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Bottleneck-based heuristic dispatching rule for optimizing mixed TDD/IDD performance in various factories

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Abstract Most research on scheduling problems focuses on increasing production efficiency. For instance, the shortest processing time (SPT) and earliest due date (EDD) dispatching rules perform well in minimizing mean flow time and reducing maximum tardiness, respectively. However, those indices ignore the financial impact (material cost and order price) on the factory. Previous studies focused mainly on cycle time and due date. However, the theory of constraint (TOC) considers not only the effect of time, but also financial factors. Therefore, TOC addresses the concepts of throughput-dollar-day (TDD) and inventory-dollar-day (IDD). The former index (TDD) represents penalties for tardy deliveries, while the latter index (IDD) refers to the material holding cost. Based on these two indices, this investigation creates a novel mixed TDD/IDD weighted value (Z_{value}) to replace the other traditional indices for taking measurements in various factories. This study also designs a heuristic dynamic scheduling algorithm (mixed TDD/IDD dispatching rule) for reducing the system Z_{value} . Some traditional dispatching rules are compared with the proposed rule in terms of TDD, IDD, and Z_{value} . Analytical results indicate that the mixed TDD/IDD dispatching rule is feasible and generally outperforms other conventional dispatching rules in terms of Z_{value} under various factories.

Keywords Heuristic · Scheduling · Dispatching · Performance · TDD · IDD

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1 Introduction

Most research considers the problem of scheduling for time-based improvements. For example, both shortest processing time (SPT) and earliest due date (EDD) dispatching rules can minimize the mean flow time and reduce the maximum tardiness, respectively [1]. However, those indices neglect the importance of financial factors. Although the mean flow time (or cycle time) and the tardiness (or due date) are two critical indicators for various factories [2, 3], the theory of constraint (TOC) regards them only as local optimum indices for the factory as a whole. TOC considers that financial factors should be also incorporated into performance measurement. Therefore, two performance indices, throughput-dollar-day (TDD) and inventory-dollar-day (IDD), are addressed here [4]. The former represents the punishment of tardy delivery and the latter refers to the inventory holding cost. TDD is the summation of all of the values of late orders that have not been shipped multiplied by the number of days of lateness. The TDD index is used to measure the reliability of product delivery. The higher the value of the summations, the worse the performance. The target for the summation of TDD is zero. Generally, adequate stocks can immediately satisfy consumer demand, and a lack of stock risks losing orders. Raw material is generally released into a system early to avoid late orders, but this policy causes higher work-in-progress (WIP) levels and increases flow time. Therefore, the IDD index attempts to determine inventory effectiveness. The definition of IDD is the summation of the value of all raw materials multiplied by the number of days of storage or processing. The target is not zero; however, we do our best to reduce the holding cost and the IDD value simultaneously.

Whereas the indices of TDD and IDD have the same scales of days and dollars, these two indices can be integrated into a single measurement. Additionally, various factories identify a significant difference in measurement between TDD and IDD. For instance, a make-to-order

(MTO) factory is extremely concerned with customer service to the extent that it pays more attention to tardiness cost than holding cost. First, the improvement of performance is focused on the measurement of TDD, and then IDD should be minimized. However, the opposite situation applies for the make-to-stock (MTS) factory. A MTS factory seeks to reduce the holding cost to improve the performance of IDD and, thus, reduce the value of TDD. Meanwhile, a make-to-assemble (MTA) factory focuses on reducing holding cost in former assembly periods and avoids tardy order delivery during latter shipment periods. Therefore, the MTA factory emphasizes that reducing IDD is as important as reducing TDD. Consequently, for various factories, this study adopts two weighted factors for adjusting the importance of measurement. The novel mixed TDD/IDD index is shown in Eq. (1):

$$Z_{value} = \alpha \left(\sum TDD_i \right) + \beta \left(\sum IDD_i \right) \quad (1)$$

where α and β are non-negative factors and $\alpha + \beta = 1$.

