行政院國家科學委員會專題研究計畫成果報告

塊稀疏矩陣、連續最小化技巧及其在解非線性大規模多貨品網路流量問題之研究

(Two Techniques and a Method for Large Nonlinear Multicommodity Network Flow Problems)

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一、中文摘要

在這個計畫中，我們研究出了兩個技巧—塊稀疏矩陣技巧及連續最小化技巧，並將此兩個技巧融入一個結合投射半牛頓法及對應型投射半牛頓法中來解非線性大規模多貨品網路流量問題。我們很嚴謹地證明了我們的方法的收斂性，我們以定性式的方式分析我們的方法的計算效，我們也同時跑了許多數值例子並與前曾發表在 Networks 期刊上的論文中的方法比較。我們得到的結論是：在大規模的網路下，我們所提出的方法的計算速度有明顯的優越性。

關鍵詞：非線性多貨品網路流量、稀疏矩陣技巧、投射、對應型最佳化。

Abstract

We present a sparse-block-matrix technique based method for solving nonlinear multicommodity network flow problems with large commodities. Our method combines a well-known projected quasi-Newton (PQN) method and a dual projected pseudo quasi-Newton (DPPQN) method, which solves the dual problem of the quadratic subproblems induced in the PQN method. In the framework of PQN method, there is a sparse block-two-element matrix property residing in the dual quadratic subproblem, and the dual function can be formulated as a scaled projection problem. To exploit these two characteristics, we propose a sparse-block-matrix technique and an n-iteration scaled projection technique to further enhance the computational efficiency of DPPQN method, especially in the case of large commodities. We demonstrate the efficiency of the DPPQN method embedded with the two new techniques by comparing with a previously developed efficient algorithm. Test results show that the proposed method outperforms the previously developed method in the case of large commodities.

Keywords: Nonlinear multicommodit network flow, sparse matrix technique, projection, dual method

二、計畫緣由與目的

1、背景說明及計畫重要性

我們將闡述本計畫所研究的問題的沿革如下：

Nonlinear multicommodity network flow problem (NMNFP) with capacity constraints is a recurring network flow problem and has important applications in transportation networks, communication networks, multiproduct distribution systems, and routing in VLSI design. This problem is formulated when one wishes to send several different commodities from their origins to their destinations along the arcs of a capacitated network with a nonlinear convex objective function formed by the sum of all arc costs, because the commodities interact when they flow on the same limited-capacity arcs. NMNFPs has been studied extensively by operations researchers, system theoretists and engineers, computer scientists, and others because of their wide applicability.

NMNFPs are typical convex programming problems, and the NMNFP solution techniques mostly originate from nonlinear programming algorithms that are specialized to exploit the linear constraint structure with various approaches such as the feasible direction approach, linear approximation algorithms, the equilibration approach reduced gradient method, dual-relaxation method, projected Newton method, truncated quadratic programming algorithms. Recently, a computationally very efficient method that combines projected Jacobi (PJ) method with a dual projected pseudo quasi-Newton (DPPQN) method to take advantage of the special structure of the inequality constraints and network sparsity was developed in [1]. This method, abbreviated as PJDQN method, has achieved a dramatic speed-up ratio for more than several hundred times faster than a typical method for NMNFPs as shown in [1].

2、研究目的

本計畫研究此問題的目的可以簡單敘述如下：

Considering a trend that the number of commodities in networks is increasing, the purpose of this paper is to deal with NMNFPs with large commodities.

To reserve the advantages of the PJDQN method, our approach will follow its framework to exploit the special structure of inequality constraints of NMNFPs; however, we will use a projected quasi-Newton (PQN) method to replace the PJ method to improve the convergence rate. The dual problem of the quadratic subproblem induced in the PQN method possesses a sparse block-two-element matrix property, which is especially eminent in NMNFPs with large commodities and has never been exploited in any existing method. In fact, it is this property that motivates us to develop the proposed sparse-block-matrix technique to further enhance the computational efficiency of the DPPQN method to solve the dual problem in the case of large commodities. Furthermore, due to using the PQN method instead of the PJ method, a scaled projection problem rather than a projection problem [1] is induced in the DPPQN method. To deal with this scaled projection problem, we propose an n-iteration scaled projection technique [2].

