Neural–Fuzzy Gap Control for a Current/Voltage-Controlled 1/4-Vehicle MagLev System

Shinq-Jen Wu, Cheng-Tao Wu, and Yen-Chen Chang

***Abstract*—A magnetically levitated (MagLev) vehicle prototype has independent levitation (attraction) and propulsion dynamics. We focus on the levitation behavior to obtain precise gap control of a 1/4 vehicle. An electromagnetic leviation system is highly nonlinear and naturally unstable, and its equilibrium region is severely restricted. It is therefore a tough task to achieve high-performance vehicle-levitated control. In this paper, a MagLev system is modeled by two self-organizing neural–fuzzy techniques to achieve *linear* and *affine* Takagi–Sugeno (T–S) fuzzy systems. The corresponding linear-type optimal fuzzy controllers are then used to regulate both physical systems (voltage- and current-controlled systems). On the other hand, an affine-type fuzzy control design scheme is proposed for the affine-type systems. Control performance and robustness to an external disturbance are shown in simulation results. Affine T–S fuzzy representation provides one more adjustable parameter in the neural–fuzzy learning process. Therefore, an affine T–S-based controller possesses better performance for a current-controlled system since it is nonlinear not only to system states but also to system inputs. This phenomenon is shown in simulation results. Technical contributions include a nonlinear affine-type optimal fuzzy control design scheme, self-organizing neural-learning-based linear and affine T–S fuzzy modeling for both MagLev systems, and the achievement of an integrated neural–fuzzy technique to stabilize current- and voltage-controlled MagLev systems under minimal energy-consumption conditions.**

***Index Terms*—Affine Takagi–Sugeno (T–S) system, linear T–S system, modeling index, neural–fuzzy.**

I. Introduction

An electromagnetic suspension system suspends an object without mechanical contact (frictionless). It therefore has attractive potential in flywheels, wind tunnel model suspensions, magnetic vibration isolation systems, and high-speed rotation magnetic bearing machinery. Magnetically levitated (MagLev) transport systems have recently been constructed, tested, and have made improvements in noise, vibration, and maintenance-cost reduction [1]–[4]. A vehicle possesses three translation motions (i.e., surge, sway, and heave) and three rotation motions (i.e., roll, pitch, and yaw). A MagLev vehicle system possesses independent lift, guide, and propulsion dynamics [1]. In this paper, we focus on the levitation behavior. A current-controlled MagLev system is more attractive than a voltage-controlled system in terms of its lower dimension and zero sensitivity to coil copper resistance. Despite its good potential, MagLev systems are, however, naturally unstable and highly nonlinear. The current-controlled MagLev is particularly nonlinear not only to the system state but also to the system input. Therefore, it is difficult to construct a system to achieve high-performance gap control. Several nonlinear robust technologies are adopted to stabilize MagLev systems, such as sliding model control, H_2, linear–quadratic–Gaussian, mixed H_2/H_∞, H_∞/μ synthesis control, and Q-parameterization control [6]–[13].

Research of fuzzy modeling and fuzzy control has come of age [14]–[17]. There are two model-based approaches to theoretically construct a Takagi–Sugeno (T–S) fuzzy system of a nonlinear system. One is from a local linear approximation, which generates a linear singleton-included rule consequence (*affine* T–S fuzzy system). The other is via a sector nonlinearity concept [18]–[20], which, in general, results in a linear singleton-free rule consequence (*linear* T–S fuzzy system). Both fuzzy systems are demonstrated to be universal approximations of any smooth nonlinear systems [21]–[23]. Although both consequence parts are represented by linear equations, there exists a constant singleton in the fuzzy rule consequence for an affine T–S fuzzy model. The linear-type T–S system is the most popular fuzzy model due to its further intrinsic analysis: The linear matrix inequality (LMI)-based fuzzy controller minimizes the upper bound of a performance index [21]; structure-oriented and switching fuzzy controllers are developed for more complicated systems [20], [24], [25]; the optimal fuzzy control technique is used to minimizes the performance index from local-concept or global-concept approaches [26]–[28].

However, it is impractical to theoretically convert a mathematical model into a T–S fuzzy model if the nonlinear system is too complex to describe. More and more researchers attempt to learn fuzzy modeling from input–output data [29]–[32]. The approach of model-free nonlinear systems to guarantee the

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proposed fuzzy model under limited modeling error and the corresponding fuzzy control with desirable implementation is still in development. An affine T–S fuzzy model is much more preferred over a linear type in providing one more adjustable parameter for neural–fuzzy learning. However, few affine-type controllers are proposed. Kim *et al.* synthesize the affine-type fuzzy controller via the convex optimization technique and recast it into an LMI problem [33], [34]. They further specialize in an affine T–S fuzzy system with a *constant input matrix* and transform the regulating problem into bilinear matrix inequalities [35]. Bergsten *et al.* have tried to derive an affine-type observer, but the constant-term consequence in an affine T–S fuzzy system is only a trivial term for observer derivation, and the simulation is, in fact, a typical linear-type formulation [36].

We propose an optimal fuzzy control design scheme for the learned affine T–S-based MagLev systems. Both current- and voltage-controlled MagLev systems are first modeled by two self-organizing neural–fuzzy inference networks to achieve two types of T–S fuzzy systems (i.e., linear and affine). Then, the corresponding optimal fuzzy controllers are used to regulate the ball position under minimal energy consumption conditions. The neural–fuzzy network can self-learn Gaussian-type membership functions and fuzzy subsystems' parameters for the consequence of each rule. Furthermore, an integrated neural-based fuzzy modeling and optimal fuzzy controlling algorithm is introduced to ensure that the neural-learning-based fuzzy models can achieve limited modeling error and that the designed fuzzy controllers can efficiently stabilize MagLev systems [32]. In our paper, an affine-type optimal fuzzy control scheme is derived in Section II. Section III includes the integrated neural–fuzzy modeling and gap controlling for both current- and voltage-controlled MagLev systems, the derived optimal fuzzy controllers, and the performance of four proposed neural-based fuzzy controllers. Section IV summarizes the results of our research and suggests areas for further research.

II. Optimal Fuzzy Control Scheme

In this section, we propose an affine-type fuzzy control scheme and summarize our previously proposed linear-type fuzzy controller. These two control schemes are incorporated within the corresponding neural–fuzzy modeling techniques (Figs. 4 and 5) and then integrated into an algorithm in Fig. 1 to ensure limited modeling error and to guarantee good control performance.

A. Affine-Type Optimal Fuzzy Control Scheme

Fig. 4 describes a six-layer structure for realizing an affine T–S fuzzy model. By this self-constructing neural–fuzzy inference network, a nonlinear system can be realized as the following affine T–S fuzzy system:

$$\begin{aligned} R^i: \quad & \text{IF } x_1 \text{ is } T_{1i}(m_{1i},\sigma_{1i}),\ldots,x_n \text{ is } T_{ni}(m_{ni},\sigma_{ni}) \\ & \text{THEN } \dot{X}(t) = A_iX(t)+B_iu(t)+D_i \\ & \qquad Y(t)=CX(t), \qquad i=1,\ldots,r \end{aligned} \tag{1}$$

where R^i denotes the ith rule of the fuzzy model; $x_1,\ldots,x_n$ are system states; $T_{ji}(m_{ji},\sigma_{ji})$, $j=1,\ldots,n$ is the fuzzy term of the input fuzzy variable x_j in the ith rule, with m_{ji} and σ_{ji} as the mean and standard deviation, respectively, of the Gaussian membership function; $X(t)=[x_1,\ldots,x_n]^t \in \Re^n$ is the state vector, $Y(t)=[y_1,\ldots,y_{n'}]^t\in\Re^{n'}$ is the system output vector; $u(t)\in\Re^m$ is the system input (i.e., control output); and A_i, B_i, C, and D_i are $n\times n$, $n\times m$, $n'\times n$, and $n\times 1$ matrices, respectively. The problem of minimal energy consumption for the fuzzy gap control of two MagLev systems in (17) and (18) is to design rule-based fuzzy controllers, i.e.,

$$\begin{aligned} R^i: \quad & \text{IF } y_1 \text{ is } S_{1i},\ldots,y_{n'} \text{ is } S_{n'i} \\ & \text{THEN } u(t)=r_i(t), \qquad i=1,\ldots,\delta \end{aligned} \tag{2}$$

to minimize the quadratic cost function

$$J(u(\cdot)) = \int_{t_0}^{\infty} \left[X^t(t)LX(t)+u^t(t)u(t)\right]dt \tag{3}$$

where $X^t(t)LX(t)$ are the state-trajectory penalties, with L belonging to the symmetric positive semidefinite $n\times n$ matrices, and $u^t(t)u(t)$ denotes energy consumption; $y_1,\ldots,y_{n'}$ are elements of the output vector $Y(t)$; $S_{1i},\ldots,S_{n'i}$ are input fuzzy terms in the ith control rule; and the plant input (i.e., control output) vector $u(t)$ or $r_i(t)$ is in $\Re^m$ space. In other words, the neural–fuzzy control of MagLev systems in (17) and (18) can be approximately formulated as a quadratic optimal fuzzy control problem, as follows.

Problem 1: Given an affine T–S fuzzy systems in (1) with $X(0)=X_0\in\Re^n$ and a rule-based fuzzy controller in (2), find the individual optimal control law $r_i^*(\cdot)$, $i=1,\ldots,\delta$ such that the composed optimal controller $u^*(\cdot)$ can minimize a quadratic cost function $J(u(\cdot))$ in (3) over all possible inputs $u(\cdot)$.

We note that the entire T–S-type fuzzy system in (1) can be represented as

$$\dot{X}(t)=\sum_{i=1}^{r} h_i(X(t))(A_iX(t)+B_iu(t)+D_i) \tag{4}$$

with $u(t)=\sum_{i=1}^{\delta} w_i(Y(t))r_{i(t)}$, and $X_0=X_0\in\Re^n$; $h_i(X(t))$ denotes the normalized firing strength of the ith rule of the fuzzy system ($h_i(X(t))=\alpha_i/\sum_{i=1}^{r}\alpha_i$, with $\alpha_i=\Pi_{j=1}^{n}\mu_{T_{ji}}(X(t))$, where $\mu_{T_{ji}}(X(t))$ is the membership function of the fuzzy term T_{ji}.

