行政院國家科學委員會專題研究計畫成果報告

以遞迴式反向運算元設計變週期之反覆學習控制器

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一、中文摘要

本計劃針對系統遭受緩慢變動週期之窄頻雜訊，研究一套利用回授方式的控制方法，以消除雜訊。對於固定週期之窄頻雜訊，反覆控制(Repetitive Control)或學習控制(Learning Control)已經提供相當多的研究，然而面對雜訊週期的變化，往往無法有效的產生相對應的控制訊號。尤其是以數位控制實現時，常常有雜訊週期非採樣週期整數倍的情況，導致數位週期與實際週期無法匹配。有別於傳統反覆控制對問題的解決方式，本計劃將提出以遞迴式反向運算元(Iterative Inverse Operator)為出發點的控制訊號合成機制，以一般型延遲濾波器(general delay filter)取代較為簡單的延遲運算元，進而探討以調整延遲時間來因應週期變化的機制，而達到變週期窄頻雜訊消除的結果。

關鍵詞：反覆控制、學習控制

Abstract

A constructive derivation of repetitive control is obtained, through attempting to derive a control law for asymptotic rejection of periodic disturbances. This derivation suggests a unified design method for a learning control algorithm. Also, based on the observation, digital repetitive control can be generalized to reject periodic disturbance whose period is not exactly an integer multiple of the sampling interval. This study introduces a delay filter in the digital repetitive control law, which optimally interpolates the signal between samples, thus effectively reconstructing the signal of the previous period and making the learning process of repetitive control successful. The simulations on active noise cancellation within a duct confirm the superiority of this tuning method.

Keywords: Repetitive Control, Delay Operator, Iterative Learning

二、緣由與目的

Repetitive control is effective in asymptotic tracking or rejection of periodic signals [1,2]. It consists of a wide variety of applications, such as noncircular cutting [3], disk drive tracking [4], and active noise cancellation [5]. Repetitive control was explained and proved by the internal model principle (see [1,2]). This work, however, proposes a constructive derivation based on the operator theory. First, the disturbance rejection problem is formulated as a question for solving an operator equation. The derivation begins with the iterative inversion of an operator by the Neumann series, which results in a Neumann series solution as well as a sufficient existence condition for an operator inversion problem. Additionally, this solution can be alternatively represented by successive iteration of an equation, which provides a good insight into deriving the learning algorithm for an operator equation.

Repetitive control repeatedly generates the present control force $u(t)$ by learning from the previous period of the control force $u(t-T)$ and the tracking or disturbance rejection error $\mathring{a}(t-T)$. However, in a discrete-time system where only the sampled signals are given, $u(t-T)$ and $\mathring{a}(t-T)$ are unavailable unless that the signal period T is exactly an integer multiple of the sampling interval. When the signal period is precisely known and fixed, integer multiple condition can be easily achieved. However, it becomes a difficult task when the signal period varies. There are two types of methods for solving this problem. The first method (see [6,7]) uses $u(t-NT_s)$ and $\mathring{a}(t-NT_s)$ to approximate $u(t-T)$ and $\mathring{a}(t-T)$, respectively, where NT_s is the nearest integer multiple of sampling interval to the signal period T. The second method (see [7,8]) alters the sampling rate on-line while maintaining a fixed controller. The disadvantage of the first tuning scheme is the inevitable period mismatch due to the roundoff of the actual signal period. This mismatch may result in undesirable remaining oscillating errors, thus deteriorating the steady-state performance. As for the second tuning scheme, the most serious problem is that changing the sampling rate without changing the controller can affect system robustness, and even cause instability.

Based on idea of the delay operator, this study attempts to provide an alternative method for a discrete-time repetitive controller. With the fixed sampling interval, a delay filter, which aims to optimally interpolate $u(t\text{-}T)$ and $\mathring{d}(t-T)$ between samples, is introduced to enhance the steady-state performance. According to distinct signal periods, this optimal filter can be updated via only a small amount of computations. This delay filter tuning method is applied to active noise cancellation within a duct. Simulations illustrate the effective enhancement of the steady-state performance.