Thus, the contribution of this project is we propose two new techniques which can be embedded in the DPPQN method to incorporate with a PQN method for solving NMNFPs with large commodities efficiently.

三、結果與討論

我們將我們所提出的方法以及在 Network 所發表的 PJDQN 法用四個柵欄式(grid type)的網路例子來模擬。此四個柵欄式的網路例子各別是 6×6 的 36 nodes, 120 branches, 7×7 的 49 nodes, 168 branches, 8×8 64 nodes, 224 branches 及 9×9 的 81 nodes, 288 branches 的網路,在各個網路上我們個別測試了不同貨品數量的 cases,分別是 3,6,9,12,15 及 18 個貨品數。在所有的例子中,我們的成本函數是 $\frac{1}{2}\sum_{l}\left(\sum_{k} f_l^k\right)^2$ 其中 f_l^k 這個代表流經 branch l 貨品 k 的流量。

我們在 sparc-10 workstation 上,用我們的方法及 PJDQN 法來解前面所提的 24 個例子。所得結果列於下列四個表中。

Table I. Comparison of out method with the PJDQN method for solving NMNFPs on the 6×6 network

No. of	Our method		PJDQN method		Speed-
Commo dities	Objective values	CPU times (t1)seconds	Objective values	CPU times (t2)seconds	up ratio
3	1.378	0.59	1.378	0.55	0.93
6	3.193	0.96	3.193	1.07	1.15
9	5.629	1.34	5.629	1.66	1.24
12	7.123	1.56	7.123	2.37	1.52
15	9.037	7.81	9.037	3.00	1.66
18	11.861	2.52	11.861	5.12	2.03

Table II. Comparison of our method with the PJDQN method for solving NMNFPs on the 7×7 network

No. of	Our method		PJDQN method		Speed-
Commo dities	Objective values	CPU times (t1)seconds	Objective values	CPU times (t2)seconds	up ratio
3	1.377	0.92	1.337	0.77	0.84
6	2.896	1.26	2.896	1.63	1.29
9	4.544	1.66	4.544	2.81	1.69
12	5.955	2.15	5.955	3.93	1.83
15	7.457	2.92	7.457	5.80	1.99
18	9.004	3.27	9.004	7.75	2.37

Table II. Comparison of our method with the PJDQN method for solving NMNFPs on the 8×8 network

No. of	Our method		PJDQN method		Speed-
Commo dities	Objective values	CPU times (t1)seconds	Objective values	CPU times (t2)seconds	up ratio
3	1.377	1.42	1.377	1.14	0.80
6	2.812	2.28	2.812	3.01	1.58
9	4.156	1.90	4.156	3.84	1.68
12	5.757	3.17	5.757	6.19	1.95
15	7.347	4.22	7.374	8.88	2.10
18	8.972	6.09	8.972	14.48	2.37

Table II. Comparison of our method with the PJDQN method for solving NMNFPs on the 9×9 network

No. of	Our method		PJDQN method		Speed-
Commo dities	Objective values	CPU times (t1)seconds	Objective values	CPU times (t2)seconds	up ratio
3	1.376	2.05	1.376	1.95	0.95
6	2.945	2.84	2.945	3.92	1.38
9	4.595	3.15	4.595	4.57	1.45
12	5.929	4.83	5.929	8.90	1.84
15	7.356	5.93	7.356	14.93	2.52
18	8.681	7.4	8.681	21.17	2.86

從以上四個表可以看出,當貨品數是3的時候,我們的方法甚至比 PJDWN method 來得慢。然而,當貨品數逐漸增加時,我們的方法對 PJDQN 法的 speed-up ratio 亦隨之增加。由此可見,我們所提出之方法是非常地適合解非

線性大規模多貨品網路流量問題。

四、計畫成果自評

本計畫主要目的是在提出一個解非線性大規模多貨品網路流量的方法。我們不但已成功地提出並做了許多擬模來證明它的優越性。同時我們也已將此研究成果撰寫成論文投稿到著名的 Operations Research 期刊。

五、參考文獻

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