Theorem 1 (Affine T–S Type): For the affine T–S fuzzy system in (1) and the fuzzy controller in (2), if A_i is nonsingular, (A_i,B_i) is completely controllable (c.c.), and (A_i,C) is completely observable (c.o.) for $i=1,\ldots,r$, then the local optimal fuzzy control law is

$$r_i^*(t)=-B_i^t\bar{\pi}_iX^*(t)+\bar{r}_i^s, \qquad i=1,\ldots,r \tag{5}$$

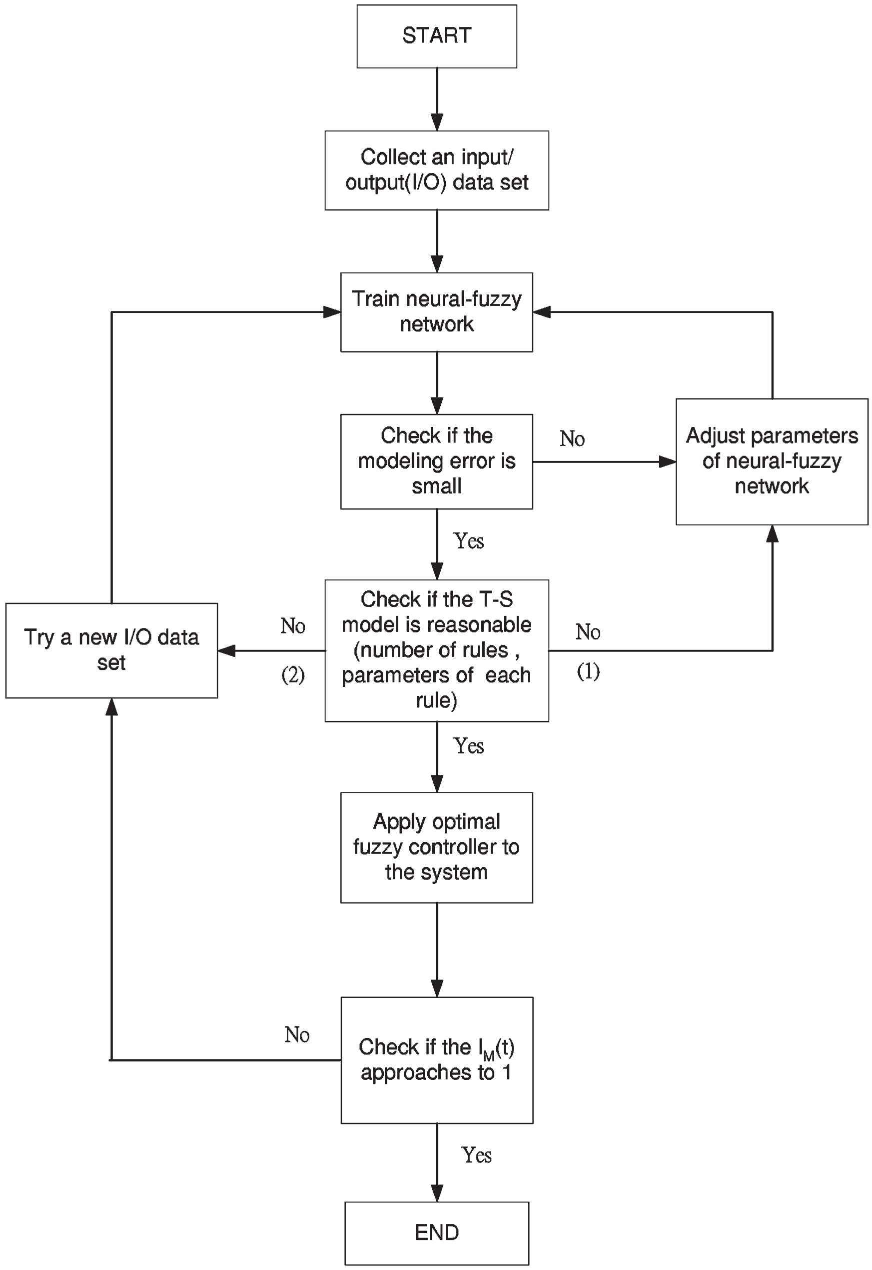

Fig. 1. Integrated neural–fuzzy modeling and controlling algorithm [32].

where $\bar{r}_i^s = -B_i^t(\bar{\pi}_i \bar{X}_i^s + \bar{b}_i^s)$ and $\bar{X}_i^s = A_i^{-1} D_i$; their "blending" global optimal fuzzy controller is

$$u^*(t) = \sum_{i=1}^{r} h_i \left(X^*(t)\right) \left[-B_i^t \bar{\pi}_i X^*(t) + \bar{r}_i^s\right] \tag{6}$$

which minimizes $J(u(\cdot))$ in (3), where

$$\bar{b}_i^s = -\int_0^\infty e^{\left[A_i - B_i B_i^t \bar{\pi}_i\right]^t \tau} d\tau \cdot L\bar{X}_i^s \tag{7}$$

and $\bar{\pi}_i$ is the unique symmetric positive semidefinite solution of the Riccati equation

$$K_i A_i + A_i^t K_i - K_i B_i B_i^t K_i + L = 0. \tag{8}$$

Proof:

1) From the essence of the dynamic programming formalism, we know that

$$\min_{u_{[t_0,\infty)}} J\left(u(\cdot)\right) = \min_{u_{[t,\infty)}} \left\{ \int_t^\infty \left(X_l^t L X_l + u_l^t u_l\right) dl + \min_{u_{[t_0,t]}} \int_{t_0}^t \left(X_l^t L X_l + u_l^t u_l\right) dl \right\} \tag{9}$$

where the lower index is used to denote the time dependence for notation simplification (X_l for $X(l)$). Therefore, our problem is turned into successively finding the optimal global decision (global optimal fuzzy controller) u_t^* for minimizing the cost function

$$J_t(u_t) = \int_t^\infty \left(X_l^t L X_l + u_l^t u_l\right) dl, \qquad t \in [t_0, \infty) \tag{10}$$

and estimating $X_{t^+}^*$ with regard to the initial state X_t^*, where t^+ denotes the time instant slightly later than time t and then with the new initial state $X_{t^+}^*$, resolving $u_{t_+}^*$ to minimize $J_{t_+}(u_{t^+})$.

2) At any time instant t, the optimal local decision (local optimal fuzzy control law) stems from minimizing $J_t(u_t)$ with regard to the fuzzy subsystem

$$\dot{X}_l = A_i X_l + B_i u_l + D_i, \qquad l \in [t, \infty), \qquad i = 1, \ldots, r \tag{11}$$

and the optimal global decision results from minimizing $J_t(u_t)$ with regard to the entire fuzzy system

$$\dot{X}_l = \sum_{i=1}^r h_i(X_l)\left(A_i X_l + B_i u_l + D_i\right), \qquad l \in [t, \infty). \tag{12}$$

Since u_t^* is only a variable to be solved, regardless of the aforementioned local optimization problem or the global optimization issue, we can use $r_{i_t}^*$ to denote the optimal local decision of the ith fuzzy subsystem. Now, let $\zeta_l(X_l, u_l)$ and $\zeta_{i_l}(X_l, r_{i_l})$, $i = 1, \ldots, r$ denote the total energy and local energy, respectively, at any time instant l, $l \in [t, \infty)$. Then, $J_t(u_t) = \int_t^\infty \zeta_l(X_l, u_l) dl$, and $J_t(r_{i_t}) = \int_t^\infty \zeta_{i_l}(X_l, r_{i_l}) dl$. At any time instant, the energy of the entire fuzzy system is some kind of (nonlinear) combination of the fuzzy subsystems' energy $(\zeta_l(X_l, u_l) = \sum_{i=1}^r h_i'(X_l)\zeta_{i_l}(X_l, r_{i_l}))$. At time instant t with the initial condition X_t^*, let $r_{i_t}^*$ denote the optimal local decision to minimize $J_t(r_{i_t})$ for all $i = 1, \ldots, r$, i.e.,

$$\begin{aligned}\frac{\partial J_t(r_{i_t})}{\partial r_{i_t}}\Big|_{r_{i_t}^*} &= \frac{\partial}{\partial r_{i_t}} \int_t^\infty \zeta_{i_l}(X_l, r_{i_l}) dl \Big|_{r_{i_t}^*} \\ &= \frac{\partial \zeta_{i_t}\left(X_t^*, r_{i_t}\right)}{\partial r_{i_t}}\Big|_{r_{i_t}^*} = 0 \\ \frac{\partial^2 J_t(r_{i_t})}{\partial r_{i_t}^2}\Big|_{r_{i_t}^*} &= \frac{\partial^2 \zeta_{i_t}\left(X_t^*, r_{i_t}\right)}{\partial r_{i_t}^2}\Big|_{r_{i_t}^*} > 0.\end{aligned}$$

Then, their corresponding global decision $\breve{u}_t = \sum_{i=1}^r h_i(X_t^*) r_{i_t}|_{r_{i_t}^*}$ can satisfy

$$\begin{aligned}\frac{\partial J_t(u_t)}{\partial u_t}\Big|_{\breve{u}_t} &= \frac{\partial}{\partial u_t} \int_t^\infty \zeta_l(X_l, u_l) dl \Big|_{\breve{u}_t} \\ &= \frac{\partial}{\partial u_t} \int_t^\infty \sum_{i=1}^r h_i'(X_l) \zeta_{i_l}(X_l, r_{i_l}) dl \Big|_{\breve{u}_t} \\ &= \sum_{i=1}^r h_i'\left(X_t^*\right) \frac{\partial \zeta_{i_t}\left(X_t^*, r_{i_t}\right)}{\partial u_t}\Big|_{\breve{u}_t} \\ &= \sum_{i=1}^r h_i'\left(X_t^*\right) \frac{\partial \zeta_{i_t}\left(X_t^*, r_{i_t}\right)}{\partial r_{i_t}} \cdot \frac{\partial r_{i_t}}{\partial u_t}\Big|_{\breve{u}_t} = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 J_t(u_t)}{\partial u_t^2}\Big|_{\breve{u}_t} &= \sum_{i=1}^r h_i'\left(X_t^*\right) \frac{\partial^2 \zeta_{i_t}\left(X_t^*, r_{i_t}\right)}{\partial r_{i_t}^2} \cdot \left(\frac{\partial r_{i_t}}{\partial u_t}\right)^2 \Big|_{\breve{u}_t} \\ &\quad + \sum_{i=1}^r h_i'\left(X_t^*\right) \frac{\partial \zeta_{i_t}\left(X_t^*, r_{i_t}\right)}{\partial r_{i_t}} \cdot \frac{\partial^2 r_{i_t}}{\partial u_t^2}\Big|_{\breve{u}_t} > 0\end{aligned}$$

i.e., $\breve{u}_t = u_t^*$. Therefore, at any time instant t, if we can find $r_{i_t}^*$ to minimize $J_t(r_{i_t})$, then it follows that their composed global decision u_t^* can be the global minimizer of the total cost $J_t(u_t)$.

