



PII: S0038-1098(98)00201-4

CRITICAL CURRENT OF A BALLISTIC ASYMMETRIC SNSNS JUNCTION

Victor C.Y. Chang and C.S. Chu *

Department of Electrophysics, National Chiao Tung University, Hsinchu 30050, Taiwan,
Republic of China*(Received 20 March 1998; accepted 15 April 1998 by A.H. MacDonald)*

The critical current through an asymmetrically stacked, double superconductor–normal-metal–superconductor (SNS) junction is studied in the mesoscopic and ballistic regime. The asymmetry in such SNSNS junctions excludes any intuitive choice for the phase ϕ_2 of the middle superconductor S_2 . We propose to determine ϕ_2 from the condition that the currents in the two normal regions are the same. The Bogoliubov-de Gennes equation is solved analytically within the Andreev approximation. In the long S_2 limit, we find that the current-phase relation (CPR) can be constructed directly from the CPRs of the two constituent SNS junctions, of which the longer SNS junction imposes a cutoff to the critical current. In short S_2 cases, the cutoff feature remains, but the cutoff value is affected by the Andreev-level tunneling. The dependencies of the SNSNS critical current on the length of S_2 , and the asymmetry in the configuration are studied. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: A. nanostructures, A. superconductors, D. electronic transport, D. tunneling.

Due to the progress of microtechnology, the nanostructures consisting of both normal metal (N) and superconductor (S) have caused renewed interests in the study of superconducting junctions. Numerous efforts, both theoretical and experimental, are devoted to explore such superconducting mesoscopic systems, where not only the Cooper pairs in S regions but also the quasiparticles in N regions are coherent. The interplay between these two kinds of coherence induces peculiar transport properties in the mesoscopic regime. For example, the critical current fluctuation in a dirty mesoscopic SNS junction [1, 2] and the critical current, which is quantized, in a ballistic superconducting quantum point contact (SQPC) [3, 4], are of entirely different nature from their classical counterparts, of which the transverse dimension W are much greater than the Fermi wavelength λ_F .

On the other hand, these superconducting mesoscopic features can find their analogous phenomena in the corresponding mesoscopic normal systems. The

quantization in the critical current of an SQPC is analogous to the quantized conductance of a normal QPC, which width $W \sim \lambda_F$. This issue of analogy has prompted recent study of symmetric SNSNS junctions via an effective Ginzburg-Landau model [5] or a microscopic approach [6]. It is found that the analogy with resonant transmission is marginal [5], but the feature associated with Andreev-level tunneling is prominent [6]. The Andreev levels are current-carrying states formed in the junction. Each Andreev level is associated with a quasiparticle process — either a right-going electronlike quasiparticle or a right-going holelike quasiparticle.

Recently, there are many experiments probing the physical characteristics of Andreev reflection and Andreev levels. For instance, the nonsinusoidal current-phase relation (CPR) for an atomic-size SQPC has been observed in a mechanical break junction formed out of a superconducting loop [7]. This nonsinusoidal CPR can be understood as the changes in an Andreev level of the process type it associates with [6]. In ad-

* To whom all correspondence should be sent.

dition, the resonant tunneling feature of a NSNSN structure has been observed and is attributed to the existence of (quasi-)bound Andreev levels formed between the superconducting barriers [8]. All these results demonstrate the experimental realization of mesoscopic superconducting structures and the success in understanding the features of these structures using Andreev reflections.

In this paper, we study an asymmetric SNSNS junction of which the lengths of the two normal regions are different. We also consider the case when the energy gap of the middle superconductor is different from that of the superconducting end-electrodes. For a symmetric SNSNS junction, an intuitive choice for the phases of the superconductors is that the phase differences between consecutive superconductors are the same. For an asymmetric SNSNS junction, however, no intuitive choice exists, and the only thing that we are certain is that the phase ϕ_2 of the middle superconductor (S_2) is a function of the phase difference ϕ between the two superconducting end-electrodes.