In the MTO factory, the value of the α factor should be significantly greater than the value of the β factor because the TDD index is more important than the IDD index to evaluate customer satisfaction. Conversely, for the MTS factory, we set $\beta > \alpha$. Based on the factory type, factors can be assigned to stress the importance of different cost ratios. Nevertheless, the question remains as to whether a proper dispatching rule exists for solving the problem of minimizing Z_{value} . Unfortunately, neither a proper procedure nor an algorithm can be guaranteed to adequately solve the complexity problem. Based on the literature review [12], the minimizing integration of ΣTDD_i and ΣIDD_i may be regarded as to solve the problem of minimizing the sum of weighted tardiness ($\Sigma W_i T_i$) and weighted flow time ($\Sigma W_i F_i$), if these weighted values in $\Sigma W_i T_i$ and $\Sigma W_i F_i$ represent the sales value of orders and the material cost, respectively [5–7]. (Restated, $\Sigma W_i T_i = \Sigma m_i T_i = \Sigma TDD_i$ and $\Sigma W_i F_i = \Sigma h_i F_i = \Sigma IDD_i$. Those notations are defined later in Section 2.) Although minimizing $\Sigma W_i T_i$ and $\Sigma W_i F_i$ is known as an NP-hard problem, the apparent tardiness cost (ATC) and weighted shortest processing time (WSPT) dispatching rules still mostly perform well, respectively, under the single-machine model [1, 8, 9]. Therefore, a heuristic dispatching rule is developed for minimizing Z_{value} according to the WSPT and ATC rules. In the meanwhile, Z_{value} also is decomposed and its composite element analyzed.

2 Mixed TDD/IDD-based dispatching rule

We use the notations as follows:

TDD_i	Throughput-dollar-day of order i
IDD_i	Inventory-dollar-day of order i
α	Adjustment factor of ΣTDD
β	Adjustment factor of ΣIDD
P_{ij}	Process time of order i on machine j
T_i	Tardiness of order i
m_i	Sale price of order i
h_i	Material cost of order i
W_i	Given weighted value of order i
C_i	Completion time of order i
d_i	Due date of order i
Id_{ij}	Idle time of order i on machine j
A_i	Start time of order i
S_i	Slack time of order i
F_i	Flow time of order i
ccr	Capacity constraint resource or bottleneck of system
TH_i	Throughput of order i , marginal contribution of order i , $TH_i = m_i - h_i$

This study assumes that k orders are processed on n machines without considering machine breakdown nor order preemption. Each machine can handle at most one order at a time. The tardiness of order i is defined as $T_i = \max(0, C_i - d_i)$, where C_i is the completion time of order i . The flow time refers to the time span from the order being processed until it is completed, and equals the processing time plus the idle time on each machine. For simplicity, all orders are assumed here to be available at the start of scheduling. That is, $A_i = 0$ for all orders. The machines and orders are assumed to be available from time 0. Therefore, the completion time and the flow time for order each are equal. According to the definition, the order completion time is formulated as follows:

$$\begin{aligned} F_i &= C_i \\ &= [(P_{i1} + Id_{i1}) + (P_{i2} + Id_{i2}) + \dots + (P_{in} + Id_{in})] \\ &= \sum_{j=1}^n P_{ij} + \sum_{j=1}^n Id_{ij} \end{aligned} \quad (2)$$

The slack time S_i equals $d_i - \sum_{j=1}^n P_{ij}$ and Z_{value} is decomposed as shown in Eq. 3:

$$\begin{aligned}
Z_{\text{value}} &= \alpha \left(\sum_{i=1}^k m_i T_i \right) + \beta \left(\sum_{i=1}^k h_i F_i \right) \\
&= \alpha \sum_{i=1}^k [m_i \times \max(0, C_i - d_i)] + \beta \sum_{j=1}^k h_i (C_i - A_i) \\
&= \alpha \sum_{i=1}^k \max \left[0, m_i \times \left(\sum_{j=1}^n P_{ij} + \sum_{j=1}^n Id_{ij} - d_i \right) \right] \\
&\quad + \beta \sum_{i=1}^k h_i \left(\sum_{j=1}^n P_{ij} + \sum_{j=1}^n Id_{ij} - A_i \right) \\
&= \alpha \sum_{i=1}^k \max \left\{ 0, m_i \times \left[\sum_{j=1}^n Id_{ij} - \left(d_i - \sum_{j=1}^n P_{ij} \right) \right] \right\} \\
&\quad + \beta \sum_{i=1}^k \left[h_i \times \left(\sum_{j=1}^n P_{ij} + \sum_{j=1}^n Id_{ij} - A_i \right) \right] \\
&= \alpha \sum_{i=1}^k \max \left\{ 0, m_i \times \left[\sum_{j=1}^n Id_{ij} - S_i \right] \right\} \\
&\quad + \beta \sum_{i=1}^k \left[h_i \times \sum_{j=1}^n (P_{ij} + Id_{ij}) \right] \quad (\text{set } A_i = 0)
\end{aligned} \tag{3}$$