3) We further assume that A_i, $i = 1, \ldots, r$, is nonsingular, and let $\hat{X}(t) = X(t) + \bar{X}_i^s$, where $\bar{X}_i^s = A_i^{-1} D_i$. Then, the local affine-type quadratic problem is further reformulated as follows: Given a fuzzy subsystem $\dot{\hat{X}}_l = A_i \hat{X}_l + B_i r_{i_l}$, $l \in [t, \infty)$, $i = 1, \ldots, r$, with the initial state $\hat{X}_{0_t} = \hat{X}_t^*$ and the normalized firing strength $h(\hat{X}_l) = h(X_l)$, find the optimal local decision at time instant t, i.e., $r_{i_t}^*$, for minimizing the cost functional $J_t(r_{i_t}) = \int_t^\infty ((\hat{X}_l^t - \bar{X}_i^s) L (\hat{X}_l - \bar{X}_i^s) + r_{i_l}^t r_{i_l}) dl$. Since the fuzzy subsystem is linear, its quadratic optimization problem is the same as the general linear–quadratic tracking issue. Therefore, we obtain the regulating law $r_i^*(t) = -S^{-1} B_i^t [\bar{\pi}_i (X^*(t) + \bar{X}_i^s) + b_i^s(t)]$ and the fuzzy subsystem $\dot{X}^*(t) = (A_i - B_i S^{-1} B_i^t \bar{\pi}_i)(X^*(t) + \bar{X}_i^s) - B_i S^{-1} B_i^t b_i^s(t)$, where $b_i^s(t)$ satisfies

$$\dot{b}_i^s(t) = -\left(A_i - B_i S^{-1} B_i^t \bar{\pi}_i\right)^t b_i^s(t) + L \bar{X}_i^s \tag{13}$$

with $b_i^s(\infty) = \mathbf{0}_{n \times 1}$. Furthermore, we have $b_i^s(t) = \bar{b}_i^s$ in (7). This completes the proof. ■

B. Linear-Type Optimal Fuzzy Control Scheme

We use a neural–fuzzy structure in Fig. 5 to achieve the following linear-type T–S fuzzy model:

$$\begin{aligned} R^i: \quad & \text{IF } x_1 \text{ is } T_{1i}(m_{1i}, \sigma_{1i}), \ldots, x_n \text{ is } T_{ni}(m_{ni}, \sigma_{ni}) \\ & \text{THEN } \dot{X}(t) = A_i X(t) + B_i u(t) \\ & \qquad Y(t) = C X(t), \qquad i = 1, \ldots, r. \end{aligned} \tag{14}$$

The problem of minimal energy consumption for the fuzzy gap control of MagLev systems in (17) and (18) is to design rule-based fuzzy controllers in (2) to minimize the quadratic cost function in (3) with regard to the linear-type T–S fuzzy system in (14). Here, we summarize our previously proposed optimal fuzzy control design scheme for a linear-type T–S fuzzy system, as follows.

Proposition 1 (Linear T–S Type) [26]: For the T–S fuzzy system in (14) and the fuzzy controller in (2), if (A_i, B_i) is c.c. and (A_i, C) is c.o. for $i = 1, \ldots, r$, then the local optimal fuzzy control law is

$$r_i^*(t) = -B_i^t \bar{\pi}_i X^*(t), \qquad i = 1, \ldots, r \tag{15}$$

and their "blending" global optimal fuzzy controller is

$$u^*(t) = \sum_{i=1}^r h_i\left(X^*(t)\right)\left[-B_i^t \bar{\pi}_i X^*(t)\right] \tag{16}$$

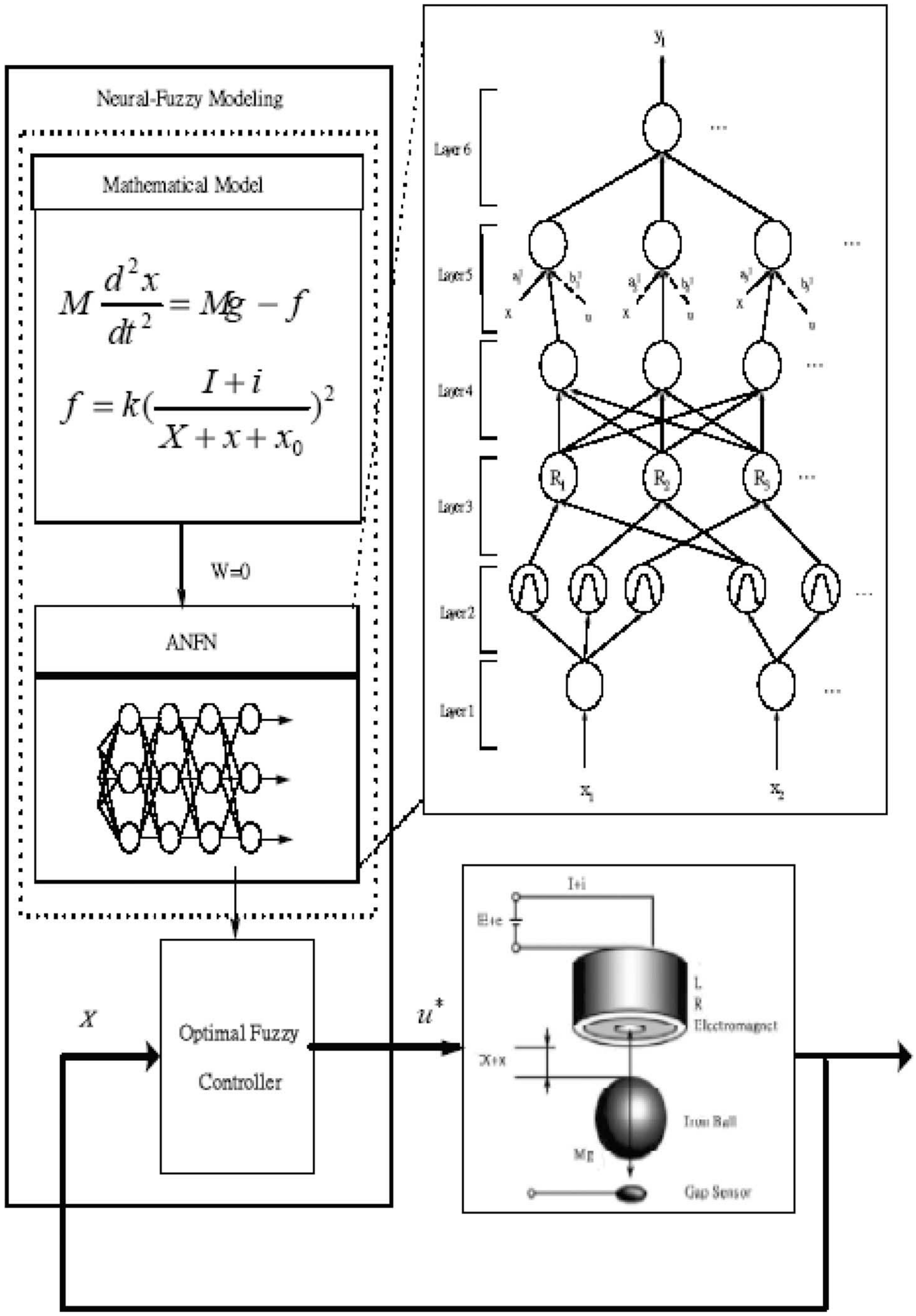

Fig. 2. Self-learning optimal intelligent suspension system.

which minimizes $J(u(\cdot))$ in (3), where $\bar{\pi}_i$ is the unique symmetric positive semidefinite solution of the Riccati equation in (8).

III. Neural-Learning-Based Fuzzy Gap Control

In this section, we use two neural–fuzzy networks (Figs. 4 and 5) to learn MagLev systems' behavior on the basis of minimizing the square error between training data and predicted values. To further solve the modeling failure from insufficient training data, we define parameter $I_M = (\hat{Y}_{cl}(t) + \theta)/(Y_{cl}(t) + \theta)$ to denote the modeling error index, where $\hat{Y}_{cl}(t)$ is the output of the T–S fuzzy closed-loop system, $Y_{cl}(t)$ is the output of the real physical closed-loop system, and θ is only a small constant. The integrated neural–fuzzy modeling and controlling algorithm in Fig. 1 is used to guarantee that the generated affine/linear T–S fuzzy model can almost perfectly approximate MagLev systems [32]. Therefore, the self-learning optimal intelligent suspension framework in Fig. 2 can efficiently regulate highly nonlinear, complex, and uncertain MagLev systems. We also compare the performance of these two optimal fuzzy controllers (affine and linear types).

A. Electromagnetic Leviation System

A vehicle possesses three translation motions (i.e., surge, sway, and heave) and three rotation motions (i.e., roll, pitch, and yaw). The dynamic behavior can be described as three

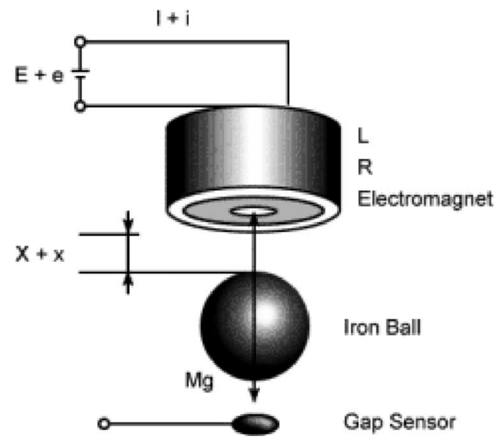

Fig. 3. Electromagnetic suspension system.

independent (i.e., lift, guide, and propulsion) systems. We now focus on the levitation system. A MagLev prototype uses independent electromagnetic attraction forces to lift a vehicle. Therefore, the vehicle levitation behavior can be further described as four decoupled systems. Fig. 3 shows the quarter section of an electromagnetic levitated vehicle [37]. An electromagnet is located at the upper part of a MagLev system. The control problem stably levitates an iron ball by an electromagnetic force. We now have the following assumptions: A1) The magnetic flux density and magnetic field do not have any hysteresis, and they are not saturated; A2) the magnetic permeability of the electromagnet is infinity; and A3) the eddy current in the magnetic pole can be neglected. Then, a current-controlled MagLev system can be described as

$$M\frac{d^2x(t)}{dt^2} = Mg - f(t)$$
$$f(t) = k\left(\frac{I_g + i(t)}{\bar{X} + x(t) + x_0}\right)^2 \tag{17}$$

where $M = 1.75$ kg is the mass of the iron ball; $x(t)$ is the position deviation from its steady-state air-gap $\bar{X} = 0.005$ m; $i(t)$ is the current deviation from its gravity compensation current $I_g = 1.15$ A; $f(t)$ is electromagnetic force; and $k = 0.000284$ and $x_0 = -0.000286$ m are constant coefficients. We note that the systems are nonlinear with regard to both the system state and the system input (current). It is difficult to design appropriate controllers to stabilize such a system.