三、結果與討論

Two main results are obtained in the following. Let **H** be a linear vector space of complex-valued functions $x(t)$, $0 \le t < \infty$, spanned by $\{e^{\hat{j}ù\, t},\ ù \in \mathbf{R}\}$, with the norm

$$\|x\| = \left(\lim_{T\to\infty} \frac{1}{T}\int_0^T |x(t)|^2\, dt \right)^{1/2} < \infty \qquad (1)$$

and the inner product

$$< x, y >= \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\overline{y(t)}dt, \quad \forall x, y \in \mathbf{H} \qquad (2)$$

where $\overline{y(t)}$ represents the complex conjugate of $y(t)$. let **M** be a collection of all such signals:

$$\mathbf{M} = \left\{ x:\ \lim_{T\to\infty} \frac{1}{T}\int_0^T |x(t)|^2\, dt = 0 \right\} \qquad (3)$$

It is easy to show that **M** is a closed set. Thus, the union of **H** and **M**, denoted by $\mathbf{H} \cup \mathbf{M}$, makes a new complete linear vector space, which includes both transient and almost periodic functions. Suppose that two functions x, $y \in \mathbf{H} \cup \mathbf{M}$ are said to belong to the same class if the difference $x\text{-}y$ belongs to **M**, then the set of all such classes is called the quotient space of $\mathbf{H} \cup \mathbf{M}$ relative to **M**, represented by **E**. This quotient space **E** is a Hilbert space with the inner product defined in the following way: Given two elements of **E**, i.e., two classes $\hat{\imath}$ and $ç$, we choose a representative from each class, say x from $\hat{\imath}$ and y from $ç$, then

$$< \hat{\imath}, ç >= \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)\overline{y(t)}dt \qquad (4)$$

In a completely analogous manner, the norm can be defined as follows

$$\|\hat{\imath}\| = \left(\lim_{T\to\infty} \frac{1}{T}\int_0^T |x(t)|^2\, dt \right)^{1/2} \qquad (5)$$

where x is any representative from the class $\hat{\imath}$ in **E**. Let $e_{ù}$ be the class containing the function $e^{jù\, t}$ in **E**, the set $\{e_{ù},\ ù \in \mathbf{R}\}$ forms an orthonormal basis in **E**.

Similarly, linear transformations on **E** can also form a normed space. Let $\boldsymbol{L}$ (**E**) be a space of all LTI stable systems P on **E**, with the operator norm

$$\|P\| = \sup_{\|\hat{\imath}\|=1,\, \hat{\imath} \in \mathbf{E}} \|P\hat{\imath}\| \qquad (6)$$

The frequency-response function of an operator $P \in \boldsymbol{L}(\mathbf{E})$ is defined by

$$P(ù) =< Pe_{ù}, e_{ù} >, \qquad ù \in \mathbf{R} \qquad (7)$$

Proposition 1. (Solution to Asymptotic Rejection of Periodic Disturbances) For an operator $P \in \boldsymbol{L}$ (**E**), if another operator $C \in \boldsymbol{L}$ (**E**) exists, such that $\| I - CP \| < 1$, asymptotic rejection of periodic disturbances can be achieved for any periodic d with finite RMS norm, and the corresponding control signal class can be expressed in the following

$$\hat{\imath} = \left\{ u:\ u = -\sum_{n=0}^{\infty} (I - CP)^n Cd + x,\ \ \forall x \in \mathbf{M} \right\} \in \mathbf{E} \qquad (9)$$

This is known as the repetitive control law, which automatically generates u belonging to the solution class presented in Proposition 1. This solution-generating algorithm can be proved in a straightforward manner by applying the fixed-point theorem.

Proposition 2. (Repetitive Control Law) For an operator $P \in \boldsymbol{L}$ (**E**), if another operator $C \in \boldsymbol{L}$ (**E**) exists, such that $\| I - CP \| < 1$, asymptotic rejection of any periodic disturbance d with finite RMS norm, can be achieved by the control signal u generated by the following control law

$u(t) = [Du](t) - [CD\mathring{a}](t)$, for $t \in [0, \infty)$

where D denotes an delay operator, which performs a T-second delay with zero initial states, i.e., $(Du)(t) = u(t-T)$, and $u(t-T) = 0$ for $t - T < 0$, and $\mathring{a}$ denotes the error vector $Pu+d$. Additionally, u is bounded in the RMS sense.