According to Eq. 3, parameters α , β , m_i , h_i , P_{ij} , and S_i are given, and the reduction of Z_{value} is obviously affected by the variable Id_{ij} . The difference in idle time is used to decide the dispatch rule adoption. Restated, the minimum idle time of orders in the buffer before the machine will reduce Z_{value} . However, the question remains of which algorithms can be effectively employed to reduce the idle time. The TOC theory mentions that system throughput is affected by its bottleneck, and the bottleneck can be regarded as a single machine within a complicated job shop. While reducing the idle time of orders remaining in the bottleneck and promoting the utility of a bottleneck, the factory increases profits and, correspondingly, reduces the Z_{value} in system. Furthermore, the ATC and WSPT dispatching rules can effectively minimize $\sum W_i T_i$ and $\sum W_i F_i$, respectively. From the two rules, some key elements that significantly influence Z_{value} are adopted and presented below (the Appendix shows the priority indices of those dispatching rules [10, 12]):

- $\log \left(d_i - \sum_{j=1}^n P_{ij} \right)$: This element, which affects the TDD index, represents the tightness of the due date. The element of slack time ($d_i - \sum P_{ij}$) is obtained via the ATC dispatching rule. Here, this element is formulated into the logarithmic scale.

- $h_i / \sum_{j=1}^n P_{ij}$: This element, which affects the IDD index, represents the diminishing cumulative cost per unit processing time and is determined by the WSPT dispatching rule.
- $\frac{(m_i - h_i)}{P_{iccr}}$: This element, which affects Z_{value} represents the effective throughput of the bottleneck. This element increases the speed of profit-making and promotes bottleneck efficiency according to the TOC theory.
- α : This element adjusts the ratio of TDD to IDD, and is assigned a non-negative value according to the factory type.
- β : This element adjusts the ratio of TDD to IDD, and is assigned a non-negative value according to the factory type.

Those elements above are considered to form a composite dispatching rule [11]. The priority index is developed in Eq. 4:

$$\begin{aligned}
P.I.(i) &= \frac{TH_i}{P_{iccr}} \left(\frac{h_i}{\sum_{j=1}^n P_{ij}} \right)^\beta / \left[\log \left(d_i - \sum_{j=1}^n P_{ij} \right) \right]^\alpha \\
&= \frac{(m_i - h_i)}{P_{iccr}} \left(\frac{h_i}{\sum_{j=1}^n P_{ij}} \right)^\beta / \left[\log \left(d_i - \sum_{j=1}^n P_{ij} \right) \right]^\alpha
\end{aligned} \tag{4}$$

A greater P.I. index of an order implies a higher priority to process the order. Furthermore, based on the given factors α and β for various plants, the rule can dynamically adjust order priorities for entrance into a bottleneck to reduce the ratio of TDD to IDD. The application of a composite P.I. index is called the mixed TDD/IDD dispatching rule.

If a system is a single-machine model, the machine is regarded as the bottleneck. In the job shop factory, the rule

Table 1 Simulation orders data

Receipt order	Process time (day)	Due day (day)	Sales*	Raw cost
#1	6	12	\$100	\$40
#2	10	20	\$150	\$60
#3	5	7	\$50	\$10
#4	8	18	\$450	\$150
#5	4	8	\$80	\$20
#6	14	17	\$500	\$120

*The sales entry is equal to the summary of each order price multiplied by its quantity

Table 2 Order priorities by SPT rule (#5–#3–#1–#4–#2–#6)

Task order	Total sales	Materials cost	Process time	Flow time	Due day	Order tardiness	Order lateness
#5	\$80	\$20	4	4	8	0	-4
#3	\$50	\$10	5	4+5=9	7	2	2
#1	\$100	\$40	6	9+6=15	12	3	3
#4	\$450	\$150	8	15+8=23	18	5	5
#2	\$150	\$60	10	23+10=33	20	13	13
#6	\$500	\$120	14	33+14=47	17	30	30
Σ	\$1,330	\$400	47	131		53	49

$$\sum \text{TDD} = \$50 * 2 + \$100 * 3 + \$450 * 5 + \$150 * 13 + \$500 * 30 = \$19,600$$

$$\sum \text{IDD} = 4 * \$20 + 9 * \$10 + 15 * \$40 + 23 * \$150 + 33 * \$60 + 47 * \$120 = \$11,840$$

$$Z_{\text{value}} = 0.5 * (\sum \text{TDD} + \sum \text{IDD}) = 0.5 * (19,600 + 11,840) = \$15,720 (\text{if } \alpha = \beta = 0.5)$$

must identify the capacity constraint resource (bottleneck machine) for determining bottleneck throughput. Generally, the machine with the highest breakdown rate, WIP levels, and lowest available capacity or dedicated machines are a potential bottleneck. If no bottleneck exists, it means that sufficient capacity is available for processing all orders. Here, the ΣTDD_i index is set to zero and, then, the orders are ranked according to their $(h_i / \Sigma P_{ij})^\beta$ value. A larger $(h_i / \Sigma P_{ij})^\beta$ value corresponds to a higher priority. Simultaneously, $Z_{\text{value}} = \beta * \Sigma \text{IDD}_i$ is revised.

