行政院國家科學委員會專題研究計畫 成果報告

喪失銷貨存貨系統經驗訂購政策之研究
研究成果報告(精簡版)

計畫類別：個別型
計畫編號：NSC 100-2410-H-009-007-
執行期間：100年08月01日至101年07月31日
執行單位：國立交通大學管理科學系（所）

計畫主持人：姜齊

計畫參與人員：博士班研究生-兼任助理人員：許惠蘭

公開資訊：本計畫可公開查詢

中華民國 101年07月02日

中文摘要： 大多數存貨系統假設需求若無法立即滿足可後補。實務上需求若無法立即滿足常喪失。與後補存貨模式之研究相比，喪失銷貨存貨模式之文獻較少。本計畫探討前置時間為正數但短於盤存周期之喪失銷貨存貨系統。2006 年姜齊已針對此存貨系統研擬出最適訂購政策，但使用需回復計算的動態規劃方法。如Silver 於2008 年指出，存貨管理理論與實務存在明顯差異，實務上很少人使用難於執行的方法，反之他們使用經驗或簡而易懂的政策。本計畫針對上述存貨系統提出兩個經驗訂購政策: 一為訂購後水準政策，另一為修正固定訂單政策。訂購後水準政策為後補存貨模式之最佳政策，而固定訂單政策近來為一些學者所研究。本計畫研擬一個設定訂購後水準的方法，並比較上述兩個政策與最佳訂購政策的績效。研究結果顯示所提出的兩個經驗政策為不錯之訂購政策，特別是修正固定訂單政策在大多數情況下表現優於訂購後水準政策。實務界人士因此至少有一個簡單易執行的訂購政策可使用。

中文關鍵詞： 喪失銷貨、存貨模式、定期盤存、經驗政策、動態規劃

英文摘要： Most inventory systems assume that demand not filled immediately is backlogged. In practice, demand not filled at once is often lost. The literature on lost-sales inventory models is scarce, compared to the studies on backlogged inventory models. In this research, we consider lost-sales periodic review inventory systems where the lead-time is positive but shorter than the period length (for example, the lead-time is two or three days while the review period is one week). Chiang [European Journal of Operational Research, 170 (2006) 44-56] actually has developed optimal ordering policies for such systems. However, he used the dynamic programming approach which entails recursive computation. As noted by Silver [Infor, 46 (2008) 15-27], there is a significant gap between theory and practice in inventory management. Practitioners usually do not use difficult-to-program (though optimal) methods； instead, they prefer heuristic (though not optimal) ordering policies. In this research, we propose two heuristic ordering policies for the lost-sales periodic review systems: one is an order-up-to policy and the other is a revised constant-order policy. The order-up-to policy is known to be optimal for the backlogged inventory models, while the constant-order policy was recently examined by various scholars. In this research, we devise a method to set the order-up-to level for the lost-sales models. We also compare the performance of these two heuristics with the optimal ordering policy proposed by Chiang. We show that these two heuristics perform well under a wide range of input parameters. In particular, the revised constant-order policy performs better than the order-up-to policy under most parameter settings. Thus, practitioners has at least a simple ordering policy to use.

英文關鍵詞： Lost-sales, Inventory model, Periodic review, Heuristic policy, Dynamic programming

喪失銷貨存貨系統經驗訂購政策之研究

國立交通大學管理科學系姜齊

I. Introduction

Periodic-review inventory systems are commonly found in practice (e.g., Prasad et al., 2005; Silver et al., 1998), especially if many different items are purchased from the same supplier and the coordination of ordering and transportation is important. In a recent survey (Simchi-Levi et al., 2003), material managers indicate the effectiveness of periodic-review systems for reducing inventory levels in a supply chain.

Although most studies on periodic-review inventory models have (implicitly) assumed that the review periods are as small as one day (see, e.g., Porteus, 1990 and references therein), periodic-review systems in practice often have the review periods (i.e., *replenishment cycles* or simply *cycles*) that are a few days or weeks long and regular orders are placed at a review epoch (see, e.g., Chiang, 2003 and Chiang and Gutierrez, 1998 for periodic systems where an emergency order can be placed at a review epoch or virtually at any time between two review epochs). For such periodic-review systems, it is appropriate to compute holding and shortage costs based on respectively, the average inventory of a replenishment cycle and the duration of shortage (assuming that demand not immediately filled is backlogged). Due to the difficulties involved in exact analysis, the approximate treatment of such systems is often used in textbooks (e.g., Hadley and Whitin, 1963 and Silver et al., 1998) to obtain simple solutions. Chiang (2006) thus proposed a dynamic programming model in which a cycle consists of a number of small periods and holding and shortage costs will be computed based on the ending inventory of small periods. However, as the dynamic programming approach entails recursive computation, it may not be easily implemented in practice. As noted by Silver (2008), there is a significant gap between theory and practice in inventory management. Practitioners usually do not use difficult-to-program, though optimal methods; instead, they prefer heuristic, though suboptimal ordering policies.

