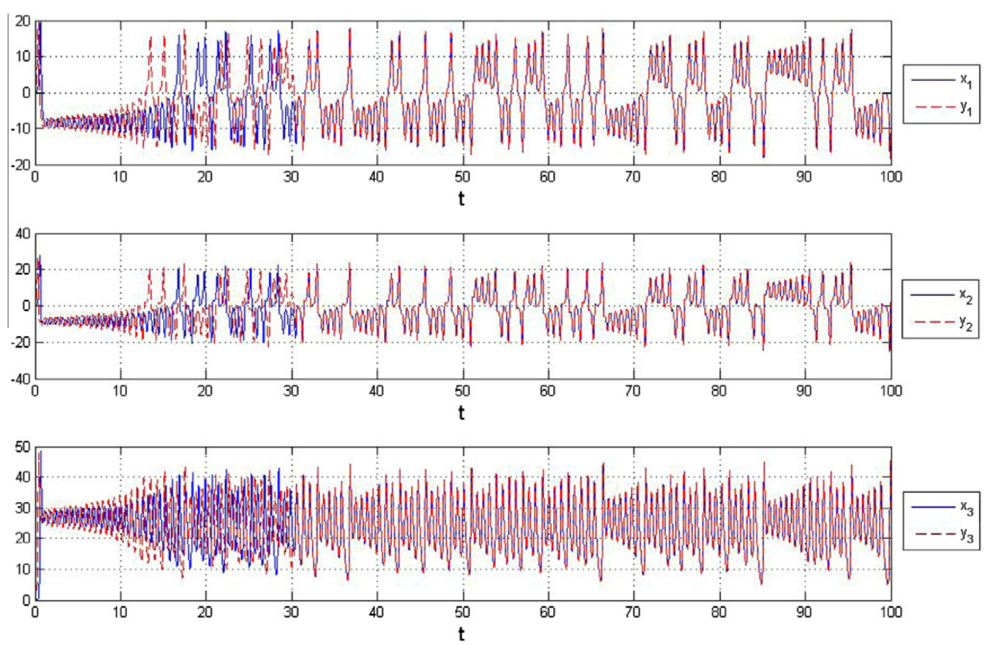

Fig. 6. Time histories of states for Example 1- the FLCC is coming into after 30s.

3.2. Example 2: generalized synchronization of different order chaotic system- Lorenz and New Chen–Lee system

Chen & Lee (2004) gave a new chaotic system, which is now called the Chen–Lee system (Tam & Tou, 2008). The system is described by the following nonlinear differential equations and is denoted as system (1):

$$\begin{cases} \frac{dz_1(t)}{dt} = -z_2(t)z_3(t) + a_1 z_1(t) \\ \frac{dz_2(t)}{dt} = z_1(t)z_3(t) + b_1 z_2(t) \\ \frac{dz_3(t)}{dt} = \frac{1}{3} z_1(t)z_2(t) + c z_3(t) \end{cases} \tag{3-2-1}$$

where z_1, z_2 and z_3 are state variables, and a_1, b_1 and c_1 are three system parameters. When $(a_1, b_1, c_1) = (5, -10, -3.8)$, system (3-2-1) is a chaotic attractor. The positive Lyapunov exponent of this attractor is $\lambda_1 = 0.88$, while the other ones are $\lambda_2 = 0$ and $\lambda_3 = -13.57$, respectively. It is clear that the Chen–Lee system is a regular chaotic system. For more-detailed dynamics of the Chen–Lee system, see Chen & Lee (2004).

It is known that in order to obtain hyper-chaos, there are two important requisites: (1) the minimal dimension of the phase space that embeds a hyper-chaotic attractor should be at least four, which requires a minimum of four couple first-order autonomous ordinary differential equations; and (2) the number of terms in the couple equations giving rise to instability should be at least two, of which at least one should be a nonlinear function. In (Chen et al., 2009), Chen and Lee introduce a nonlinear feedback controller to the third equation of system (3-2-1), the following dynamic system can be obtained:

$$\begin{cases} \frac{dz_1(t)}{dt} = -z_2(t)z_3(t) + a_1 z_1(t) \\ \frac{dz_2(t)}{dt} = z_1(t)z_3(t) + b_1 z_2(t) \\ \frac{dz_3(t)}{dt} = \frac{1}{3}z_1(t)z_2(t) + c_1 z_3(t) + \frac{1}{5}z_4(t) \\ \frac{dz_4(t)}{dt} = d_1 z_1(t) + \frac{1}{2}z_2(t)z_3(t) + \frac{1}{20}z_4(t) \end{cases} \tag{3-2-2}$$

where d is a constant, determining the dynamic behaviors of the system (3-2-2) and a_1, b_1, and c_1 are three system parameters. Thus, controller z_4 causes chaotic system (3-2-1) to become a four-dimensional system, which has four Lyapunov exponents. This may lead to a hyper-chaotic system. When $(a_1,b_1,c_1) = (5,-10,-3.8)$ and we choose $d = 1.3$, system (3-2-2) is a hyper-chaotic attractor. The projection of phase portraits of system (3-2-2) with hyper-chaotic behaviors is shown in Fig. 7.