We further introduce an RC current into (17) and assume the following: A4) No leakage flux exists; and A5) coil inductance is constant around an operating point, and the electromotive force from the iron ball motion is negligible. We then obtain the following voltage-controlled MagLev system, which is nonlinear with regard to the system state but is linear with regard to the system input (voltage):

$$M\frac{d^2x(t)}{dt^2} = Mg - f(t)$$
$$f(t) = k\left(\frac{I_g + i(t)}{\bar{X} + x(t) + x_0}\right)^2$$
$$L\frac{di(t)}{dt} + R\left(I_g + i(t)\right) = E + e(t) \tag{18}$$

where $e(t)$ is the voltage deviation from its steady-state voltage $E = 30.59$ V, and $L = 0.558$ H and $R = 26.6\ \Omega$ are the inductance and resistance, respectively.

We setup two input–output data sets from these two MagLev systems and, respectively, feed them into neural–fuzzy inference networks to achieve two T–S fuzzy systems (affine and linear types).

B. Current-Controlled MagLev System

In this section, we first develop affine and then linear T–S fuzzy models for a current-controlled MagLev system in (17). Then, their corresponding controllers are proposed to stabilize the MagLev system with minimal current consumption.

Fig. 4 describes a self-constructing neural–fuzzy inference network for realizing an affine T–S fuzzy model [29]. The input variables x_l are directly transmitted into the network in Layer 1. They are converted into fuzzy variables in Layer 2, whose corresponding term sets T_{lj} have Gaussian membership functions with mean m_{lj} and standard deviation σ_{lj}. Layer 3 estimates the firing strength of each fuzzy rule. Layer 4 achieves normalization. Layer 5 obtains a singleton-included linear-type rule consequence. Finally, Layer 6 performs defuzzification functions. *Structure learning* includes both precondition and consequence identification of a fuzzy IF–THEN rule. *Precondition identification* starts from input-space partition and is reformulated into an optimization problem to minimize the number of generated rules and the number of fuzzy term sets for each input fuzzy variable. The input space is partitioned in a flexible way via an aligned clustering-based algorithm. *Consequence identification* decides the significant terms (states and inputs) to be added by using a projected-based correlation measure. For *parameter learning*, based on a supervised learning technique, we use a least mean square method to adjust the parameters of each rule consequence and a backpropagation algorithm to minimize a given cost function to adjust the parameters of each rule precondition. The combined precondition and consequence structure identification scheme is automatically set up within an efficient and dynamic growth network. Finally, the neural–fuzzy structure possesses a self-construction ability to generate its own rule nodes and term-set nodes with associated linking weights.

The control scheme in Theorem 1 is integrated with this neural–fuzzy modeling technique into an integrated algorithm in Fig. 1 to ensure limited modeling error and guaranteed control performance. We feed 2300 training pattern generated from (17) into this network with the learning rate set at 0.005. After training, we obtain the following affine T–S fuzzy model:

$$R^1: \quad \text{IF } x(t) \text{ is } T_{11}(0.002, 0.21)$$
$$\text{THEN } \dot{X}(t) = A_1^a X(t) + B_1^a u(t) + D_1^a$$
$$R^2: \quad \text{IF } x(t) \text{ is } T_{12}(-0.18, 1.76)$$
$$\text{THEN } \dot{X}(t) = A_2^a X(t) + B_2^a u(t) + D_2^a \tag{19}$$

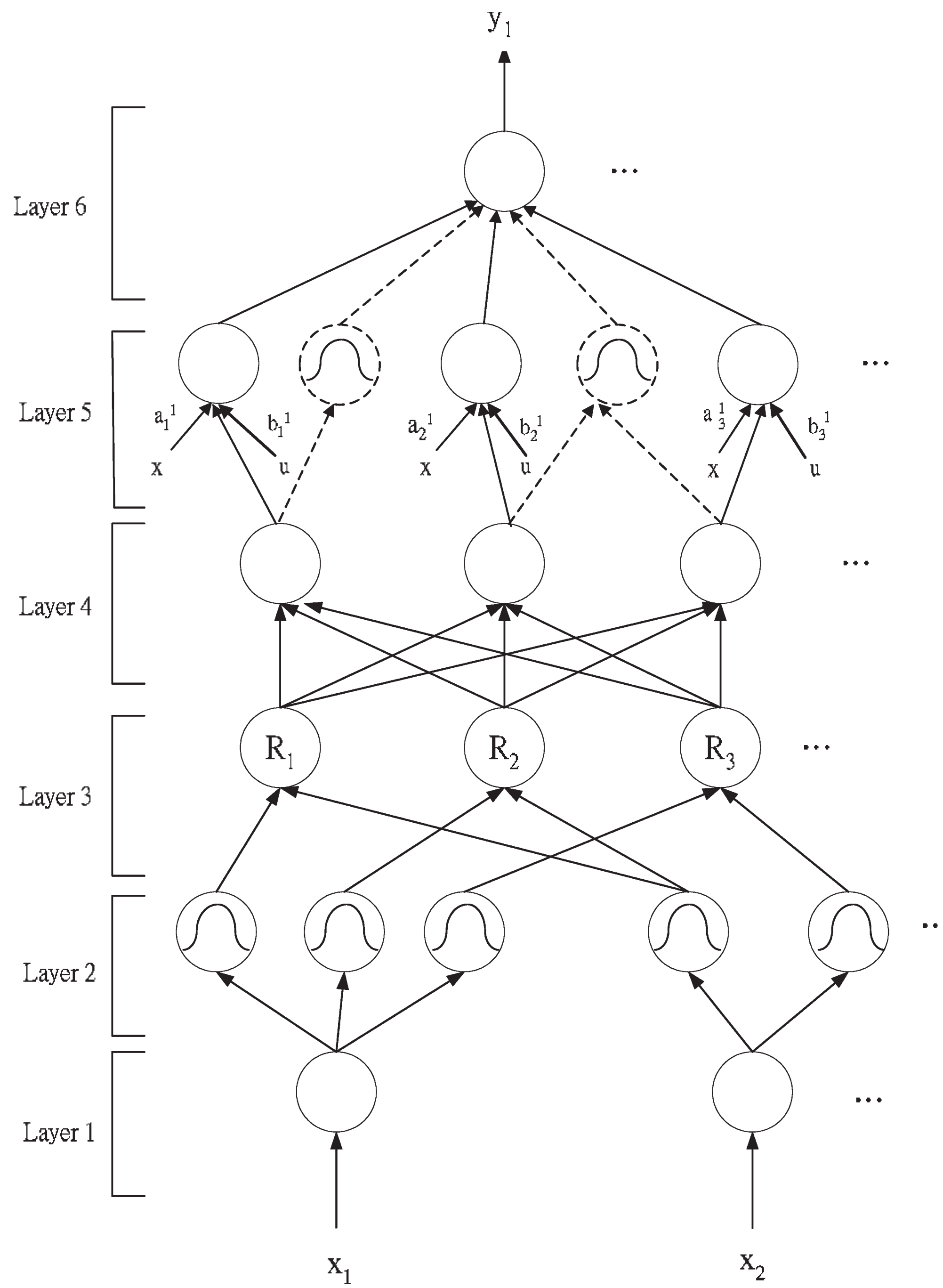

Fig. 4. Neural–fuzzy inference network for an affine T–S fuzzy model.

where $X(t) = [x(t), \dot{x}(t)]^t$, and $u(t) = i(t)$; $T_{1i}(m_{1i}, \sigma_{1i}) = \exp[-(x - m_{1i})^2/\sigma_{1i}^2]$; $A_1^a = \begin{bmatrix} 0 & 1 \\ 3204.8 & 0 \end{bmatrix}$, $A_2^a = \begin{bmatrix} 0 & 1 \\ 3553.5 & 0 \end{bmatrix}$, $B_1^a = \begin{bmatrix} 0 \\ -12.8 \end{bmatrix}$, $B_2^a = \begin{bmatrix} 0 \\ -14.35 \end{bmatrix}$, $D_1^a = \begin{bmatrix} 0 \\ 0.012 \end{bmatrix}$, and $D_2^a = \begin{bmatrix} 0 \\ -0.024 \end{bmatrix}$; and $Y = CX$ for each rule.

This affine T–S fuzzy model is c.c. and c.o. for each subsystem. Therefore, the affine-type optimal fuzzy controller is $i(t) = u^*(t) = \sum_{i=1}^{2} h_i(X^*(t)) r_i^*(t)$, where $r_i^*(t) = -B_i^{at} \bar{\pi}_i^{ac} X^*(t) + \bar{r}_i^s$, $\bar{r}_i^s = -B_i^{at}(\bar{\pi}_i^{ac} \bar{X}_i^s + \bar{b}_i^s)$, and $\bar{X}_i^s = A_i^{a-1} D_i^a$ with the solution of the steady-state Riccati equation (SSRE), $\bar{\pi}_1^{ac} = \begin{bmatrix} 2228.9 & 29.123 \\ 39.123 & 0.696 \end{bmatrix}$, and $\bar{\pi}_2^{ac} = \begin{bmatrix} 2073 & 34.526 \\ 34.526 & 0.583 \end{bmatrix}$. We first rewrite the current-controlled MagLev system in (17) into state-space representation with state $X(t) = [x(t), \dot{x}(t)]^t$ and input $u(t) = i(t)$. Then, we substitute the derived optimal fuzzy controller (current) into the system. Fig. 6 shows the evolution of the state responses and the optimal control current for four initial conditions: $X(0) = (0.003, 0)^t, (0.002, 0)^t, (0.001, 0)^t$, and $(-0.001, 0)^t$.