In this research, we extend the work of Chiang and propose two heuristic policies for the lost-sales periodic review system with positive fractional lead time (i.e., lead time is smaller than the period length): one is an order-up-to policy and the other is a revised standing-order (or constant-order) policy. We assume the zero cost of ordering as in Chiang. The order-up-to policy is known to be optimal for the backlogged inventory models, while the standing-order policy was recently examined by various scholars (e.g., Chiang, 2007 and Bolton and Katok, 2008). We advocate that firms use the proposed heuristics for obtaining easy-to-implement ordering policies.

II. Lost-sales Periodic Review Models

We suppose that a replenishment cycle, whose length is exogenously determined, consists of m periods, each of identical but arbitrary length. Let ξ be a generic demand variable and in particular, let ζ denote the demand of a cycle. Also, let $\varphi^k(\cdot)$ be the probability density function of k-period's demand (the superscript may be omitted for brevity if k = 1). Assume first that all demand not immediately satisfied is lost. Demand is assumed to be non-negative and independently distributed in disjoint time intervals. In addition, the following notation is used.

λ = mean arrival rate.

τ = the (deterministic) supply lead-time, which is an integral multiple of a period.

c = the unit procurement cost.

h = the inventory cost per unit held per period.
π = the shortage cost per unit.
L = the holding and shortage costs of a replenishment cycle.
α = the one-period discount factor (i.e., discounting the cost incurred in one period from now to the present time), $0 < \alpha \le 1$.
X = the starting inventory on hand at a review epoch.

Assume that $\tau \le m$ (i.e., at most one order is outstanding at any time). Let $V_{n,0}(X, 0) \equiv V_n(X)$, and $V_{n,j}(X, Y)$, for $j \neq 0$, denote the expected discounted cost with n cycles and j periods remaining when the starting on-hand and on-order inventory are X and Y, respectively. $V_{n,j}(X, Y)$ is simply $V_{n,j}(X, 0)$ for $j = 1, \ldots, m - \tau$. $V_{n,j}(X, Y)$ satisfies the functional equations (Chiang, 2006)

$$V_{n,0}(X, 0)$$

$$= \min_{Z \ge 0}\{\alpha^{\tau} cZ + L(X) + \alpha\int_0^X V_{n-1,m-1}(X - \xi, Z)\varphi(\xi)d\xi + \alpha V_{n-1,m-1}(0, Z)\int_X^\infty \varphi(\xi)d\xi\} \quad (1)$$

$$V_{n,j}(X, Y) = L(X) + \alpha\int_0^X V_{n,j-1}(X - \xi, Y)\varphi(\xi)d\xi + \alpha V_{n,j-1}(0, Y)\int_X^\infty \varphi(\xi)d\xi$$

$$j = 1, \ldots, m - 1, \text{ and } j \neq m - \tau + 1 \quad (2)$$

$$V_{n,m-\tau+1}(X, Y) = L(X) + \alpha\int_0^X V_{n,m-\tau}(X - \xi + Y, 0)\varphi(\xi)d\xi + \alpha V_{n,m-\tau}(Y, 0)\int_X^\infty \varphi(\xi)d\xi, \quad (3)$$

where $V_{0,0}(X, 0) \equiv 0$, $L(X)$ is the one-period holding and shortage costs given by

$$L(X) = \int_0^X h(X - \xi)\varphi(\xi)d\xi + \int_X^\infty \pi(\xi - X)\varphi(\xi)d\xi \quad (4)$$

and Z (the decision variable) is the quantity ordered at a review epoch which becomes inventory on order thereafter. If $\tau = 1$, there is only one state variable X and (1)-(3) reduce to

$$V_{n,0}(X) = \min_{Z \ge 0}\{\alpha cZ + L(X) + \alpha\int_0^X V_{n-1,m-1}(X - \xi + Z)\varphi(\xi)d\xi + \alpha V_{n-1,m-1}(Z)\int_X^\infty \varphi(\xi)d\xi\} \quad (5)$$

$$V_{n,j}(X) = L(X) + \alpha\int_0^X V_{n,j-1}(X - \xi)\varphi(\xi)d\xi + \alpha V_{n,j-1}(0)\int_X^\infty \varphi(\xi)d\xi, \qquad j = 1, \ldots, m - 1 \quad (6)$$