Eq. (3-1-2) is chosen as slave system to be synchronized with the master system (3-2-2). Our goal is $[e] = [e_1(t), e_2(t), e_3(t)] = [z_1(t)-y_1(t), z_3(t)-y_2(t), z_4(t)-y_3(t)]$. As a result, we get the following error dynamics:

$$\begin{cases} \dot{e}_1 = -z_2 z_3 + a_1 z_1 - (a(y_2 - y_1) + u_1) \\ \dot{e}_2 = \frac{1}{3}z_1 z_2 + c_1 z_3 - ((cy_1 - y_1 y_3 - y_2) + u_2) \\ \dot{e}_3 = d_1 z_1 + \frac{1}{2}z_2 z_3 + \frac{1}{20}z_4 - ((y_1 y_2 - by_3) + u_3) \end{cases} \tag{3-2-3}$$

Choosing Lyapunov function as:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{3-2-4}$$

Its time derivative is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(-z_2 z_3 + a_1 z_1 - (a(y_2 - y_1) + u_1)) \\ &\quad + e_2\left(\frac{1}{3}z_1 z_2 + c_1 z_3 - ((cy_1 - y_1 y_3 - y_2) + u_2)\right) \\ &\quad + e_3\left(d_1 z_1 + \frac{1}{2}z_2 z_3 + \frac{1}{20}z_4 - ((y_1 y_2 - by_3) + u_3)\right) \end{aligned} \tag{3-2-5}$$

We divide Eq. (3-2-5) into three parts as follows:

Assume $V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) = V_1 + V_2 + V_3$, then $\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 = \dot{V}_1 + \dot{V}_2 + \dot{V}_3$, where $V_1 = \frac{1}{2}e_1^2$, $V_2 = \frac{1}{2}e_2^2$ and $V_3 = \frac{1}{2}e_3^2$.

Part 1 : $\dot{V}_1 = e_1\dot{e}_1 = e_1(-z_2 z_3 + a_1 z_1 - (a(y_2 - y_1) + u_1))$
Part 2 : $\dot{V}_2 = e_2\dot{e}_2 = e_2(\frac{1}{3}z_1 z_2 + c_1 z_3 - ((cy_1 - y_1 y_3 - y_2) + u_2))$
Part 3 : $\dot{V}_3 = e_3\dot{e}_3 = e_3(d_1 z_1 + \frac{1}{2}z_2 z_3 + \frac{1}{20}z_4 - ((y_1 y_2 - by_3) + u_3))$

Part 1: FLCC in *Part 1* can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_1$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 8. We choose f_4 to be the upper bound value and g_4 to be the lower bound value of $\dot{e}_1$ (without any controller), they are satisfied with $f_4 < \dot{e}_1$ (without any controller) $< g_4$ and f_4, g_4 are all constants.

Rule 1: *if* e_1 *is P, then* $\dot{e}_1$ *is N and we take* $u_{11} = f_4$
Rule 2: *if* e_1 *is N, then* $\dot{e}_1$ *is P and we take* $u_{12} = g_4$
Rule 3: *if* e_1 *is Z, then* $\dot{e}_1$ *is Z and we take* $u_{13} = 0 = e_1$

where $f_4 = -g_4 = constant = 2000$ and we choose $u_{13} = 0 = e_1$ when e_1 approaches to zero. We take *Rule* 1 ∼ 3 in Part 1, $\dot{V}_1 = e_1\dot{e}_1$, for explaining:

Rule 1 : *if* e_1 *is P, then* $\dot{e}_1$ *is N and we take* $u_{11} = f_4$:

$$\dot{V}_1 = e_1\dot{e}_1 = e_1(-x_2 x_3 + a_1 x_1 - a(y_2 - y_1) - f_4)$$

where $e_1 > 0$ and $(-z_2 z_3 + a_1 z_1 - a(y_2 - y_1) - f_4) = (\dot{e}_1(\textit{without controller}) - f_4) < 0$. Therefore, $\dot{V}_1 = e_1\dot{e}_1 = e_1(-z_2 z_3 + a_1 z_1 - a(y_2 - y_1) - f_4) < 0$ and is going to approach asymptotically stable.