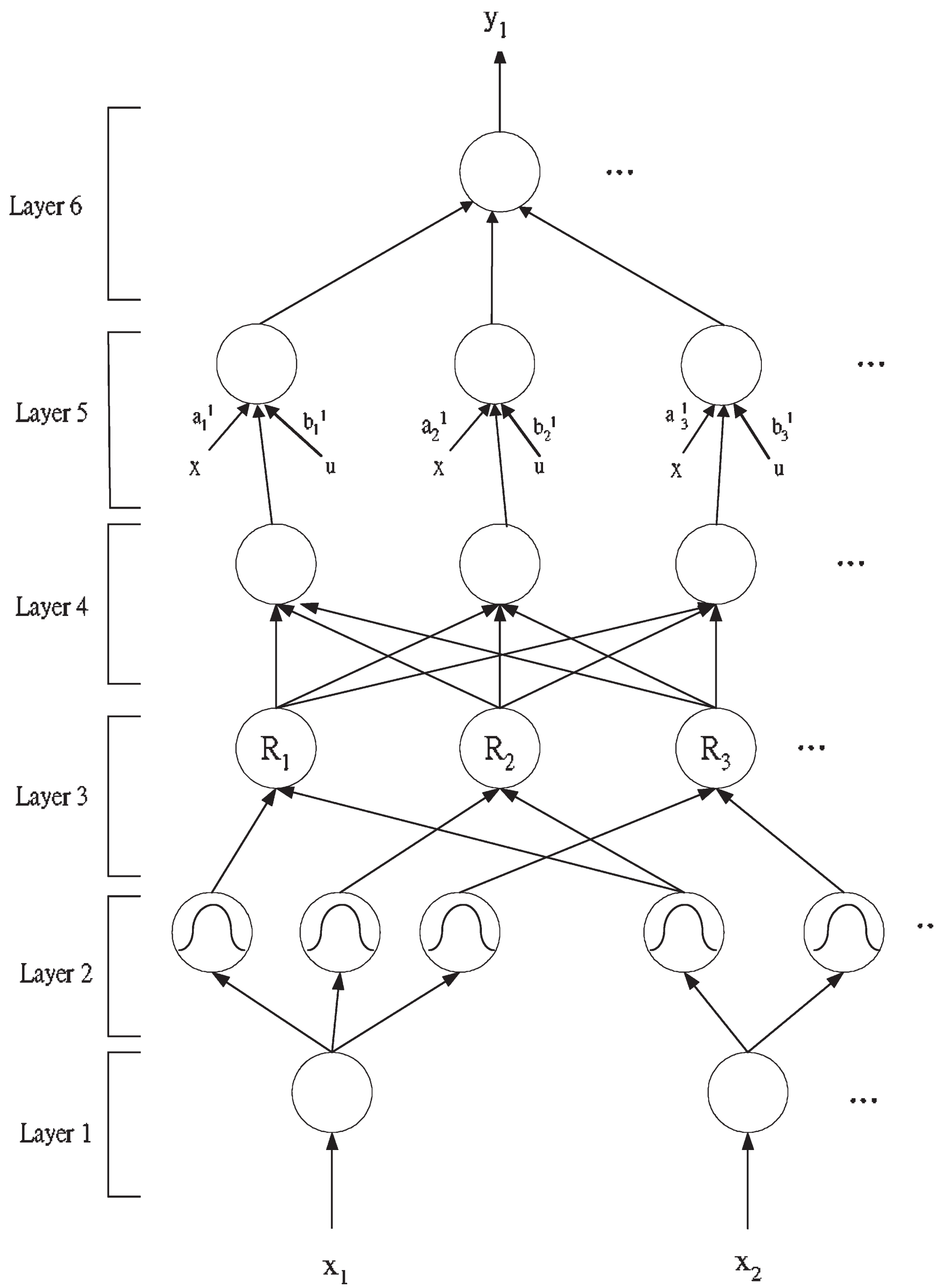

Fig. 5. Neural–fuzzy inference network for a linear T–S fuzzy model.

In the remainder of this section, we shall design a linear-type optimal fuzzy controller for a MagLev system in (17). Although a linear T–S fuzzy system can be obtained via the aforementioned neural–fuzzy inference network [29] by viewing system input variables as augmented state variables. However, fuzzy set D_i in Fig. 4 is the basic consequence of the ith rule; state $X(t)$ and input $u(t)$ are optionally added. Therefore, we could not use the same inference network to construct a linear T–S fuzzy system by only directly deleting the singleton from the generated rules, which will diverge the learning process. We modify the inference network in Fig. 4 [29] into Fig. 5 [32] such that the new learning process focuses on generating input- and state-dependent terms. Fig. 5 is a six-layer structure for realizing a linear T–S fuzzy model, which is similar to Fig. 4 in structure, except for the rule representation in the fifth layer. The node outputs of Layer 4 are only the basic node inputs of Layer 5, i.e., u_i^5, for storing firing strength information. Not only the input variables of Layer 1 but also the external inputs of the physical system are included as the node inputs to generate the consequence condition of Layer 5. In other words, the activity function in this layer is

$$f = \sum_j a_{ji} x_j + \sum_m b_{mi} u_m, \quad a^5(f) = f \cdot u_i^5.$$

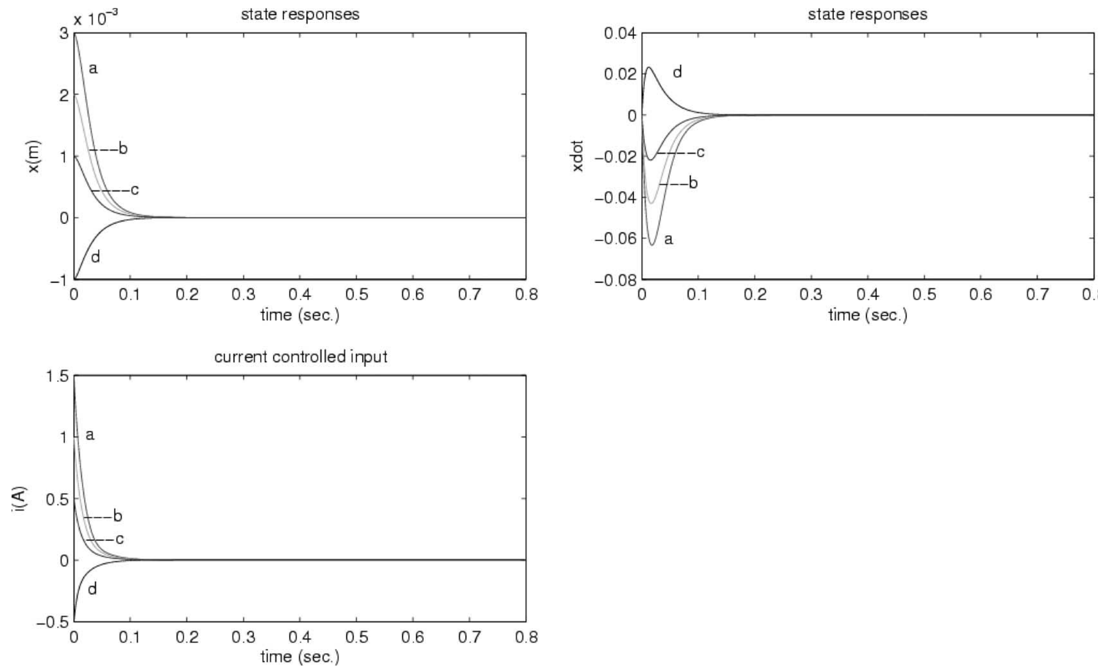

Fig. 6. State responses and optimal control current for the affine-type closed-loop current-controlled MagLev system with four initial conditions. (a) $X(0) = (0.003, 0)^t$. (b) $X(0) = (0.002, 0)^t$. (c) $X = (0.001, 0)^t$. (d) $X = (-0.001, 0)^t$.

We note that the structures in Fig. 4 (affine type) and Fig. 5 (linear type) are similar, but the learning spirit is totally different. The singleton is the key term for the network in Fig. 4, and state-dependent terms are only optionally added for compensation. However, for the network in Fig. 5, input- and state-dependent terms are the key terms. In addition, for parameter reduction, we eliminate trivial system states and system inputs by using a projected- or stochastic-based correlation measure. A squared-error term is used as the cost function $(E(\cdot) = (1/2)\sum_{i=1}^{n}(d_i - y_i)^2)$ in both networks, and the connection weights are updated according to a gradient-descent method.

The same 2300 training patterns are used, and the learning rate is set at 0.005. The generated linear T–S fuzzy model is

$$\begin{aligned} R^1: &\quad \text{IF } x(t) \text{ is } T_{11}(-0.326, 0.041) \\ &\quad \text{THEN } \dot{X}(t) = A_1^l X(t) + B_1^l u(t) \\ R^2: &\quad \text{IF } x(t) \text{ is } T_{12}(0.421, 0.0023) \\ &\quad \text{THEN } \dot{X}(t) = A_2^l X(t) + B_2^l u(t) \end{aligned} \tag{20}$$

where $X(t) = [x(t), \dot{x}(t)]^t$ and $u(t) = i(t)$; $T_{1i}(m_{1i}, \sigma_{1i}) = \exp[-(x - m_{1i})^2/\sigma_{1i}^2]$; $A_1^l = \begin{bmatrix} 0 & 1 \\ 425.05 & 0 \end{bmatrix}$, $A_2^l = \begin{bmatrix} 0 & 1 \\ 1411.66 & 0 \end{bmatrix}$, $B_1^l = \begin{bmatrix} 0 \\ -0.813 \end{bmatrix}$, and $B_2^l = \begin{bmatrix} 0 \\ 3832.15 \end{bmatrix}$; and $Y = CX$ for each rule.