We need a physical condition that allows us to find $\phi_2(\phi)$. Towards this end, we note that the local current in the normal regions of the junction can be calculated from a microscopically derived expression (see below). The current depends explicitly on both ϕ and ϕ_2 . But for arbitrary ϕ_2 , the current in the two normal regions of the junction can be different, even though physically they should be the same. This shows that ϕ_2 cannot be arbitrary for a given ϕ . Instead, equating the currents in the two normal regions is the physical condition that we should impose in order to determine $\phi_2(\phi)$. From this we can obtain the transport characteristics of asymmetric SNSNS junctions.

For junctions in which S_2 is long, we find that the CPR of the junctions can be constructed directly from the CPRs of the two constituent SNS junctions. This simply demonstrates that the SNSNS junctions behave like two independent SNS junctions connected in series. In addition, the SNS junction that has the longer N region imposes a cutoff to the critical current of the entire junction. For junctions in which S_2 is short, the cutoff feature remains. But because of the Andreev-level tunneling, the two SNS junctions are no longer independent. Consequently, the cutoff-current value is changed. These cutoff features, together with their dependencies on the length of S_2 and the asymmetry in the configuration, manifest directly in the critical current I_c of the asymmetric SNSNS junctions.

The pair potential for the asymmetric $S_1N_1S_2N_2S_3$ junction is given by

$$\Delta(x) = \begin{cases} \Delta_1 e^{i\phi_1}, & x < 0 \\ 0, & 0 < x < L_1 \\ \Delta_2 e^{i\phi_2}, & L_1 < x < L_1 + L_2 \\ 0, & L_1 + L_2 < x < L_1 + L_2 + L_3 \\ \Delta_1 e^{i\phi_3}, & x > L_1 + L_2 + L_3, \end{cases} \quad (1)$$

where the energy gap magnitudes Δ_1 of the two superconducting end-electrodes are the same while the energy gap Δ_2 of the middle superconductor can be different. Here we consider the case $\Delta_2 \leq \Delta_1$. The quasiparticles of the system are described by the Bogoliubov-de Gennes (BdG) equation

$$\begin{bmatrix} H(x) & \Delta(x) \\ \Delta^*(x) & -H^*(x) \end{bmatrix} \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} = E \begin{bmatrix} u(x) \\ v(x) \end{bmatrix}, \quad (2)$$

where $H(x) = p_x^2/2m - \mu$ is the single-electron hamiltonian, and μ , the chemical potential, is assumed to be the same throughout the structure.

For quasiparticles with energies $E > \Delta_1$, they are scattering states resulting from quasiparticles incident from one of the superconducting end-electrodes. These quasiparticles can be electronlike or holelike. The scattering states are obtained by matching the wavefunctions at the SN interfaces. By invoking the Andreev approximation, we obtain the analytic expressions for the scattering coefficients [6].

For $E \leq \Delta_1$, the quasiparticles are confined in between the two end-electrodes and their energies are quantized. Furthermore, when $E < \Delta_2$, the quasiparticles are confined within, but can tunnel between, the two normal regions. We have obtained the analytic expression for the quantization conditions for these bound states. There are two kinds of such bound states, according to the processes involved. The p -process bound state is set up by right-going electronlike quasiparticles and the n -process bound state is set up by right-going holelike quasiparticles [9].

The supercurrent can be obtained using the current density expression

$$j(x) = \frac{e}{m} \sum_l \{ f(E_l) u_l^*(x) \hat{p}_x u_l(x) + [1 - f(E_l)] \times v_l(x) \hat{p}_x v_l^*(x) \} + \text{c.c.}, \quad (3)$$

where l refers to all the quasiparticle states, with $E_l > 0$, and with the wavefunctions given by $[u_l(x), v_l(x)]^T$. Here $e = -|e|$, and $\hat{p}_x = -i\hbar d/dx - (e/c)A_x(x)$. The vector potential $A_x(x) = 0$ in our case. The function $f(\epsilon) = [1 + \exp(\epsilon/k_B T)]^{-1}$ is the Fermi function. In this paper, the temperature T is taken to be zero. In one dimension, the current density becomes the current. Since equation (3) is local, containing the location at which the current is to be evaluated, we can calculate the currents in the two normal regions of the asym-