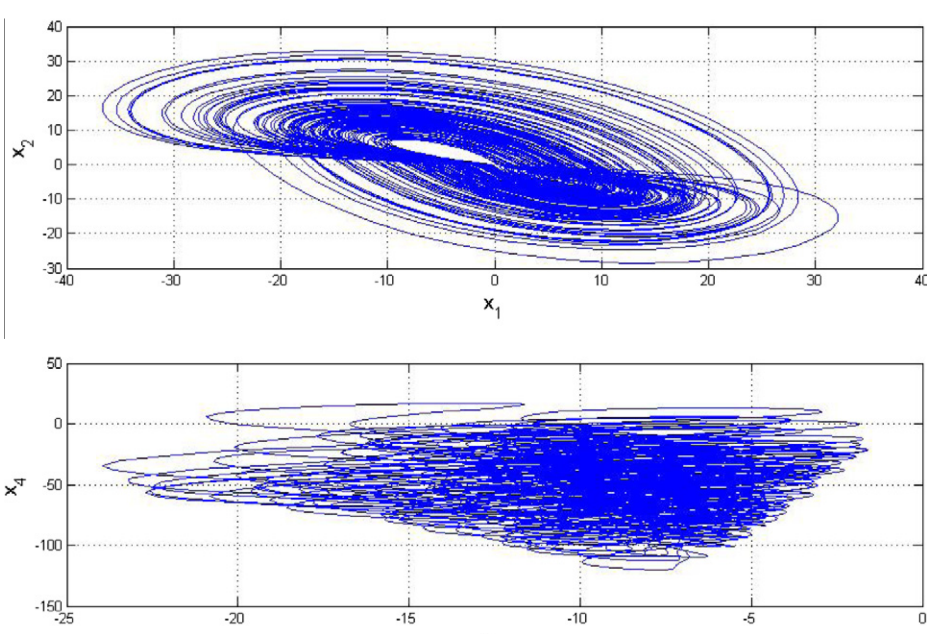

Fig. 7. Projections of phase portrait of chaotic Chen–Lee system.

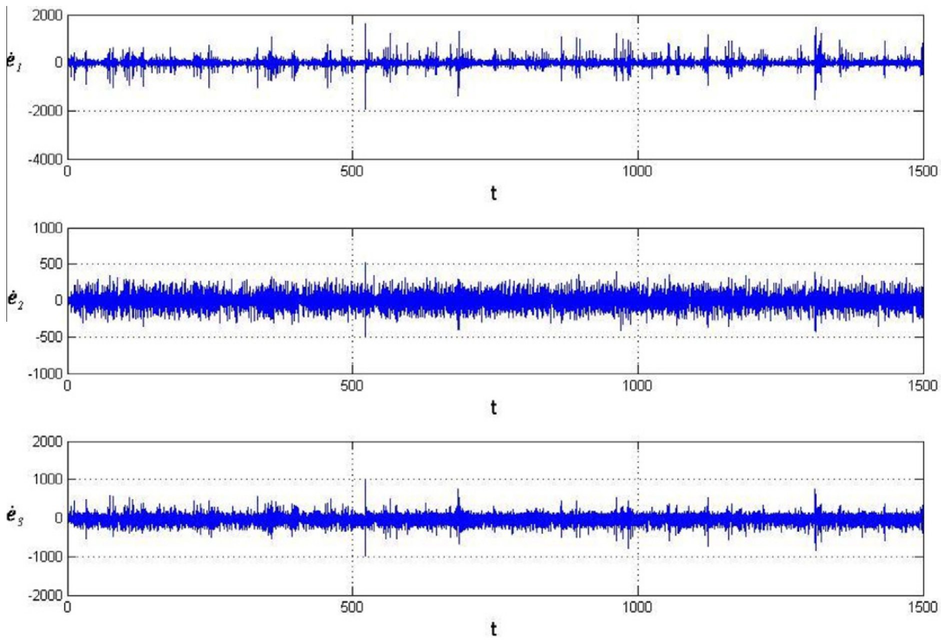

Fig. 8. Time histories of error derivatives for master and slave chaotic systems without controllers.

$$Rule\ 2: if\ e_1\ is\ N,\ then\ \dot{e}_1\ is\ P\ and\ we\ take\ u_{12} = g_4$$
$$\dot{V}_1 = e_1\dot{e}_1 = e_1(-x_2x_3 + a_1x_1 - a(y_2 - y_1) - g_4)$$

where $e_1 < 0$and $(-x_2x_3 + a_1x_1 - a(y_2 - y_1) - g_4) = (\dot{e}_1(without\ controller) - g_4) > 0$. Therefore, $\dot{V}_1 = e_1\dot{e}_1 = e_1(-x_2x_3 + a_1x_1 - a(y_2 - y_1) - g_4) < 0$ and is going to approach asymptotically stable.