According to the linear T–S fuzzy representation in (20), we know that each fuzzy subsystem is c.c. and c.o. ($rank[B_i^l \quad A_i^l B_i^l] = 2$ and $rank[C^t \quad {A_i^l}^t C^t]_t = 2$, for $i = 1, 2$.). Therefore, for each subsystem, there exists a unique SSRE solution, i.e.,

$$\bar{\pi}_1^{lc} = \begin{bmatrix} 26487 & 1284 \\ 1284 & 62 \end{bmatrix}, \quad \bar{\pi}_2^{lc} = \begin{bmatrix} 1.0661 & 0.0004 \\ 0.0004 & 0.0003 \end{bmatrix}.$$

The linear-type optimal fuzzy controller is $i(t) = u^*(t) = \sum_{i=1}^{2} h_i(X^*(t)) r_i^*(t)$, where $r_i^*(t) = -{B_i^l}^t \bar{\pi}_i^{lc} X^*(t)$. Fig. 7 shows the simulation results for four initial conditions: $X(0) = (0.003, 0)^t, (0.002, 0)^t, (0.001, 0)^t$, and $(-0.001, 0)^t$.

For current-controlled MagLev systems, nonlinearity is with regard not only to system states but also to system inputs. Therefore, control of such a physical system is very difficult. Since affine-type fuzzy modeling provides one more adjustable parameter than linear-type fuzzy modeling during the neural-learning process, the controlling effect of affine-type controllers is supposed to be better than that of linear-type controllers. We examine this by comparing the results in Figs. 6 and 7.

Furthermore, a fuzzy approach, as we know, possesses some degree of robustness. We therefore simulate the tendency of closed-loop MagLev systems to be subject to an abrupt external disturbance. An external disturbance is added at 0.2 s, whereas a sinusoidal disturbance is added between 0.8 and 1.2 s. Fig. 8 is the disturbance-interrupted ball position and control-current evolution for affine- and linear-type compensated systems. We note that the affine type possesses better robustness to compensate for an external disturbance.

C. Voltage-Controlled MagLev System

In this section, we shall be concerned with a voltage-controlled MagLev system in (18). Two thousand training patterns generated from (18) are used to train neural–fuzzy network in Fig. 4 with the learning rate set at 0.005. We obtain the following affine T–S fuzzy model:

$$\begin{aligned} R^1: &\quad \text{IF } x(t) \text{ is } T_{11}(1.897, 0.002) \\ &\quad \text{and } i(t) \text{ is } T_{31}(-0.114, 1.507) \\ &\quad \text{THEN } \dot{X}(t) = A_1^a X(t) + B_1^a u(t) + D_1^a \end{aligned}$$

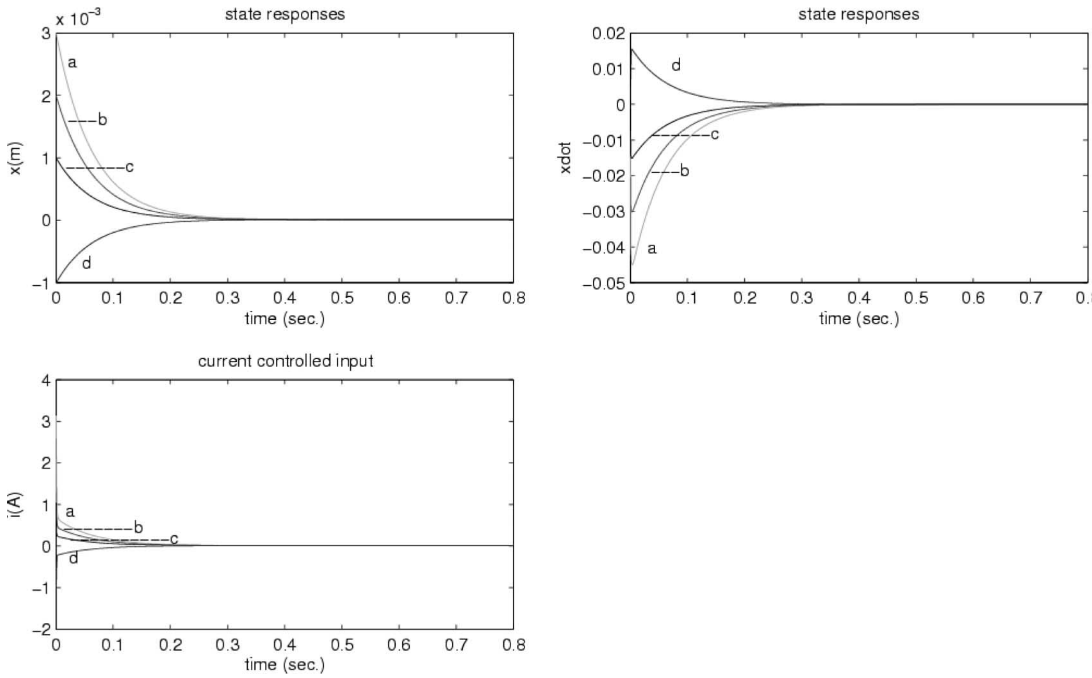

Fig. 7. State responses and optimal control current for the linear-type closed-loop current-controlled MagLev system with four initial conditions. (a) $X(0) = (0.003, 0)^t$. (b) $X(0) = (0.002, 0)^t$. (c) $X = (0.001, 0)^t$. (d) $X = (-0.001, 0)^t$.

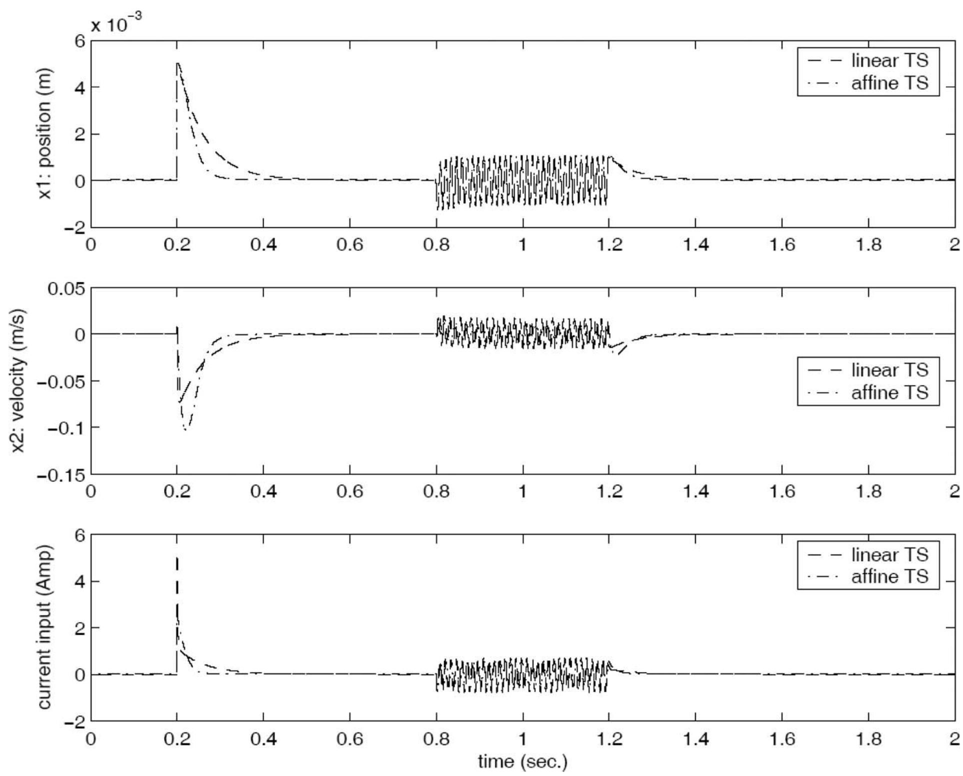

Fig. 8. Disturbance-interrupted response for a current-controlled MagLev system compensated by affine- and linear-type optimal fuzzy controllers, where an external disturbance is added at 0.2 s and a sinusoidal disturbance is added between 0.8 and 1.2 s.

$$
\begin{aligned}
R^2: \quad & \text{IF } x(t) \text{ is } T_{12}(-0.264, 2.23) \\
& \quad \text{and } i(t) \text{ is } T_{32}(0.031, 1.108) \\
& \text{THEN } \dot{X}(t) = A_2^a X(t) + B_2^a u(t) + D_2^a \\
R^3: \quad & \text{IF } i(t) \text{ is } T_{33}(-1.509, 0.002) \\
& \text{THEN } \dot{X}(t) = A_3^a X(t) + B_3^a u(t) + D_3^a \qquad (21)
\end{aligned}
$$

where $X(t) = [x(t), \dot{x}(t), i(t)]^t$ and $u(t) = e(t)$; $T_{ji}(m_{ji}, \sigma_{ji}) = \exp[-(x' - m_{ji})^2/\sigma_{ji}^2]$; $A_1^a = \begin{bmatrix} 0 & 1 & 0 \\ 549.716 & 0 & -4.281 \\ 0 & 0 & -47.67 \end{bmatrix}$,

$$A_2^a = \begin{bmatrix} 0 & 1 & 0 \\ 3402.003 & 0 & -13.809 \\ 0 & 0 & -47.67 \end{bmatrix}, A_3^a = \begin{bmatrix} 0 & 1 & 0 \\ 14.271 & 0 & -39.846 \\ 0 & 0 & -47.67 \end{bmatrix},$$