3 Illustrative example

The single-machine model is applied to a practical example. The mixed TDD/IDD rule is compared with six traditional dispatching rules (shown in the Appendix) in terms of Z_{value} . The example sets $\alpha = \beta = 0.5$. Each machine can process six different orders ($6! = 720$ outcomes). Table 1 lists various data, such as sales value, process time, material cost, and due date. The determination of order priority and the performance evaluation of the SPT EDD and WSPT dispatching rules are shown in Tables 2, 3, and 4, respectively. Moreover, another three rules relating to minimum slack time (MST), ATC ($\theta = 5$), and T profit (maximum total profit) have their results merely listed. Table 5 lists the mixed TDD/IDD procedure. Finally, Table 6 summarizes all of the performance indices.

According to Table 6, despite the mixed TDD/IDD dispatching rule being poor in traditional performance (for example, \bar{F} , \bar{T} , and $\text{Max } T$), the rule performs well in terms of ΣIDD , ΣTDD , and Z_{value} . Analysis indicates that the heuristic rule can increase system profit and significantly reduce the order tardiness punishment and material holding cost. When the various α and β are given, the procedure modifies the order priority via the P.I. index to adjust the ratio of TDD to IDD. Table 7 lists the Z_{value} under various factors of α and β . Theoretically, another interesting issue about the mixed TDD/IDD rule is that, when the value of α is close to β , a trade-off relationship should exist between ΣTDD and ΣIDD . However, the result shown in Table 7 appears insignificant for the trade-off complementary relationship.

4 Conclusion

By integrating the time factor and dollar value into industrial scenarios, the proposed Z_{value} index achieves more accurate measurements than traditional methods (due date and cycle time). In various factories, traditional dispatching rules perform well in terms of due date and cycle time, but may incur a higher Z_{value} , which represents a high penalty in the global view. The theory of constraint (TOC) focuses on bottleneck scheduling to expedite the marginal contribution unit time of the system. The

Table 3 Order priorities by EDD rule (#3–#5–#1–#6–#4–#2)

Task order	Total sales	Materials cost	Process time	Flow time	Due day	Order tardiness	Order lateness
#3	\$50	\$10	5	5	7	0	-2
#5	\$80	\$20	4	5+4=9	8	1	1
#1	\$100	\$40	6	9+6=15	12	3	3
#6	\$500	\$120	14	15+14=29	17	12	12
#4	\$450	\$150	8	29+8=37	18	19	19
#2	\$150	\$60	10	37+10=47	20	27	27
Σ	\$1,330	\$400	47	142		62	60

$$\sum \text{TDD} = \$80 * 1 + \$100 * 3 + \$500 * 12 + \$450 * 19 + \$150 * 27 = \$18,980$$

$$\sum \text{IDD} = 5 * \$10 + 9 * \$20 + 15 * \$40 + 29 * \$120 + 37 * \$150 + 47 * \$60 = \$12,680$$

$$Z_{\text{value}} = 0.5 * (\sum \text{TDD} + \sum \text{IDD}) = 0.5 * (18,980 + 12,680) = \$15,830 (\text{if } \alpha = \beta = 0.5)$$

Table 4 Order priorities by WSPT rule (#4 #6 #1 #2 #5 #3)