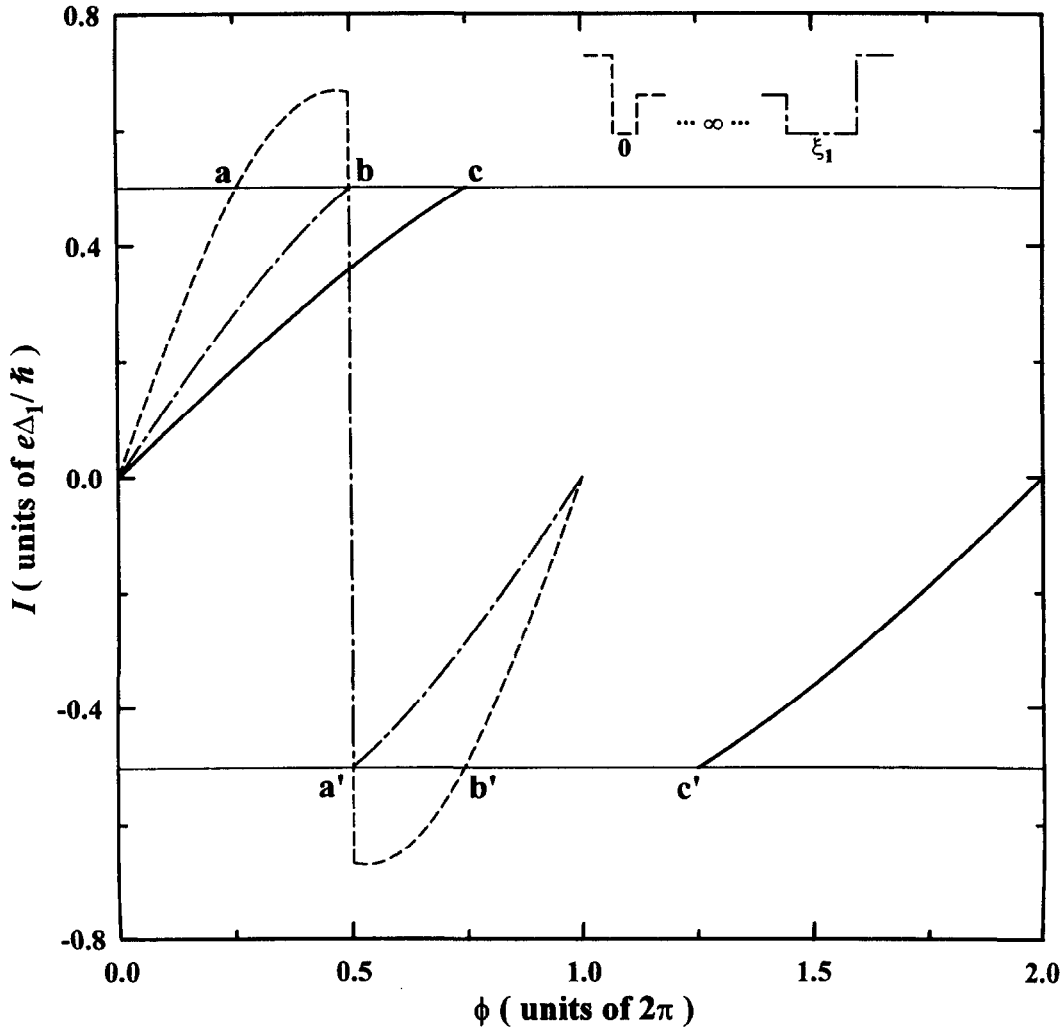


Fig. 1. I - ϕ relation of a long L_2 asymmetric SNSNS junction, and of its constituent SNS junctions. The inset shows the physical configuration of the junction. The two horizontal light lines highlight the cutoff feature. $L_2 = 10\xi_1$ is shown long enough for the junction to behave like two serially connected SNS junctions.

metric SNSNS junction. As mentioned before, we can determine ϕ_2 for a given ϕ by equating the currents in the two normal regions.