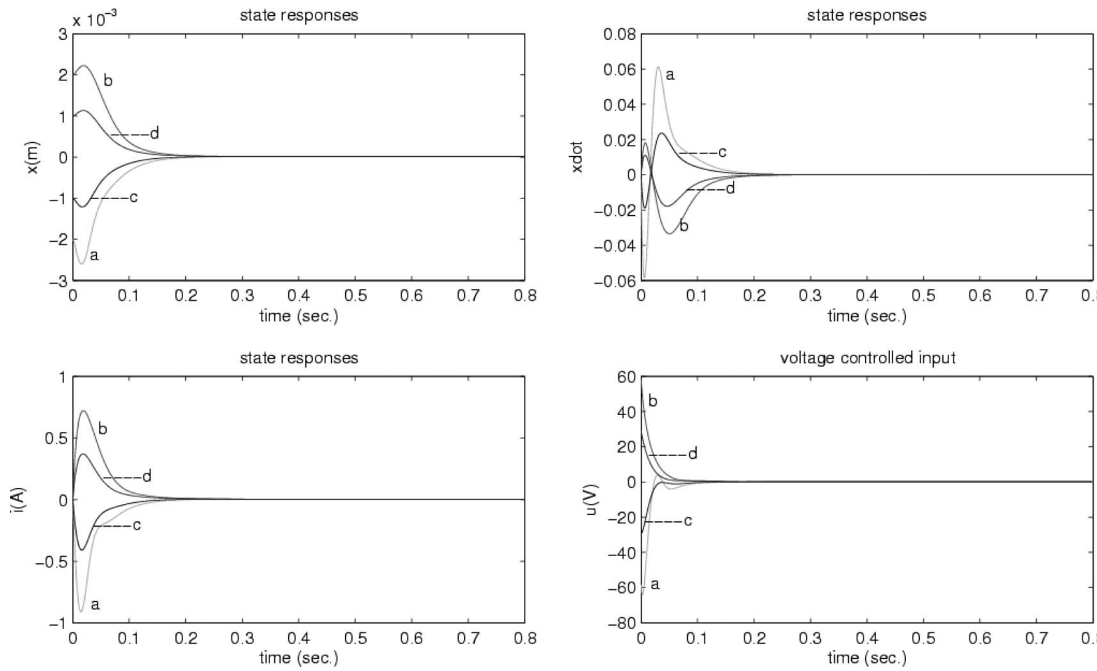

Fig. 9. State responses and optimal control voltage for the affine-type closed-loop voltage-controlled MagLev system with four initial conditions. (a) $X(0) = (-0.002, 0, 0)^t$. (b) $X = (0.002, 0, 0)^t$. (c) $X = (-0.001, 0, 0)^t$. (d) $X = (0.001, 0, 0)^t$.

$$B_1^a = B_2^a = B_3^a = \begin{bmatrix} 0 \\ 0 \\ 1.7921 \end{bmatrix}, D_1^a = \begin{bmatrix} 0 \\ -0.157 \\ 0 \end{bmatrix}, D_2^a = \begin{bmatrix} 0 \\ -0.012 \\ 0 \end{bmatrix},$$

and $D_3^a = \begin{bmatrix} 0 \\ -1.321 \\ 0 \end{bmatrix}$; and $Y = CX$ for each rule. We know that each fuzzy subsystem is c.c. and c.o. Therefore, we have the corresponding SSRE solution and the affine-type optimal fuzzy controller

$$\bar{\pi}_1^{av} = 10^3 \cdot \begin{bmatrix} 2217 & 94.556 & -5.69 \\ 94.556 & 4.033 & -0.243 \\ -5.69 & -0.243 & 0.0146 \end{bmatrix}$$

$$\bar{\pi}_2^{av} = 10^3 \cdot \begin{bmatrix} 7284.5 & 124.89 & -16.27 \\ 124.89 & 2.141 & -0.279 \\ -16.27 & -0.279 & 0.036 \end{bmatrix}$$

$$\bar{\pi}_2^{av} = \begin{bmatrix} 57.358 & 14.916 & -11.527 \\ 14.916 & 4.018 & -3.098 \\ -11.527 & -3.098 & 2.405 \end{bmatrix}$$

$e^*(t) = u^*(t) = \sum_{i=1}^3 h_i(X^*(t)) r_i^*(t)$, where $r_i^*(t) = -B_i^{a\,t} \bar{\pi}_i^{av} X^*(t) + \bar{r}_i^s$, $\bar{r}_i^s = -B_i^{a\,t}(\bar{\pi}_i^{av} \bar{X}_i^s + \bar{b}_i^s)$, and $\bar{X}_i^s = A_i^{a\,-1} D_i^a$. We rewrite the voltage-controlled MagLev system in (18) into state-space representation with state $X(t) = [x(t), \dot{x}(t), i(t)]^t$ and input $u(t) = e(t)$. Then, we substitute the derived optimal fuzzy controller (voltage) into the system. Fig. 9 shows the regulated state trajectories of the closed-loop voltage-controlled MagLev system for four different initial conditions: $X(0) = (-0.002, 0, 0)^t, (0.002, 0, 0)^t, (-0.001, 0, 0)^t$, and $(0.001, 0, 0)^t$.

As for the linear-type case, the same 2000 training patterns generated from (18) are fed into an inference network in Fig. 5 with the learning rate set at 0.005. We then have the following linear T–S fuzzy model:

$$\begin{aligned} R^1: \quad & \text{IF } x(t) \text{ is } T_{11}(-0.462, 0.043) \\ & \text{and } i(t) \text{ is } T_{31}(-0.630, 0.922) \\ & \text{THEN } \dot{X}(t) = A_1^l X(t) + B_1^l u(t) \\ R^2: \quad & \text{IF } x(t) \text{ is } T_{12}(-0.000247, 0.310) \\ & \text{and } i(t) \text{ is } T_{32}(0.748, 0.002) \\ & \text{THEN } \dot{X}(t) = A_2^l X(t) + B_2^l u(t) \\ R^3: \quad & \text{IF } i(t) \text{ is } T_{33}(-0.128, 0.942) \\ & \text{THEN } \dot{X}(t) = A_3^l X(t) + B_3^l u(t) \end{aligned} \tag{22}$$

where $X(t) = [x(t), \dot{x}(t), i(t)]^t$ and $u(t) = e(t)$; $T_{ji}(m_{ji}, \sigma_{ji}) = \exp[-(x' - m_{ji})^2/\sigma_{ji}^2]$; $A_1^l = \begin{bmatrix} 0 & 1 & 0 \\ 33.53 & 0 & 15.74 \\ 0 & 0 & -47.67 \end{bmatrix}$,

$$A_2^l = \begin{bmatrix} 0 & 1 & 0 \\ 130.89 & 0 & -0.1 \\ 0 & 0 & -47.67 \end{bmatrix}, A_3^l = \begin{bmatrix} 0 & 1 & 0 \\ 4250.62 & 0 & -17.63 \\ 0 & 0 & -47.67 \end{bmatrix},$$

and $B_1^l = B_2^l = B_3^l = \begin{bmatrix} 0 \\ 0 \\ 1.7921 \end{bmatrix}$; and $Y = CX$ for each rule. Each fuzzy subsystem is c.c. and c.o. By Proposition 1,

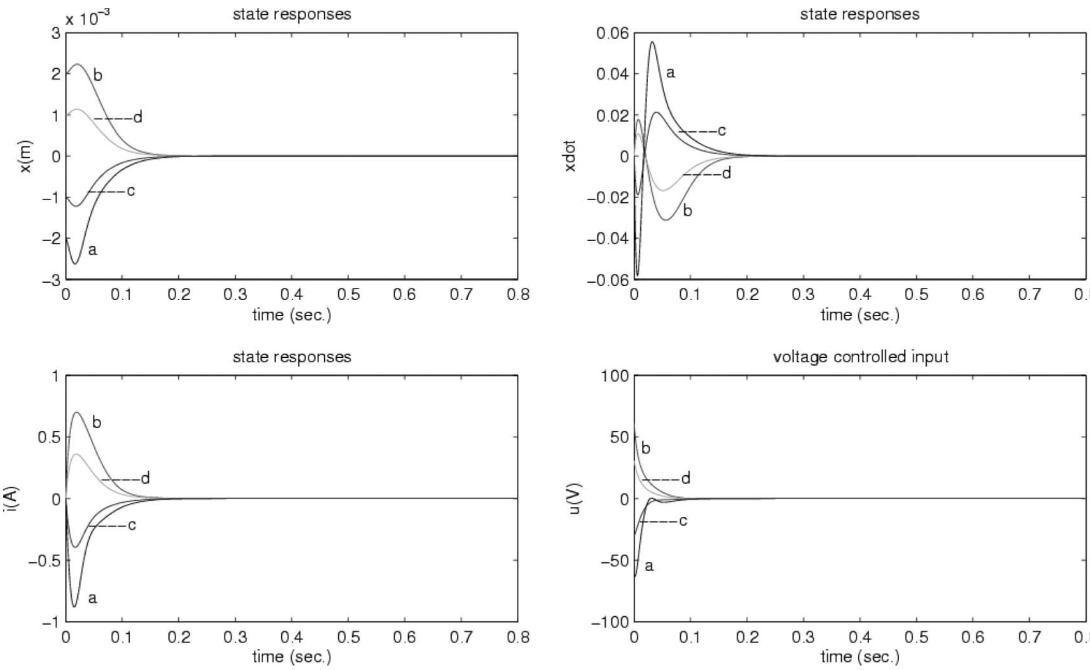

Fig. 10. State responses and optimal control voltage for the linear-type closed-loop voltage-controlled MagLev system with four initial conditions. (a) $X(0) = (-0.002, 0, 0)^t$. (b) $X = (0.002, 0, 0)^t$. (c) $X = (-0.001, 0, 0)^t$. (d) $X = (0.001, 0, 0)^t$.

we have the corresponding SSRE solution and the linear-type optimal fuzzy controller

$$\bar{\pi}_1^{lv} = \begin{bmatrix} 1399.2 & 241.4 & 71 \\ 241.4 & 41.7 & 12.3 \\ 71 & 12.3 & 3.6 \end{bmatrix}$$

$$\bar{\pi}_2^{lv} = 10^5 \cdot \begin{bmatrix} 3120.1 & 272.7 & -0.5 \\ 272.7 & 23.8 & 0 \\ -0.5 & 0 & 0 \end{bmatrix}$$

$$\bar{\pi}_2^{lv} = 10^3 \cdot \begin{bmatrix} 7081.4 & 108.6 & -17 \\ 108.6 & 1.7 & -0.3 \\ -17 & -0.3 & 0 \end{bmatrix}$$

$e^*(t) = u^*(t) = \sum_{i=1}^{3} h_i(X^*(t)) r_i^*(t)$, where $r_i^*(t) = -B_i^{l^t} \bar{\pi}_i^{lv} X^*(t)$. Fig. 10 shows the simulation results for four initial conditions: $X(0) = (-0.002, 0, 0)^t, (0.002, 0, 0)^t, (-0.001, 0, 0)^t$, and $(0.001, 0, 0)^t$.

Comparing Figs. 9 and 10, we find that both approaches show equivalent performance for a voltage-controlled MagLev system. We also examine the robustness of these two (affine-type and linear-type) closed-loop MagLev systems. Fig. 11 shows the evolution of the disturbance-interrupted ball position, the used current, and the control voltage. We note that no obvious difference exists for these two types of closed-loop fuzzy voltage-controlled MagLev system.