Task order	Total sales	Materials cost	Process time	Flow time	Due day	Order tardiness	Order lateness	W_i/P_i (H_i/P_i)
#4	\$450	\$150	8	8	18	0	-10	150/8=18.8
#6	\$500	\$120	14	8+14=22	17	5	5	120/14=8.6
#1	\$100	\$40	6	22+6=28	12	16	16	40/6=6.7
#2	\$150	\$60	10	28+10=38	20	18	18	60/10=6
#5	\$80	\$20	4	38+4=42	8	34	34	20/4=5
#3	\$50	\$10	5	42+5=47	7	40	40	10/5=2
Σ	\$1,330	\$400	47	185		113	103	

$$\sum \text{TDD} = \$500 * 5 + \$100 * 16 + \$150 * 18 + \$80 * 34 + \$50 * 40 = \$11,520$$

$$\sum \text{IDD} = 8 * \$150 + 22 * \$120 + 28 * \$40 + 38 * \$60 + 42 * \$20 + 47 * \$10 = \$8,550$$

$$Z_{\text{value}} = 0.5 * (\sum \text{TDD} + \sum \text{IDD}) = 0.5 * (11,520 + 8,550) = \$10,035 \text{ (if } \alpha=\beta=0.5\text{)}$$

Table 5 Order priority by mixed TDD/IDD rule (#4 #6 #5 #1 #2 #3)

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)=(b)-(c)	(i)=(h)/(d)	(j)	(k)	(l)
Task order	Total sales	Materials cost	Process time	Flow time	Due day	Order tardiness	TH_i	TH_i/P_{iccr} (profit/ P_{iccr})	F_1^*	F_2^{**}	P.I. index
#4	\$450	\$150	8	8	18	0	\$300	37.5	1	18.75	162.4
#6	\$500	\$120	14	8+14=22	17	5	\$380	27.1	0.47	8.57	115.7
#5	\$80	\$20	4	22+4=26	8	18	\$60	15	0.6	5	43.2
#1	\$100	\$40	6	26+6=32	12	20	\$60	10	0.78	6.67	29.3
#2	\$150	\$60	10	32+10=42	20	22	\$90	9	1	6	22.1
#3	\$50	\$10	5	42+5=47	7	40	\$40	8	0.3	2	20.6
Σ	\$1330	\$400	47	177		105	\$930				

The F_1^* factor is represented by $\log(d_i - \sum P_{ij})$ and the F_2^{**} factor is represented by $h_i / \sum P_{ij}$

The P.I. index constitutes $[(TH_i/P_{iccr}) \times (F_2)^{0.5} \div (F_1)^{0.5}]$ (if $\alpha=\beta=0.5$)

$$\sum \text{TDD} = \$450 * 0 + \$500 * 5 + \$80 * 18 + \$100 * 20 + \$150 * 22 + \$50 * 40 = \$11,240$$

$$\sum \text{IDD} = 8 * \$150 + 22 * \$120 + 26 * \$20 + 32 * \$40 + 42 * \$60 + 47 * \$10 = \$8,630$$

$$Z_{\text{value}} = 0.5 * (\sum \text{TDD} + \sum \text{IDD}) = 0.5 * (11,240 + 8,630) = \$9,935$$

Table 6 Performance indexes for various rules

Performance index	SPT	EDD	WSPT	MS	ATC	T profit	Mixed TDD/IDD
Number of tardy	5	5	5	5	4	5	5
Mean flow time (\bar{F})	131/6	142/6	185/6	160/6	169/6	195/6	177/6
Mean tardiness (\bar{T})	53/6	62/6	113/6	80/6	97/6	116/6	105/6
Max tardiness ($Max T$)	30	27	40	27	40	40	40
Global performance							
ΣTDD	19,600	18,980	11,520	16,500	10,840	10,920	11,240
ΣIDD	11,840	12,680	8,550	12,320	9,770	9,730	8,630
Z_{value}	15,720	15,830	10,035	14,410	10,305	10,325	9,935