In Fig. 1, the CPR of an asymmetric SNSNS junction that has a long S_2 is represented by the solid curve. The configuration of the junction is $(L_1, L_2, L_3) = (0, 10, 1) \xi_1$, and $\Delta_2/\Delta_1 = 0.5$. Here ξ_1 is the coherence length of the superconducting end-electrodes. The CPRs of the asymmetric SNS junctions with $L_N = 0$ (the dashed curve) and ξ_1 (the dashed-dotted-dashed curve) are presented to illustrate their connection with the CPR of the entire SNSNS junction. If the two SNS junctions were independent, but serially connected, we would expect that, for a given current flowing through the system, the total phase difference across the entire junction is the sum of the phases across each SNS junction. We find that our graphs do conform to this

construction scheme. For example, $\phi_c = \phi_a + \phi_b$, and $\phi_{c'} = \phi_{a'} + \phi_{b'}$. Based on this scheme, the CPR branch for the entire junction that includes point c (c') can be constructed from the CPRs of the constituent SNS junctions that include points a and b (a' and b'). Thus, we conclude that the long S_2 asymmetric junction is equivalent to two independent SNS junctions connected in series. Based on this understanding, the cutoff feature in the CPR comes about naturally.

The cutoff feature, indicated by the two horizontal light lines, can be understood as the bound to the magnitude of the supercurrent that can flow in the junction. The critical currents of the two constituent SNS junctions are different, but the bound to the current in the entire junction is determined by the SNS junction that has the longer normal region. Hence the two horizontal light lines coincide with the smaller critical