D. Stability Analysis

The used neural–fuzzy networks (Figs. 4 and 5) learn systems' behavior on the basis of minimizing the square error between training data and predicted values. To further solve the modeling failure from insufficient training data, an integrated neural–fuzzy modeling and controlling algorithm (Fig. 1) with modeling error index $I_M = ((\hat{Y}_{cl}(t) + \theta)/(Y_{cl}(t) + \theta))$ is proposed. This index is constrained to ensure admissible deviation of the real physical closed-loop system in Fig. 2 from the fuzzy closed-loop system. In other words, this algorithm guarantees that the generated affine/linear T–S fuzzy model can almost perfectly approximate MagLev systems. Therefore, the stability of the closed-loop physical system in Fig. 2 is guaranteed by the following closed-loop fuzzy systems:

$$\begin{aligned} \dot{X}^*(t) = &\sum_{i=1}^{r} h_i\left(X^*(t)\right) \\ &\times \left[\left(A_i - B_i S^{-1} B_i^t \bar{\pi}_i\right) \right. \\ &\quad \left. \times X^*(t) + B_i \bar{r_i^s} + D_i\right] \quad \text{(affine)} \end{aligned} \tag{23}$$

$$\begin{aligned} \dot{X}^*(t) = &\sum_{i=1}^{r} h_i\left(X^*(t)\right) \\ &\times \left[A_i - B_i S^{-1} B_i^t \bar{\pi}_i\right] X^*(t) \quad \text{(linear)}. \end{aligned} \tag{24}$$

We regard $\bar{X}_i^s$ as an artificial target and $\bar{U}_i^{\text{art}} = B_i \bar{r_i^s} + D_i$ as an artificial target associated constant input. Then, the stability of an affine-type feedback fuzzy system in (23) concurs with a linear-type system in (24). We define the Lyapunov function $V(X) = X^t P X$, where P is a symmetric positive matrix. Via (8), we have $A_i - B_i B_i^t \bar{\pi}_i = -\bar{\pi}_i^{-1}(L + A_i^t \bar{\pi}_i)$. Therefore

$$\begin{aligned} \dot{V}(X) = &\dot{X}^t P X + X^t P \dot{X} \\ = &\left[\sum_{i=1}^{r} h_i\left(X(t)\right) X^t \left(A_i - B_i B_i^t \bar{\pi}_i\right)^t\right] P X + X^t P \\ &\cdot \left[\sum_{i=1}^{r} h_i\left(X(t)\right) \left(A_i - B_i B_i^t \bar{\pi}_i\right) X\right] \end{aligned}$$

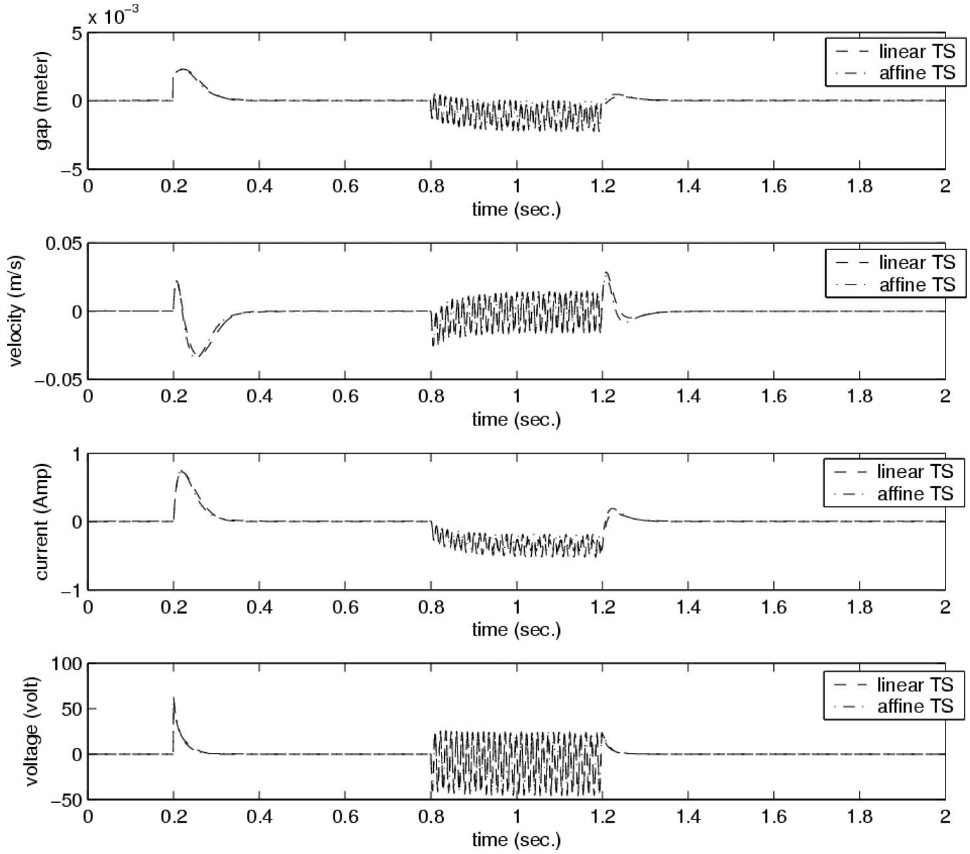

Fig. 11. Disturbance-interrupted response for a voltage-controlled MagLev system compensated by affine- and linear-type optimal fuzzy controllers, where an external disturbance is added at 0.2 s, and a sinusoidal disturbance is added between 0.8 and 1.2 s.

$$= -\sum_{i=1}^{r} h_i\left(X(t)\right)\left\{X^t\left[\left(L + A_i^t\bar{\pi}_i\right)^t\bar{\pi}_i^{-1}P + P\bar{\pi}_i^{-1} \cdot \left(L + A_i^t\bar{\pi}_i\right)\right]X\right\}$$

$$= -2\sum_{i=1}^{r} h_i\left(X(t)\right)\left[X^tP\bar{\pi}_i^{-1}\left(L + A_i^t\bar{\pi}_i\right)X\right]. \quad (25)$$

Furthermore, by choosing $P = I > 0$, we obtain

$$\dot{V} = -2\sum_{i=1}^{r} h_i\left(X(t)\right)\left[X^t\bar{\pi}_i^{-1}\left(L + A_i^t\bar{\pi}_i\right)X\right] < 0 \quad (26)$$

for $\bar{\pi}_i^{-1}(L + A_i^t\bar{\pi}_i) > 0$, since $h_i(X(t))$ is always a positive number.

Via the aforementioned Lyapunov-based stability analysis, we know that if A_i is nonsingular, (A_i, B_i) is c.c., (A_i, C) is c.o., and $\bar{\pi}_i^{-1}(L + A_i^T\bar{\pi}_i) > 0$ for each rule, then the designed closed-loop system is stable. The simulation results in Figs. 6, 7, 9, and 10 show that the proposed controllers can efficiently stabilize the physical systems in a very short time span. Furthermore, we know that a fuzzy approach possesses some degree of robustness. We therefore simulate the tendency of the closed-loop systems to be subject to an abrupt external disturbance. From the simulation results in Figs. 8 and 11, we conclude that the proposed controllers can efficiently stabilize the physical systems in a very short time span. In addition, an affine-type controller possesses better control and robust performance for current-controlled MagLev systems. We now compare our results with the methods contained in the current research. Fig. 12 is the air-gap evolution for a MagLev system as controlled by Mizutani's controller [38] and Muthairi's controllers [13]. Not only does our controller possess a rapid response, but it also has a smooth trajectory.

IV. Conclusion

Two neural–fuzzy inference networks are used to capture the dynamic behavior of both current- and voltage-controlled MagLev systems. The proposed affine-type controller and our previously proposed controller are, respectively, integrated with these two neural–fuzzy networks into integrated neural–fuzzy modeling and controlling algorithms to achieve control of both MagLev systems with limited modeling error and guaranteed performance. A current-controlled MagLev system is more complex than a voltage-controlled system. An affine T–S fuzzy model possesses one more freedom than the linear T–S type during the neural–fuzzy learning process. Therefore, the performance of the affine type is better than the linear type for current-controlled MagLev systems but is equivalent for voltage-controlled systems. This phenomenon is demonstrated in simulation results. Via simulation, we also observe that the proposed closed-loop MagLev systems are entirely robust to an external disturbance. In this paper, we have demonstrated the stability of the controllers. In the future, we shall theoretically

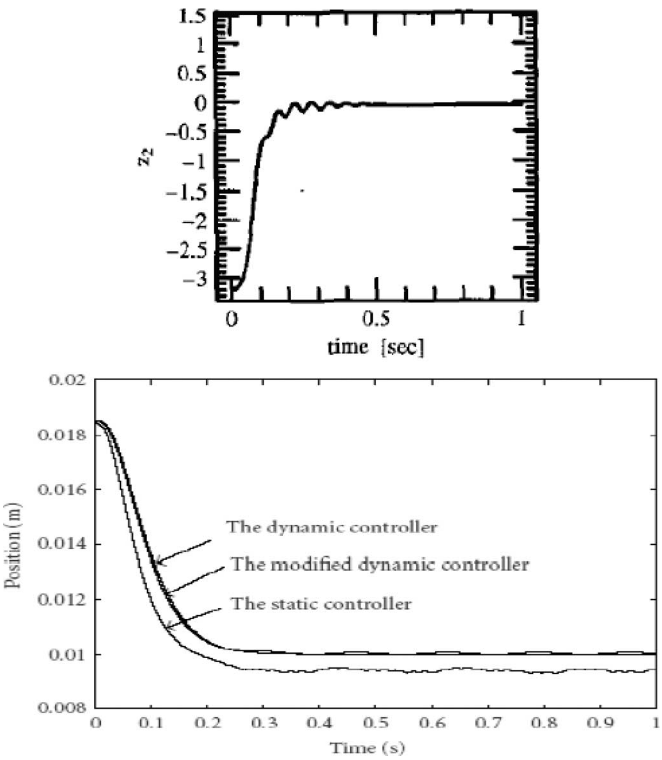

Fig. 12. Air-gap evolution for a voltage-controlled MagLev system as controlled by Mizutani's controller (top) [38] and Muthairi's controllers (bottom) [13].

demonstrate the robustness of the proposed controllers and the stability of the used inference networks.

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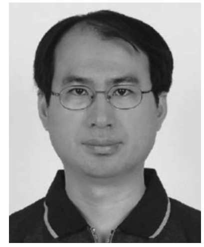

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