Table 7 Z_{value} under various α and β for seven dispatching rules

Z_{value}	SPT	EDD	WSPT	Min ST	ATC	T profit	Mixed TDD/IDD	Factories
$\alpha=1.0,$ $\beta=0.0$	19,600	18,980	11,520	16,500	<u>10,840</u>	10,920	11,190 $\Sigma\text{TDD}=11,190, \Sigma\text{IDD}=10,170$ #6_#4_#3_#5_#1_#2	MTO factory ($\alpha>\beta$)
$\alpha=0.9,$ $\beta=0.1$	18,824	18,350	11,223	16,082	<u>10,733</u>	10,801	10,902 $\Sigma\text{TDD}=10,990, \Sigma\text{IDD}=10,110$ #6_#4_#5_#3_#1_#2	
$\alpha=0.8,$ $\beta=0.2$	18,048	17,720	10,926	15,664	<u>10,626</u>	10,682	10,814 $\Sigma\text{TDD}=10,990, \Sigma\text{IDD}=10,110$ #6_#4_#5_#3_#1_#2	
$\alpha=0.7,$ $\beta=0.3$	17,272	17,090	10,629	15,246	<u>10,519</u>	10,563	10,874 $\Sigma\text{TDD}=11,690, \Sigma\text{IDD}=8,970$ #4_#6_#5_#3_#1_#2	
$\alpha=0.6,$ $\beta=0.4$	16,496	16,460	<u>10,332</u>	14,828	10,412	10,444	10,426 $\Sigma\text{TDD}=11,490, \Sigma\text{IDD}=8,830$ #4_#6_#5_#1_#3_#2	MTA factory (α is close to β)
$\alpha=0.5,$ $\beta=0.5$	15,720	15,830	10,035	14,410	10,305	10,325	9,935 $\Sigma\text{TDD}=11,240, \Sigma\text{IDD}=8,630$ #4_#6_#5_#1_#2_#3	
$\alpha=0.4,$ $\beta=0.6$	14,944	15,200	9,738	13,992	10,198	10,206	9,674 $\Sigma\Sigma\text{TDD}=11,240, \Sigma\text{IDD}=8,630$ #4_#6_#5_#1_#2_#3	
$\alpha=0.3,$ $\beta=0.7$	14,168	14,570	9,441	13,574	10,091	10,087	9,413 $\Sigma\text{TDD}=11,240, \Sigma\text{IDD}=8,630$ #4_#6_#5_#1_#2_#3	MTS factory ($\alpha<\beta$)
$\alpha=0.2,$ $\beta=0.8$	13,392	13,940	<u>9,144</u>	13,156	9,984	9,968	9,152 $\Sigma\text{TDD}=11,240, \Sigma\text{IDD}=8,630$ #4_#6_#5_#1_#2_#3	
$\alpha=0.1,$ $\beta=0.9$	12,616	13,310	<u>8,847</u>	12,738	9,877	9,849	8,891 $\Sigma\text{TDD}=11,240, \Sigma\text{IDD}=8,630$ #4_#6_#5_#1_#2_#3	
$\alpha=0.0,$ $\beta=1.0$	11,840	12,680	<u>8,550</u>	12,320	9,770	9,730	8,630 $\Sigma\text{TDD}=11,240, \Sigma\text{IDD}=8,630$ #4_#6_#5_#1_#2_#3	

Order priority of SPT is #5_#3_#1_#4_#2_#6 and $\Sigma\text{TDD}=19,600, \Sigma\text{IDD}=11,840$
 Order priority of EDD is #3_#5_#1_#6_#4_#2 and $\Sigma\text{TDD}=18,980, \Sigma\text{IDD}=12,680$
 Order priority of WSPT is #4_#6_#1_#2_#5_#3 and $\Sigma\text{TDD}=11,520, \Sigma\text{IDD}=8,550$
 Order priority of Min ST is #3_#6_#5_#1_#4_#2 and $\Sigma\text{TDD}=16,500, \Sigma\text{IDD}=12,320$
 Order priority of ATC is #6_#4_#2_#5_#1_#3 and $\Sigma\text{TDD}=10,840, \Sigma\text{IDD}=9,770$
 Order priority of T profit is #6_#4_#2_#1_#5_#3 and $\Sigma\text{TDD}=10,920, \Sigma\text{IDD}=9,730$
 The underlined values represent the minimum number in each row

analytical results here demonstrate that the mixed TDD/IDD dispatching rule is feasible and outperforms other dispatching rules in terms of Z_{value} index for various factories.

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1 Appendix

1.1 Priority index for various dispatching rules

Rule	Definition	Priority index
SPT	Shortest processing time	$\min (P_i)$
EDD	Earliest due date	$\min (d_i)$
WSPT	Weighted shortest processing time	$\max (W_i/P_i)$ or $\min (P_i/W_i)$

Rule	Definition	Priority index
ATC	Apparent tardiness cost	$\max \left[\frac{W_i}{P_i} \exp \left(-\frac{\max (0, d_i - t - P_i)}{\theta \bar{P}} \right) \right]$ θ is a scaling parameter and \bar{P} is the average of the processing times of the remaining orders
MS	Minimum slack time	$\min (d_i - \Sigma P_{ij})$
T-profit	Maximum total profit	$\max (m_i - h_i)$
Mixed TDD/IDD	Bottleneck-based	$\max \left\{ \frac{(m_i - h_i)}{P_{\text{cct}}} \left(\frac{h_i}{\sum_{j=1}^n P_{ij}} \right)^\beta / \left[\log \left(d_i - \sum_{j=1}^n P_{ij} \right) \right]^\alpha \right\}$

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