$$Rule\ 3: if\ e_1 is\ Z, then\ \dot{e}_1\ is\ Z\ and\ we\ take\ u_{13} = 0 = e_1$$
$$\dot{V}_1 = e_1\dot{e}_1 = e_1(-x_2x_3 + a_1x_1 - a(y_2 - y_1) - e_1)$$

where $e_1 = 0$ and we donot need any controller now. Therefore, $\dot{V}_1 = e_1\dot{e}_1 = 0$ and achieve asymptotically stable. As a results, FLCC in *Part 1* can be obtained from *Rule 1, 2 and 3*:

$$u_1 = \frac{\mu_P \times u_{11} + \mu_N \times u_{12} + \mu_Z \times u_{13}}{\mu_P + \mu_N + \mu_Z} \tag{3-2-6}$$

Part 2: FLCC in *Part 2* can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_2$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 8. We choose f_5 to be the upper bound value and g_5 to be the lower bound value of $\dot{e}_2$(without any controller), they are satisfied with $f_5 < \dot{e}_2$ (without any controller) $< g_5$ and f_5, g_5 are all constants.

Rule 1: *if* e_2 *is P, then* $\dot{e}_2$ *is N and* $u_{21} = f_5$
Rule 2: *if* e_2 *is N, then* $\dot{e}_2$ *is P and* $u_{22} = g_5$
Rule 3: *if* e_2 *is Z, then* $\dot{e}_2$ *is Z and* $u_{23} = 0 = e_2$

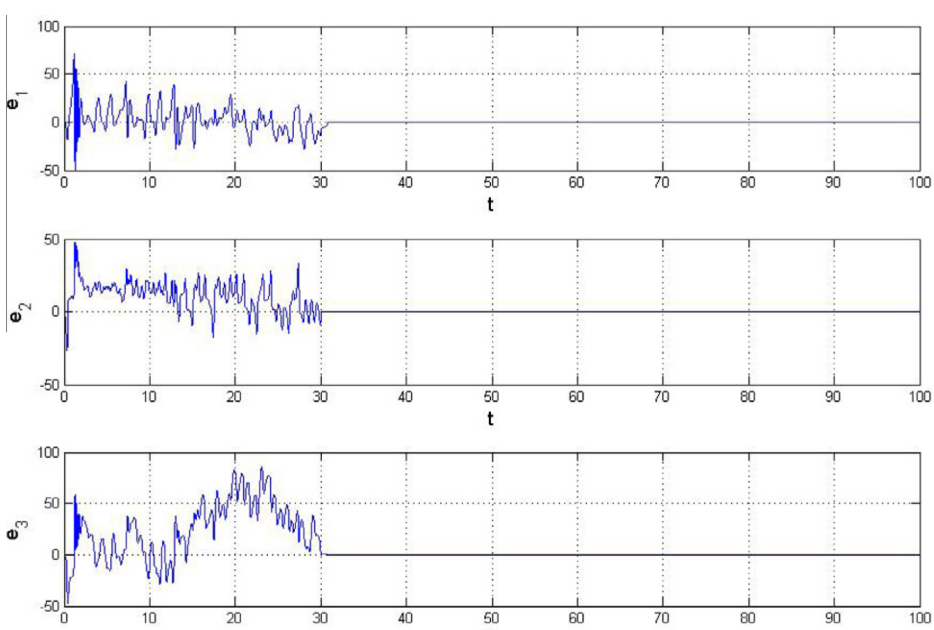

Fig. 9. Time histories of errors for Example 2- the FLCC is coming into after 30 s.