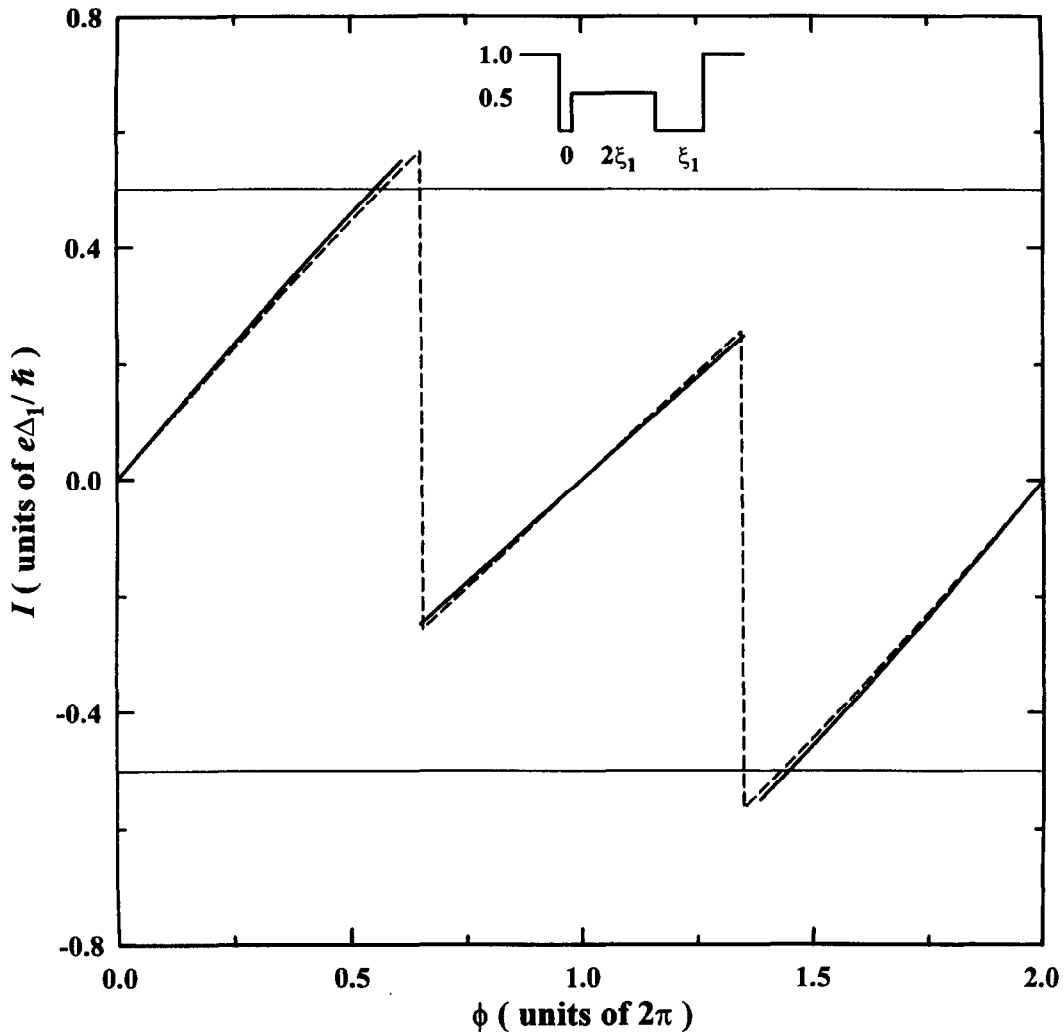


Fig. 2. I - ϕ relation of an asymmetric SNSNS junction with intermediate length for S_2 ($L_2 = 2\xi_1$). The two horizontal lines denote the critical current of the constituent SNS junction that has the longer L_N .

current of the constituent SNS junctions.

In Fig. 2, we show the CPR of an asymmetric SNSNS junction that has a shorter S_2 , where $L_2 = 2\xi_1$. The other configuration parameters are the same as in Fig. 1. The CPR is denoted by the solid curve and the CPR for the corresponding symmetric SNSNS junction, which configuration $(L_1, L_2, L_3) = (0.5, 2, 0.5)\xi_1$ and the total length L_{Total} in the normal regions is the same, is denoted by the dashed curve.

The CPR of the symmetric junction has two abrupt-current-change features, which can be shown to be the consequence of Andreev-level tunneling [6]. These two abrupt-current-change features approach each other as the length L_2 of the middle superconductor increases. In the large L_2 limit, the two abrupt-current-change features merge into one. The two abrupt-current-change features are well separated in Fig. 2, suggesting that the coupling between the

two constituent SNS junctions are important. Such coupling affects also the cutoff-current values of the asymmetric SNSNS junction. In this case, the cutoff-current values of the asymmetric junction is larger in magnitude than the critical current of the longer constituent SNS junction, which is indicated by the two horizontal lines. Except for the cutoff features in the vicinity of the abrupt-current-change features, the CPRs for the two junctions are almost the same. Since we find also that the CPR has a greater sensitivity to the changes in L_2 or L_{Total} , our results demonstrate that the CPR is least sensitive to the junction configuration change that has kept both L_2 and L_{Total} constant. The CPR of an asymmetric SNSNS junction can thus be obtained approximately by imposing a cutoff to the CPR of its corresponding symmetric junction. The sensitivities of the cutoff-current, or equivalently, the critical current, to L_2 , Δ_2/Δ_1 , and