The results are applied to digital repetitive control for active noise cancellation in ducts. The plant $P(z)$, which represents the acoustic dynamics in a $0.5 \times 0.15 \times 0.15\,\mathrm{m}$ duct as well as the dynamics of the cancellation speaker, amplifiers, and the microphone, is identified using time-domain least square algorithms with frequency weighting. In the low-frequency band, the plant has transmission zeros around 0 Hz and 700 Hz. The control bandwidth is set as 600 Hz. To stabilize the overall system, the magnitude of $D(e^{j\mu})$ is designed to roll off before the nodal frequency 700 Hz. The parameters $n = 10, \mu_0 = 0.1$, $\mu_1 = 0.1167$, $a = 0.01$ are selected. Thus, given the noise period, the filter $D(z)$ can be determined via the formula given in the proposed result. Figure 1 displays the frequency response of the optimal $D(z)$ with the desired delay of 50.4 (in the unit of sampling interval). Also, the FIR compensator $C(z)$ with the tap length 20 is designed such that the multiplication $P(z)C(z)$ has zero phase and is close to 1 in the least-squares sense. Figure 7 illustrates the design result, where $\left| D(e^{j\mu})\left(1 - C(e^{j\mu})P(e^{j\mu})\right) \right| < 1$ for any μ , except when $\mu = 0$. According to Nyquist stability criterion, the overall system is stable since the Nyquist locus does not encircle critical point $-1+0j$.

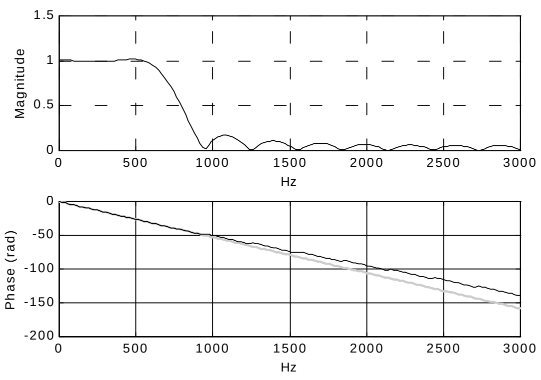

Figure 1. Frequency response of the lowpass delay filter $D(z)$ (the gray line in the phase diagram is the desired phases)

The noise period may alter as the motor or compressor is operated at different speeds. Two controller-tuning schemes are considered in the simulations. One is the integer delay tuning method proposed by Tsao and Nemani [7] and Hu [6], that is, adjusting the order of the repetitive controller according to the roundoff of the given noise period. The other is the fractional delay tuning method proposed herein. That is, as the noise period is altered, so too is the coefficient of the optimal delay filter. A periodic noise with five harmonic components is created. The noise period varied from 56.1 samples, to 45.56 samples, and finally to 50.67 samples. Figure 2 shows the simulation results for the assumption that the noise periods can be estimated, and that the repetitive controllers are updated for every 600 sampling intervals. As a steady-state noise-cancellation measure, cancellation-error to noise ratio (E/N) is defined by the RMS norm as follows

$$\mathrm{E/N} = 20\log_{10}\left(\frac{\|\text{cancellation error}\|}{\|\text{uncanceled noise}\|}\right)\ \mathrm{dB}$$

Table 1 indicates E/N values for the simulation results. Obviously, the fractional delay tuning method has superior performance over the integer delay tuning method. The remaining cancellation error for the integer delay tuning method is primarily attributed to the roundoff error of the noise period

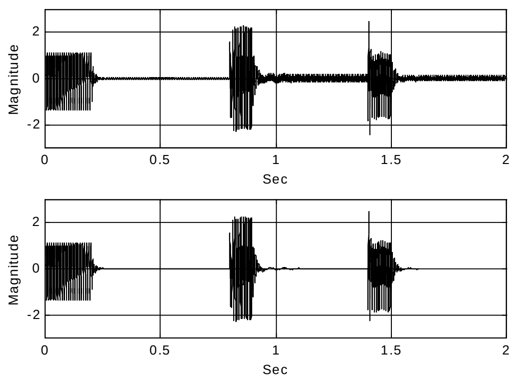

Figure 2. The cancellation error signals (above: the integer delay tuning method; below: the fractional delay tuning method)

Noise period	56.1	45.56	50.67
E/N for the integer delay tuning	-33.63 dB	-10.20 dB	-22.64 dB
E/N for the fractional delay tuning	-49.71 dB	-46.27 dB	-47.51 dB

Table 1

四、計畫成果自評

項 目	完成情況
與原計畫相符程度	100%
達成預期目標	90%
研究成果學術價值	新型控制器設計
研究成果應用價值	具實用性
學術期刊發表合適否	已發表
申請專利合適否	否
主要發現或其他價值	學習控制理論

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