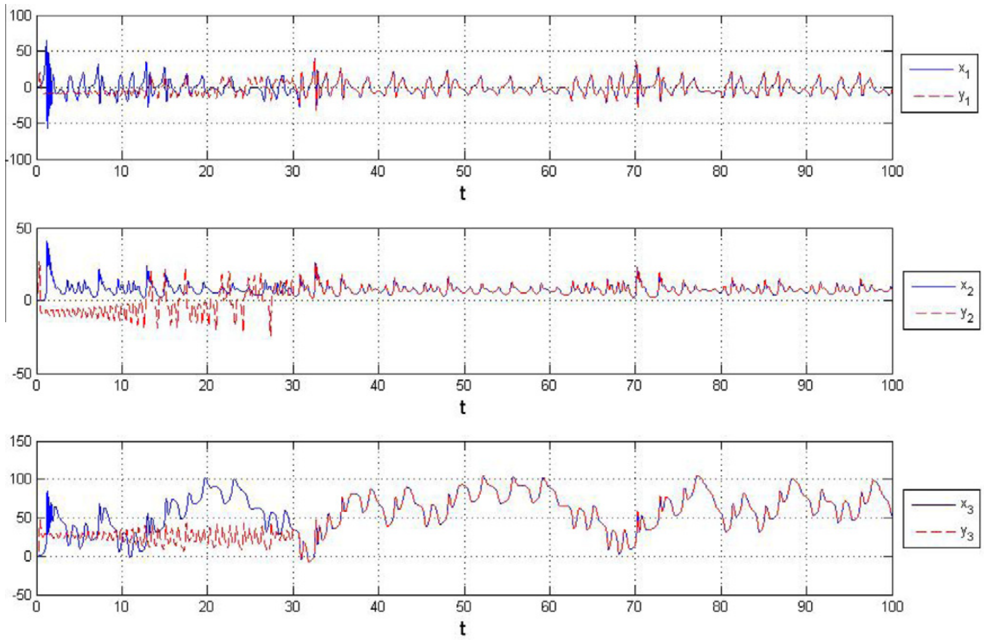

Fig. 10. Time histories of states for Example 2-the FLCC is coming into after 30 s.

where $f_5 = -g_5 = constant = 1000$ and we choose $u_{23} = 0 = e_2$ when e_2 approaches to zero. The process of FLCC designing is the same as *Part 1*, as a results, FLCC in *Part 2* can be obtained from *Rule 1, 2 and 3* and are going to take $\dot{V}_2 = e_2\dot{e}_2 < 0$:

$$u_2 = \frac{\mu_P \times u_{21} + \mu_N \times u_{22} + \mu_Z \times u_{23}}{\mu_P + \mu_N + \mu_Z} \quad (3\text{-}2\text{-}7)$$

Part 3: FLCC in *Part 3* can be obtained via the fuzzy rules in Table 1 as follows and the maxima value and minima value of $\dot{e}_3$ (without any controller) can be observed in time history of error derivatives drawn in Fig. 8. We choose f_6 to be the upper bound value and g_6 to be the lower bound value of $\dot{e}_3$(without any controller), they are satisfied with $f_6 < \dot{e}_3$ (without any controller) $< g_6$ and f_6, g_6 are all constants.

Rule 1: *if e_3 is P, then $\dot{e}_3$ is N and* $u_{31} = f_6$

Rule 2: *if e_3 is N, then $\dot{e}_3$ is P and* $u_{32} = g_6$

Rule 3: *if e_3 is Z, then $\dot{e}_3$ is Z and* $u_{33} = 0 = e_3$

where $f_3 = -g_3 = constant = 2000$ and we choose $u_{33} = 0 = e_3$ when e_3 approaches to zero. The process of FLCC designing is the same as *Part 1*, as a results, FLCC in *Part 3* can be obtained from *Rule 1, 2 and 3* and are going to take $\dot{V}_3 = e_3\dot{e}_3 < 0$:

$$u_3 = \frac{\mu_P \times u_{31} + \mu_N \times u_{32} + \mu_Z \times u_{33}}{\mu_P + \mu_N + \mu_Z} \quad (3\text{-}2\text{-}8)$$

FLCC are proposed in Eq. (3-2-6), (3-2-7) and (3-2-8) and are going to take $\dot{V}_1 = e_1\dot{e}_1 < 0$, $\dot{V}_2 = e_2\dot{e}_2 < 0$ and $\dot{V}_3 = e_3\dot{e}_3 < 0$ separately. Hence, we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 < 0$. It is clear that all of the rules in our FLC can lead the Lyapunov function to approach asymptotically stable and the simulation results are shown in Figs. 9 and 10.

4. Conclusions

In this paper, a simplest controller - fuzzy logic constant controller (FLCC) is introduced. Based on Lyapunov direct method and the upper bound and lower bound of the error derivatives, we construct the fuzzy rules and the simplest corresponding constant controllers. Complicated and nonlinear controllers would no longer appear and are replaced with simple and constant controllers through our new strategy. Simulation results in synchronization show that FLCC is effective enough and give very satisfactory results. Through this new approach, not only all cases in chaos synchronization or control can be achieved, but also the implement or experimental application of chaos synchronization could be attained much more easily.

Acknowledgment

This research was supported by the National Science Council, Republic of China, under Grant No. NSC 96-2221-E-009-145-MY3.

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