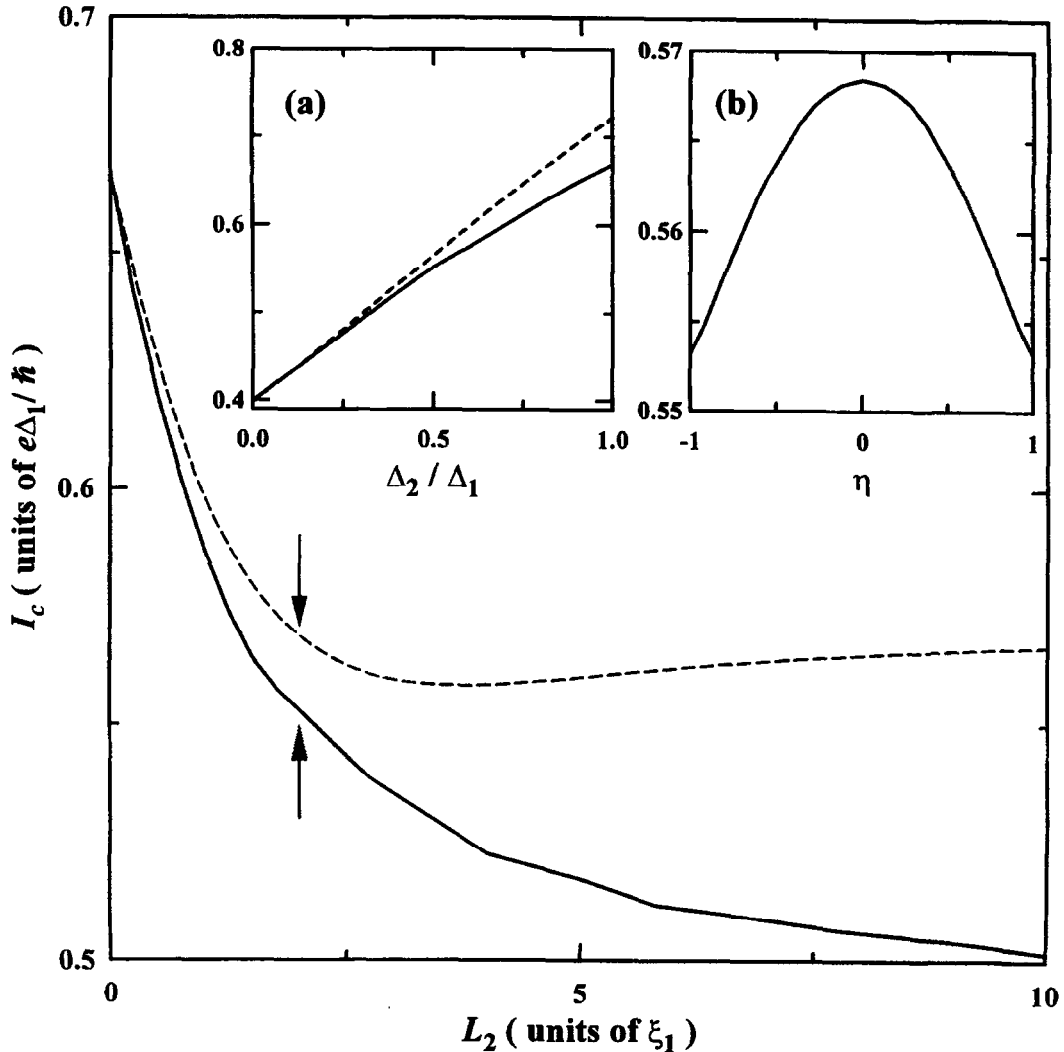


Fig. 3. Critical current I_c versus L_2 for both symmetric and asymmetric SNSNS junctions. The asymmetric case is denoted by the solid curve for $L_1 = 0$, $L_3 = \xi_1$, and $\Delta_2/\Delta_1 = 0.5$. The results for the corresponding symmetric junction is denoted by the dashed curve. Detail analysis for the structures indicated by two arrows is presented in two insets. Inset (a): I_c versus Δ_2/Δ_1 for the structures $(L_1, L_2, L_3) = (0.5, 2.0, 0.5) \xi_1$ and $(0.0, 2.0, 1.0) \xi_1$, which are represented by the dashed and the solid curves, respectively. Inset (b): I_c versus the junction asymmetry with respect to the position of S_2 . $\eta = (L_1 - L_3)/L_{\text{Total}}$ and $L_{\text{Total}} = L_1 + L_3$ is kept constant.

the asymmetry of the junction are presented in the following.