Chiang showed that $V_{n,j}(X, Y)$ is convex. Let $Z_n(X)$ be the value of non-negative Z obtained in (1) or (5) for a given X. Then the optimal policy at a review epoch with n cycles remaining is to order the amount $Z_n(X)$. Also, let R_n be the maximum possible order-up-to level at a review epoch with n cycles remaining. We are unable to develop any properties regarding $V_{n,j}(X, Y)$ or $Z_n(X)$ that can be used in the dynamic programming computation. Denote by $Z^*(X)$ and R^* the infinite-horizon optimal operational parameters obtained, i.e., to which $Z_n(X)$ and R_n respectively converge. Chiang implemented the above model and obtained optimal policies. For example, if c = \$10, m = 10, τ = 6 days, α = 0.999, h = \$0.01 (i.e., the holding cost is charged at \$0.01 per unit per day), p = \$20 (i.e., the shortage cost is charged at \$20 per unit), and μ = 2 units per day (with Poisson demand). It was found that $Z^*(X) = 29$ for $X \le 12$, $Z^*(13) = Z^*(14) = 28$, $Z^*(15) = Z^*(16) = 27$, $Z^*(17) = 26$, $Z^*(18) = 25$, $Z^*(X) = 44 - X$ for $19 \le X \le 44$ and thus $R^* = 44$.

III. Main Results

We propose two heuristic ordering policies for the studied lost-sales periodic review system with positive fractional lead time: one is an order-up-to policy and the other is a revised standing-order policy. The order-up-to policy is optimal for the backlogged inventory models (see, e.g., Porteus, 1990); however, its performance on the lost-sales models has been varied. Zipkin (2008) showed that for periodic review systems where lead-time is an integer multiple of the period length, it performed well on some parameter settings while it did not on other settings. How to set the order-up-to level seems to be the key for its performance. In this research, we suggest to set the order-up-to level at R^* obtained above for the studied periodic review systems. Taking the above example, it was found by using the suggested order-up-to policy that the total cost is increased by only 0.5%. In fact, we have designed an experiment that varied the data of the above example. It was found that the average increase in total cost is only about 0.6%.

The second heuristic policy we propose is a revised standing-order policy. The standing-order policy was recently examined by various scholars (e.g., Chiang, 2007, Bolton and Katok, 2008, and Zipkin, 2008). It seemed to perform satisfactorily for periodic review systems where lead-time is an integer multiple of the period length (Zipkin). In this research, we suggest that one uses a revised standing-order policy for the studied periodic systems with fractional lead time. To be more specific, we propose that the order quantity to be placed every cycle is the minimum of the standing order and R^* minus the on-hand inventory at review, where the size of standing orders is set to be the average cycle demand. Taking the above example, the size of standing orders will be $m\mu = 20$; thus if the on-hand inventory at review is 19, then the quantity placed will be min(20, 44 – 19) = 20. We also designed an experiment as above. It was found by using this heuristic that the average increase in total cost is only about 0.4%.

IV. Conclusion

In this research, we consider the lost-sales periodic review systems with positive fractional lead time (i.e., lead time is smaller than the period length). Previous research used dynamic programming to obtain optimal ordering policies that may not be easily implemented. On the contrast, this research devises two simple heuristic policies: an order-up-to policy and a revised standing-order policy. It was found that the average increase in total cost is less than one percent for both heuristics. In particular, the revised standing-order policy performed better than the order-up-to policy. We hope that the proposed heuristics can be applied immediately in practice.

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國科會補助計畫衍生研發成果推廣資料表

日期:2011/12/19

國科會補助計畫	計畫名稱: 喪失銷貨存貨系統經驗訂購政策之研究
	計畫主持人: 姜齊
	計畫編號: 100-2410-H-009-007- 學門領域: 生產及作業管理

無研發成果推廣資料

100 年度專題研究計畫研究成果彙整表

計畫主持人：姜齊	**計畫編號：**100-2410-H-009-007-
計畫名稱：喪失銷貨存貨系統經驗訂購政策之研究	

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	1	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

🟥	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以100字為限）

　　□實驗失敗

　　□因故實驗中斷

　　□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：□已發表 ■未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以100字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以500字為限）

本計畫延伸姜齊94年度國科會計畫之研究(彈性期間定期盤存制最適採購政策之研究)，94年度研究使用動態規劃導出定期盤存制最佳採購政策，本計畫則使用經驗訂購政策。本計畫假設採購前置時間小於盤存周期，且針對喪失銷貨存貨系統進行研究。本計畫建議兩個經驗訂購政策。研究結果顯示使用經驗訂購政策導致成本的增加平均不超過0.5%(相較最佳採購政策)。本計畫所發展的訂購政策易於為業界所應用。