In Fig. 3, the dependence of the critical current I_c on L_2 is denoted by the solid curve. The configuration parameters are the same as in Fig. 1, except that L_2 varies from 0 to $10\xi_1$. For comparison, the corresponding symmetric case is presented by the dashed line. The two curves start at the same value at $L_2 = 0$. This is reasonable because the value corresponds to the critical current of a symmetric SNS junction that has $L_N = L_{\text{Total}} = \xi_1$. However, the two curves deviate from each other monotonically as L_2 increases, and they approach to different limits. The I_c of the asymmetric junction saturates to that of an asymmet-

ric SNS junction that has $L_N = \xi_1$, whereas the I_c of the symmetric junction saturates to that of the asymmetric junction that has $L_N = 0.5\xi_1$. We conclude that the cutoff feature is quite sensitive to L_2 .

Next, in inset (a), the critical currents of two junctions — symmetric $(0.5, 2.0, 0.5) \xi_1$ and asymmetric $(0.0, 2.0, 1.0) \xi_1$ — are shown as a function of the ratio Δ_2/Δ_1 . The former is denoted by the dashed line and the latter is denoted by the solid line. When $\Delta_2 = 0$, the critical currents are reduced to that of the symmetric SNS junction that has $L_N = L_{\text{Total}}$. As shown in the figure, we have the maximum discrepancy when $\Delta_2 = \Delta_1$. We conclude that the higher the Δ_2/Δ_1 ratio is, the more obvious the cutoff feature will be.

Finally, inset (b) in Fig. 3 shows the dependence of the critical current on the asymmetry in the geometrical aspect. Such asymmetry is defined by a dimensionless parameter $\eta \equiv (L_1 - L_3)/L_{\text{Total}}$. When η goes from -1 to 1 , S_2 moves from the left-most position to the right-most position in between the two superconducting end-electrodes. The other configuration parameters are the same as in Fig. 1. The symmetric configuration has the largest critical current, and the deviation in I_c is much smaller than that in the previous figures.

In summary, we have studied the critical current through an asymmetric SNSNS junction in the mesoscopic and ballistic regime. A physical condition is proposed to determine $\phi_2(\phi)$. In contrast to its symmetric counterparts, an asymmetric SNSNS junction has a cutoff feature in the CPR. The cutoff value is the critical current of the junction. The sensitivity of the critical current to the junction configuration has been explored. Our approach can be extended to explore the cases including impurities and finite-temperature effects. The features and the method proposed in this paper can also be extended to the study of superconducting superlattices that have asymmetric unit cells.

Acknowledgements—This work was supported in part by the National Science Council of the Republic of China through Contract No. NSC87-2112-M-009-007. Computational facilities supported by the National Center for High-performance Computing are gratefully acknowledged.

REFERENCES

1. Al'tshuler, B.L. and Spivak, B.Z., *Sov. Phys. JETP*, **65**, 1987, 343.
2. Beenakker, C.W.J., *Phys. Rev. Lett.*, **67**, 1991, 3836.
3. Beenakker, C.W.J. and van Houten, H., *Phys. Rev. Lett.*, **66**, 1991, 3056.
4. Furusaki, A., Takayanagi, H. and Tsukada, M., *Phys. Rev. Lett.*, **67**, 1991, 132.
5. Zapata, I. and Sols, F., *Phys. Rev.*, **B53**, 1996, 6693.
6. Chang, V.C.Y. and Chu, C.S., *Phys. Rev.*, **B55**, 1997, 6004. In this paper, the scattering coefficients for the symmetric SNSNS junction is presented. The scattering coefficients for the asymmetric junction can be obtained similarly.
7. Koops, M.C., van Duyneveldt, G.V. and de Bruyn Ouboter, R., *Phys. Rev. Lett.* **77**, 1996, 2542.
8. Morpurgo, A.F., van Wees, B.J., Klapwijk, T.M. and Borghs, G., *Phys. Rev. Lett.* **79**, 1997, 4010.
9. Kashiwaya, S., Tanaka Y., Koyanagi M. and Kajimura K., *Jpn. J. Appl. Phys.*, **34**, 1995, 4555.